Solutions: Provided by John Noonan and Quincy Huynh.

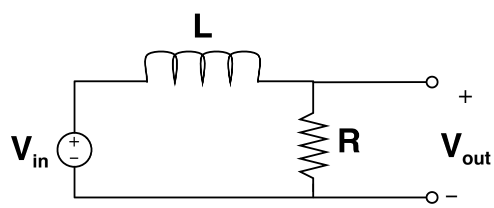
1. Proof of Induction

Given the voltage-current relationship of an inductor $V = L\frac{di}{dt}$, show that its complex impedance is $Z_L = j\omega L$.

Solutions: Since $V(t)$ is an eigenfunction and impedance is a constant, then $i(t)$ is also an eigenfunction such that $\frac{di}{dt} = j\omega \cdot i(t)$.

$$V = L\frac{di}{dt}$$
$$V = Lj\omega \cdot i(t)$$
$$V = (j\omega L)i(t)$$
$$Z_L = j\omega L$$

2. L-R Filter

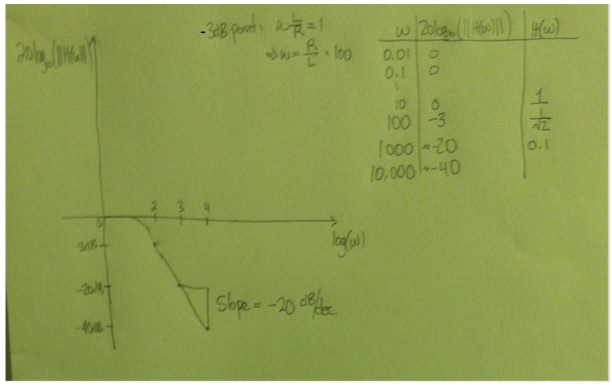


(a) Write the frequency response function $H(\omega) = \frac{V_{out}}{V_{in}}$ for the circuit.

Solutions: $H(\omega) = \frac{V_{out}}{V_{in}} = \frac{Z_R}{Z_L+Z_R} = \frac{R}{j\omega L+R} = \frac{1}{j\omega\frac{L}{R}+1}$

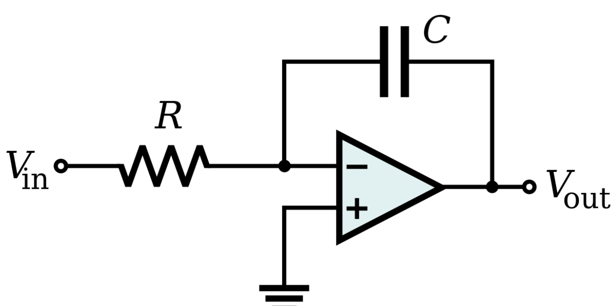
(b) If $R = 10\Omega$ and $L = 100\text{mH}$, plot the log-magnitude of $H(\omega)$ and label important magnitudes and frequencies.

Solutions: $\omega_c = \frac{R}{L} = 10/0.100 = 100\text{rad/s}$



(c) If $V_{in}(t) = \cos(10t) + \cos(100t) + \cos(1000t)$, what is $V_{out}(t)$? (Assume that $H(-\omega) = H(\omega)$.)
Caveat: We have not yet discussed how the *phase* of V_{in} is affected by the circuit. Ignore this for now; assume the phase of the output is the same as the phase of the input.
Solutions: $|H(\omega)| = \frac{1}{\sqrt{\omega^2 \frac{L^2}{R^2} + 1}} = \frac{1}{\sqrt{\frac{\omega^2}{10000} + 1}}$
$|H(10)| \approx 1$
$|H(100)| = \frac{1}{\sqrt{2}}$
$|H(1000)| = \frac{1}{10}$
$V_{out} = 1 \cdot \cos 10t + \frac{1}{\sqrt{2}} \cdot \cos(100t) + \frac{1}{10} \cdot \cos(1000t)$

3. Op-Amps: What *Can't* They Do?



(a) Write the frequency response function $H(\omega) = \frac{V_{out}}{V_{in}}$ for the circuit.
Solutions: By Golden Rules:

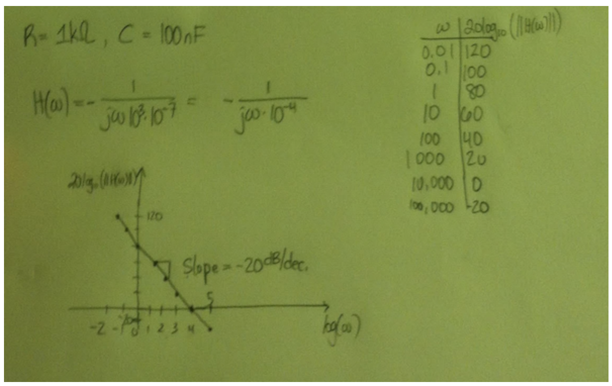
$$V_n = V_p$$
$$i_n = 0$$
$$i_p = 0$$

$V_n = 0$ by the first golden rule, and using the second rule with KCL, we find that:

$$\frac{V_{in}}{Z_R} = -\frac{V_{out}}{Z_C}$$
$$\frac{V_{out}}{V_{in}} = -\frac{Z_c}{Z_R}$$
$$\frac{V_{out}}{V_{in}} = -\frac{1}{j\omega RC}$$
$$H(\omega) = -\frac{1}{j\omega RC}$$

(b) If $R = 1\text{k}\Omega$ and $C = 100\text{nF}$, plot the log-magnitude of $H(\omega)$ and label important magnitudes and frequencies. (Again assume that $H(-\omega) = H(\omega)$.)

Solutions:



(c) **Challenge:** What does this circuit do?

Hint: You will probably need to set up the differential equation relating V_{out} and V_{in} to work this out.

Solutions:

$$\frac{V_in}{R} = -C\frac{dV_out}{dt}$$
$$-\frac{1}{RC}\int V_{in}dt = V_{out}$$

The circuit takes in V_{in} and integrates it.