System stability conditions

Discrete time systems

A discrete time system is of the form,

$$\vec{x}(t+1) = A\vec{x}(t) + B\vec{u}(t)$$

Let λ be any particular eigenvalue of A. This system is stable if $|\lambda| < 1$.

Continuous time systems

A continuous time system is of the form,

$$\frac{\mathrm{d}\vec{x}}{\mathrm{d}t}(t) = A\vec{x}(t) + B\vec{u}(t)$$

Let λ be any particular eigenvalue of A. This system is stable if $\mathrm{Re}\{\lambda\} < 0$.

Questions

1. **Discrete-time Stability**

   Determine which values of α and β will make the following discrete-time state space models stable:

   (a)
   $$x[t+1] = \alpha x[t]$$

   (b)
   $$\vec{x}[t+1] = \begin{bmatrix} \alpha & -\beta \\ \beta & \alpha \end{bmatrix} \vec{x}$$

   (c)
   $$\vec{x}[t+1] = \begin{bmatrix} 1 & \alpha \\ 0 & 1 \end{bmatrix} \vec{x}$$

2. **Continuous-time Stability**

   Consider the linearized state space model for an inverted pendulum in the up position, which we found in lecture:

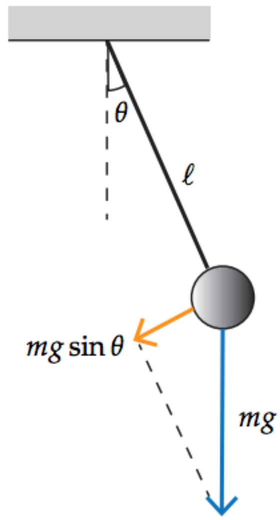

Figure 1: Pendulum Free-body Diagram

$$\frac{d}{dt}\vec{x} = \begin{bmatrix} 0 & 1 \\ \frac{g}{l} & \frac{-k}{l} \end{bmatrix} \vec{x}$$

Where

$$\vec{x} = \begin{bmatrix} \theta \\ \frac{d\theta}{dt} \end{bmatrix}$$

(a) Is this system stable?

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