SVD

The all imp. SVD eqn

\[ M = U \Sigma V^T \]

Where

- \( M \) is a matrix that represents something imp & complex to you
- \( V \) are orthogonal vectors which are eigenvectors of \( M \) \( (MV_i = \lambda V_i) \)
- In other words, \( V \) is a current domain
- \( U \) are orthogonal vectors of a co-domain
- \( \Sigma \) are weights that describe \( M \) in the co-domain

For example, if I am given some \( M \) transform

\[ M = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \]

Then my \( x + y \) becomes

\[ \begin{array}{c}
\text{0.5} \\
\text{1.0} \\
\end{array} \]

\[ \text{4.0} \]

\[ \text{5.0} \]

\[ \text{6.0} \]

\[ \text{7.0} \]

\[ \text{8.0} \]
I can describe this transformation in parts, first break $M$ into eigenvalues so that I can describe the transform in an orthogonal domain.

Next, turn those eigen vectors into unit vectors & weights:

$$M_{ij} = U_{ij} \sigma_i$$

where $U$ is the new co-domain of $u_i$ vectors & $\sigma_i$ are the weights that translate $M_{ij} = U_{ij} \sigma_i$.

Thus, we can kind of intuit what it means to break $M$ into 3 parts:

$$M = U \Sigma V^T$$
Now, why is this useful? What are the applications of this?

How did we use it in lab?

Noise reduction

- We can look at the weight of each $\sigma_i$ vector and determine what $\sigma_i$ are very imp in $M$ & which don't change very much, then we keep some number of the largest $\sigma_i$.

Example:

![Picture](image)

Imagine we describe this picture as a 1 by 1 block box & 0 for while

We can see very easily that there are only 3 vectors in this picture

When we break $M$ into parts we will get back only $\sigma_1, \sigma_2, \sigma_3$ for each of the three independent vectors, & 3 $\nu_i$ vectors.
SVD has given us a way to transform into the three most important vectors.

This is what you did in lab, but with some noise mixed in, you get back a lot of vectors, but by only taking the 20 largest $d_i$, you picked out the more "important" vectors that could describe the neuron data.

What are other applications?

Netflix recommendations
Image compression
Machine learning

Let's look at the SVD problem in your HW.

Another application of SVD is to rewrite a transform into matrices with nicer properties.

For example, you are given

$$X \frac{Y}{Z} \rightarrow \frac{Y}{Z}$$

What is a formula (w/o SVD) to describe this?
\[ y = H x \]

We can break \( H \) into SVD, what is it? 

\[ H = U \Sigma V^T \]

\[ y = U \Sigma V^T x \]

Orthogonal matrix, \( U \), \( V \) diagonal.

Now we have an equation in terms of only unitary + a diagonal matrix.