10/30/15  Lecture 15: Eldad

(We will return to Anant’s lecture on Thursday.)

Recall: From sampling theory, we know that a signal must be perfectly contained from $-f_s/2$ to $f_s/2$ to be perfectly reconstructed.

How to implement this in the real world?

\[ \text{Vin} \xrightarrow{\text{Anti-alias}} \text{DAC} \xrightarrow{\text{DSP}} \text{DAC} \xrightarrow{\text{Low-pass filter}} \text{Out} \]

- This filter ensures that our assumption is true prior to sampling.

\[ |H(e^{j\omega})| \xrightarrow{\omega} h(t) \xrightarrow{\text{filter}} (\sin(\pi t)) / t \]

- But how to construct that sinc function? It has nonzero values at $t = -\infty$, so it cannot possibly be applied far enough before $t = 0$—it’s non-causal, and can’t be fixed with a timeshift.

- A conversation/example around oversampling follows.

- How to implement the filter then? Return to our beloved RC circuit:

\[ V_{\text{Out}} = V_A e^{j\omega t} \]

- \[ V_{\text{Out}} \]
Recall: \( I = \frac{V_{in} - V_{out}}{R} = C \frac{dV_{out}}{dt} \Rightarrow V_{in} = V_{out} + RC \frac{dV_{out}}{dt} \)

- Since sinusoids are eigenfunctions of LTI systems:
  \[
  V_{in} e^{j\omega t} = H(\omega) \cdot V_{out} e^{j\omega t} + RC \cdot H(\omega) \cdot j\omega e^{j\omega t}
  \]
  \[
  \Rightarrow I = H(\omega) + j\omega H(\omega) RC \Rightarrow H(\omega) = \frac{1}{1 + j\omega RC}
  \]

- This is the response of a first-order low-pass filter

\[ \text{dB} = \text{log}_{10}(\omega) \]

For \( \omega \) "large", \( H(\omega) \approx \frac{1}{j\omega RC} \)

where \( \text{large} \) means \( \omega >> \frac{1}{RC} \)

\[ H(\frac{1}{RC}) = \frac{1}{1+j} \quad |H(\frac{1}{RC})| = \frac{1}{\sqrt{1+\frac{1}{RC}}} = 3 \text{dB} \]

- So if we place \( w_0 \) at \( \omega << \frac{1}{RC} \), we do not perfectly cut off the higher frequencies!

- Depending on where we place \( w_0 \), we either remove signal that we do want, or keep signal that we don't want.

Reminder:

\[ A_v \rightarrow 20 \log_{10}(A_v) \text{ dB} \]

So:

\[
\begin{array}{c|c}
A_v & \text{dB} \\
10V/V & 20 \text{ dB} \\
100V/V & 40 \text{ dB} \\
0.1V/V & 80 \text{ dB} \\
\frac{1}{3}V/V & 3 \text{ dB} \\
1V/V & 0 \text{ dB}
\end{array}
\]

- Finally, turn to lab:

\[ \text{Ideal:} \frac{V_{out}}{V_{in}} \quad \text{Real:} \frac{V_{out}}{V_{in}} \]
The real amplifier may have some offset, which, after gain, will roll out.
To remove this DC offset, we need a high-pass filter:

\[ I = \frac{V_{\text{out}}}{R} \quad I = C \frac{d}{dt} (V_{\text{in}} - V_{\text{out}}) \]

\[ V_{\text{out}} = \frac{1}{R} \int V_{\text{in}} \, dt - \frac{C}{R} \frac{dV_{\text{in}}}{dt} - C \frac{dV_{\text{out}}}{dt} \]

\[ V_{\text{out}} + RC \frac{dV_{\text{out}}}{dt} = RC \frac{dV_{\text{in}}}{dt} \]

\[ HV_{\text{in}} + RCjw \quad HV_{\text{in}} = RCjwV_{\text{in}} \]

\[ H(w) + jwRC \cdot H(w) = RCjw \]

**HPF:**

\[ H(w) = \frac{jwRC}{1+jwRC} \]