More AC Analysis

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Impedance and Admittance

Impedance is voltage/current

\[ Z = R + jX \]

- \( R \) = resistance = Re(Z)
- \( X \) = reactance = Im(Z)

Admittance is current/voltage

\[ Y = \frac{1}{Z} = G + jB \]

- \( G \) = conductance = Re(Y)
- \( B \) = susceptance = Im(Y)

<table>
<thead>
<tr>
<th>Component</th>
<th>Impedance ( Z )</th>
<th>Admittance ( Y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistor</td>
<td>( Z = R )</td>
<td>( Y = \frac{1}{R} )</td>
</tr>
<tr>
<td>Inductor</td>
<td>( Z = j\omega L )</td>
<td>( Y = \frac{1}{j\omega L} )</td>
</tr>
<tr>
<td>Capacitor</td>
<td>( Z = \frac{1}{j\omega C} )</td>
<td>( Y = j\omega C )</td>
</tr>
</tbody>
</table>
Impedance Transformation

(a) RL

\[ Z_1 = R_1 + j\omega L_1 \]
Voltage & Current Division

Voltage Division

\[ V_1 = \left( \frac{Z_1}{Z_1 + Z_2} \right) V_s \]

\[ V_2 = \left( \frac{Z_2}{Z_1 + Z_2} \right) V_s \]

Current Division

\[ I_1 = \left( \frac{Y_1}{Y_1 + Y_2} \right) I_s \]

\[ I_2 = \left( \frac{Y_2}{Y_1 + Y_2} \right) I_s \]
Linear circuit techniques

• We can now apply all the techniques we learned before (for dc circuits in the time domain) to ac circuits in the phase domain:
  – Superposition
  – Thevenin / Norton Equivalents
Example: Thévenin Circuit

\[ v_s(t) = 10 \cos 10^5 t \text{ (V)} \]
Example: Thévenin Circuit

\[ I_s = 2 \text{ A}, \quad R_s = 5 \]

\[ Z_2 = 3 + j4 \]

\[ Z_1 = 6 + j8 \]

\[ Z_3 = 2 - j10 \]

\[ I_s = 2 \text{ A} \]

\[ Z'_1 = 3.51 + j1.08 \]

\[ Z_3 = 2 - j10 \]

(c) \[ Z'_1 = R_s \parallel Z_1 \]
Example: Thévenin Circuit

$I_s = 2 \text{ A}$

$Z_2 = 3 + j4$

$Z_1' = 3.51 + j1.08$

$Z_3 = 2 - j10$
Example: Thévenin Circuit

\[ Z_1' = 3.51 + j1.08 \]
\[ Z_2 = 3 + j4 \]
\[ Z_3 = 2 - j10 \]

\[ Z_s' = 6.51 + j5.08 \]

(e) \[ Z_s' = Z_1' + Z_2 \]
Example: Thévenin Circuit

\[ Z_s' = 6.51 + j5.08 \]

\[ Z_3 = 2 - j10 \]

\[ Z_{Th} = Z_s' \parallel Z_3 \]

\[ \frac{(6.51 + j5.08)(2 - j10)}{(6.51 + j5.08) + (2 - j10)} = (8.42 - j1.59) \Omega \]

\[ R_{Th} = 8.42 \Omega, \]

\[ C_{Th} = \frac{1}{1.59\omega} = 6.29 \mu F \]
Solving using Phasor Diagrams

- The relationships between current and voltage for L and C are:

  **Capacitor**

  \[ I_C = j\omega C V_C \]

  **Inductor**

  \[ I_L = \frac{-jV_L}{\omega L} \]

- The relationship between current and voltage for R is trivial, obviously
Solving using Phasor Diagrams

• Consider the following circuit, with $Vs=20e^{j30}$

$$I = \frac{Vs}{R + j\omega L - \frac{j}{\omega C}}$$
Solving using Phasor Diagrams

- We can find the individual voltages graphically:
  \[ I = 2e^{j66.87^\circ} \text{ A} \]