1 Algebra Review

1.1 Eigenvectors and Eigenvalues

The eigenvector $\vec{x}$ and the its corresponding eigenvalue λ of matrix A satisfies the equation below:

$$A\vec{x} = \lambda\vec{x}$$

where $\vec{x} \neq \vec{0}$. This essentially means that if a matrix operates on its eigenvector, it will result in that same vector with only its magnitude changed.

1.2 Linear Independence

A list of vectors $\vec{x_1}, \vec{x_2} \ldots \vec{x_n}$ are linearly independent if, for scalars $a_1, a_2 \ldots a_n$

$$a_1\vec{x_1} + a_2\vec{x_2} + \cdots + a_n\vec{x_n} = 0$$

only when all $a_1, a_2 \ldots a_n = 0$. This also means that it is impossible to represent a vector $\vec{x_i}$ as a linear combination of the other vectors in the list.

1.3 Inner Product

In $\mathbb{R}^n$, the inner product (also called dot product) $\langle \vec{u}, \vec{v} \rangle$ of two vectors is defined as

$$\langle \vec{u}, \vec{v} \rangle = u_1 v_1 + u_2 v_2 + \ldots + u_n v_n$$

and

$$\vec{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ \vdots \\ u_n \end{bmatrix} \vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_n \end{bmatrix}$$

1.4 Magnitude of Vector

The 2-norm, or magnitude, of a vector $\vec{x}$ in $\mathbb{R}$ is defined as

$$||\vec{x}|| = \sqrt{\langle \vec{x}, \vec{x} \rangle}$$

1.5 Orthonormality

Two vectors $\vec{u}$ and $\vec{v}$ are orthonormal if

$$\langle \vec{u}, \vec{v} \rangle = 0$$

and

$$||\vec{u}|| = 1$$

$$||\vec{v}|| = 1$$

1. **Ice-Breaker**

   Welcome back!

2. **Who will win the election?** Candidates A, B, C are running for office. Currently, B is leading with 90% and A and C split the rest of the votes. However, the public's opinions change every day: 10% of A's supporters will revert to B; 20% and 10% of B's supporters will switch to favoring A and C, respectively; and 20% of C's supporters will switch to supporting A and another 20% will go for B.

   (a) How could we model this using the linear-algebraic tools that you have learned?

   (b) Given that the final election is very far from now, who will win this election assuming supporters keep changing in the described way? (Hint: remember that $A(t) = A(t+1)$ very far in the future)

3. **Orthonormal Vectors and Linear Independence** Let vector $\vec{u}$ and $\vec{v}$ be orthonormal vectors. Show that $\vec{u}$ and $\vec{v}$ are linearly independent.

4. **Eigenspace**

   Suppose $\lambda_1, \cdots, \lambda_m$ are distinct eigenvalues of T and $\vec{v}_1$and$\vec{v}_2$ are corresponding eigenvectors. Show that $\vec{v}_1$,and$\vec{v}_2$ are linearly independent.

5. **Op-Amp Review**

In this review problem, you will apply the golden rules to derive something you need in lab.

Figure 1 shows the equivalent model of an op-amp. It is important to note that this is the general model of an op-amp, so our op-amp golden rules cannot be applied to this unless certain conditions are met.

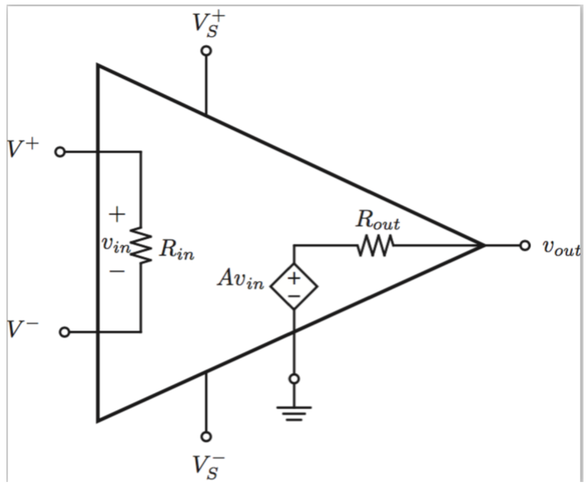


Figure 1: General Op-Amp Model

Conditions Required for Golden Rule:

- $R_{in} \to \infty$
- $R_{out} \to 0$
- $A_{v_{in}} \to \infty$
- The op-amp must be operated in negative feedback

When conditions 1-3 are met, the op-amp is considered ideal. Figure 2 shows an ideal op-amp in negative feedback, which can be analyzed using the golden rules.

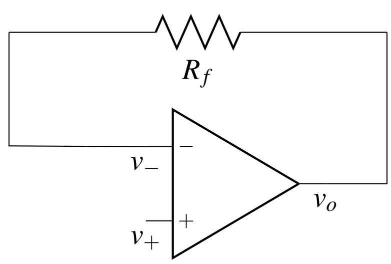
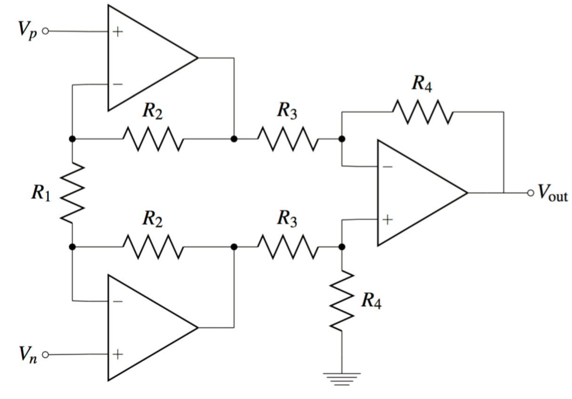


Figure 2: Ideal Op-Amp in Negative Feedback

Golden rules of ideal op-amps in negative feedback:

- No current can flow into the input terminals ($I_- = 0$ and $I_+ = 0$)
- The $(+)$ and $(-)$ terminals are at the same voltage ($V_+ = V_-$)

Now let's look at the circuit below:



(a) Write down all the branch and node equations using the golden rules of Op-Amps.

(b) Notice that there exists a symmetry between the two op-amps at the first stage of this circuit. What are the directions of the currents going through the two R_2s? How do the currents of R_2s influence the current through R_1?

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