**1. Phase Response**

Sketch the phase (in radians) vs. log $\omega$ for the filter specified below with $\omega_1 = 10^3$ rad/s and $\omega_2 = 10^5$ rad/s.

$$H(\omega) = \frac{-5}{1 + j\omega/\omega_1} \frac{1}{1 + j\omega/\omega_2}$$

**Hint:** You may want to figure out the phase responses of each component of $H(\omega)$ individually and then combine them together.

**Solutions:**

$$\angle H = \angle \frac{-5}{1 + \frac{\omega}{\omega_1}} + \angle \frac{1}{1 + \frac{\omega}{\omega_2}}$$

$$= \pi - \arctan\left(\frac{\omega}{\omega_1}\right) - \arctan\left(\frac{\omega}{\omega_2}\right)$$

As $\omega \to 0$, $\angle H \to \pi$

As $\omega \to \infty$, $\angle H \to 0$

At $\omega = \omega_1$, $\angle H \approx \frac{3\pi}{4}$

At $\omega = \omega_2$, $\angle H \approx \frac{\pi}{4}$

2. **Filter Design**

Consider a system input that has a signal at 100Hz and unwanted noise at 10kHz. For the filter described below, find the value of $\omega_c$ that attenuates the unwanted noise by a factor of 20. What is the gain of the desired signal from this filter?
3. Dynamic Voltage and Frequency Scaling

A low-power microcontroller operates in three modes. In high-performance mode, the processor operates at a supply voltage of $V_{DD} = 1.2V$ and a frequency of 1MHz and consumes 100µW of power. Assume the resistance of the gates in the microcontroller is proportional to $1/(V_{DD} - 0.4V)$.

(a) In low-power mode, the processor operates at $V_{DD} = 0.48V$. What is the highest possible operating frequency in this mode? How much power is consumed?

Solutions:

- $f_2/f_1 = V_{ddl} - 0.4/V_{ddl} - 0.4 = 0.8/0.08 = 10$, so $f_2 = 10f_1 = 100kHz$.

- $P_2 = P_1 \times \frac{f_2}{f_1} = 100µW \times \frac{10}{1} = 1000µW 
  = \frac{1000}{162.5} \approx 6 µW

(b) In sleep mode, the processor operates at a frequency of just 1kHz. What is the lowest possible operating voltage that would allow operation at this frequency? How much power is consumed?

Solutions:

- $f_3/f_1 = V_{ddl} - 0.4/V_{ddl} - 0.4$

$$
V_{ddl} = 0.4 + 8 \times 10^{-4} = 0.4008V
$$

- $P_3 = \frac{1}{1000} \times \left( \frac{V_{ddl}}{V_{ddl}} \right)^2$

$$
V_{ddl} = 0.4008V
V_{ddl} = 1.2V
P_3 = \frac{0.4008^2}{1.2^2} \approx \frac{1}{3}
\approx \frac{1}{3} \times \frac{1}{1000} \approx \frac{1}{3000}
\approx 0.333 µW
\approx 0.333 \times 2 = 0.367 µW
P_3 = \frac{1}{9000} \times 100µW = \frac{1}{90}µW = 11.1nW
$$