Frequency-Domain Analysis
Using the
Discrete Fourier Transform (DFT)

EECS 16B, Fall 2015
Time Domain

• Time-Domain Picture Doesn’t Always Tell Us

“Who Done It!”
IN VIVO MEASUREMENTS (III)

Prior to sedation

Voltage (µV)

Anesthesia/Sleep Relaxation Activity/Sensorimotor

freq 1 4 7 15 30 65 250

δ θ α β γ_{LOW} γ_{HIGH}

Sources: Muller, et al. IEEE ISSCC ’14, Muller, et al. IEEE JSSC ‘15
IN VIVO MEASUREMENTS (III)

Prior to sedation

15-minutes post-sedation

Anesthesia/Sleep  Relaxation  Activity/Sensorimotor

freq  1  4  7  15  30  65  250

Sources: Muller, et al. IEEE ISSCC ’14, Muller, et al. IEEE JSSC ‘15
IN VIVO MEASUREMENTS (III)

Prior to sedation

15-minutes post-sedation

ECoG Spectral Bands

Sources: Muller, et al. IEEE ISSCC '14, Muller, et al. IEEE JSSC '15
Prof. Emery Brown
MIT / MGH

Elizabeth Dougherty, MIT Spectrum.
Charting the unconscious: This spectrogram shows EEG recordings from a patient undergoing general anesthesia. Two doses of the intravenous anesthetic propofol lead to bursts of activity (minute seven). Then an inhaled anesthetic, isoflurane, is added, and at minute 14, a characteristic pattern of slow-wave and alpha oscillations begins. Surgery ends at minute 16, and the isoflurane is switched off. The EEG gradually shifts to high-frequency, less intense oscillations. 

EEG Probes on the Head
Prof Emery Brown’s Group (MIT / MGH)

Electrode Locations

Tracking brain states under general anesthesia by using global coherence analysis, A. Cimenser et al., PNAS 2011 108 (21) 8832-8837
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Other Uses of Frequency-Domain Analysis

• Depth of Anesthesia
• Classifying Sleep Stages
• Predicting Epileptic Seizures
• Detection and Monitoring of Brain or Spinal Injury
• Abnormal Brain States

Quantitative EEG Analysis Methods and Clinical Applications, Tong & Thanko (Eds), Artech House, 2009.
The short-time Fourier transform (STFT) is defined as the discrete-time Fourier transform evaluated over a sliding window. The STFT can be performed as

\[ X(n, \omega) = \sum_{\tau=-\infty}^{\infty} x(\tau) w(n - \tau)e^{-j\omega \tau} \]  

(2.60)

where the discrete-time index \( n \) refers to the position of the window \( w(n) \). Analogous with the periodogram, a spectrogram is defined as

\[ S_x(n, \omega) = |X(n, \omega)|^2 \]  

(2.61)

Based on the uncertainty principle, i.e. \( \sigma_t^2 \sigma_\omega^2 \geq \frac{1}{4} \), where \( \sigma_t^2 \) and \( \sigma_\omega^2 \) are respectively the time- and frequency-domain variances, perfect resolution cannot be achieved in both time and frequency domains. Windows are typically chosen to eliminate discontinuities at block boundaries.
edges and to retain positivity in the power spectrum estimate. The choice also impacts upon the spectral resolution of the resulting technique, which, put simply, corresponds to the minimum frequency separation required to resolve two equal amplitude frequency components [49].

Figure 2.12 shows the TF representation of an EEG segment during the evolution from preictal to ictal and to postictal stages. In this figure the effect of time resolution has been illustrated using a Hanning window of different durations of 1 and 2 seconds. Importantly, in this figure the drift in frequency during the ictal period is observed clearly.

**Figure 2.12** TF representation of an epileptic waveform (a) for different time resolutions using a Hanning window of (b) 1 ms and (c) 2 ms duration