EECS 16B Designing Information Devices and Systems II
Fall 2016 Murat Arcak and Michel Maharbiz Homework 0

This homework is due August 29th, 2016, at Noon.

1. **Homework process and study group**

   (a) Who else did you work with on this homework? List names and student ID's. (In case of hw party, you can also just describe the group.)

   (b) How long did you spend working on this homework? How did you approach it?

2. **KVL/KCL review**

Kirchoff's Circuit Laws are two important laws used for analyzing circuits. Kirchoff's Current Law (KCL) says that the sum of all currents entering a node must equal 0. For example, in Figure 1, the sum of all of the currents entering node 1 is $I_1 - I_2 - I_3 = 0$. Assuming I_1 and I_3 are known, we can easily obtain a solvable equation for V_x by plugging in Ohm's law: $I_1 - \frac{V_x}{R_1} - I_3 = 0$

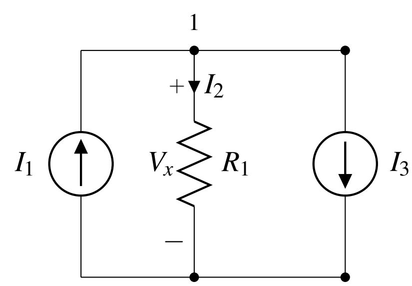


Figure 1: KCL Circuit

Kirchoff's Voltage Law (KVL) states that the sum of all voltages in a circuit loop must equal 0. To apply KVL to the circuit shown in Figure 2, we can add up voltages in the loop in the counterclockwise which yields $-V_1 + V_x + V_y = 0$. Using the relationships $V_x = i * R_1$ and $i = I_1$ we can solve for all unknowns in this circuit. You can use these two laws to solve any circuit that is planar and linear.

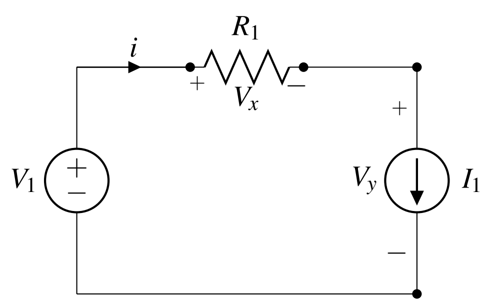


Figure 2: KVL Circuit

Use Kirchoff's Laws on the circuit below to find v_x, i_s, i_{in} and the power provided by the dependent current source. You can use $R_1 = 2\Omega$, $R_2 = 4\Omega$, and $R_3 = 2\Omega$. To help with solving the problem, we have already found the voltage difference across R_1 and R_3.

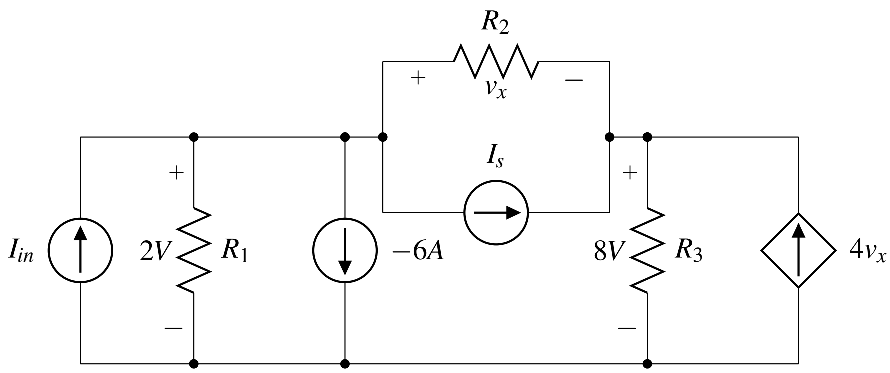


Figure 3: Example Circuit

(a) What is v_x?

(b) What is I_s?

(c) What is I_{in}?

(d) What is the power output of the dependent current source on the far right?

3. KCL

Now consider the circuit shown below:

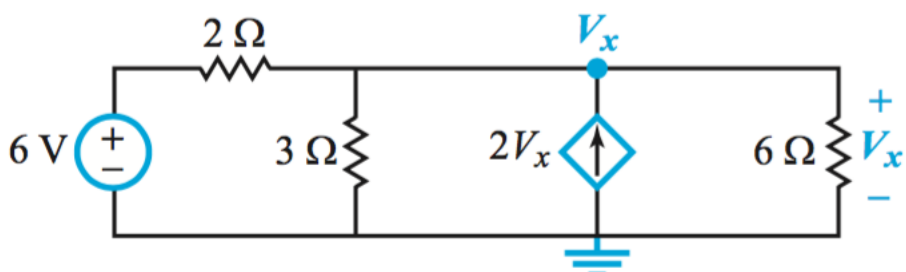


Figure 4: From Ulaby, Maharbiz, Furse. *Circuits*. Third Edition

Determine the voltage V_x.

4. KVL Now consider the circuit shown below:

Determine the amount of power supplied by the voltage source.

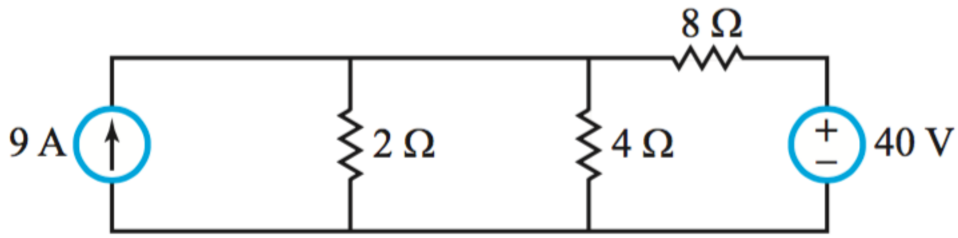


Figure 5: From Ulaby, Maharbiz, Furse. *Circuits*. Third Edition

5. Circuits and Gaussian Elimination

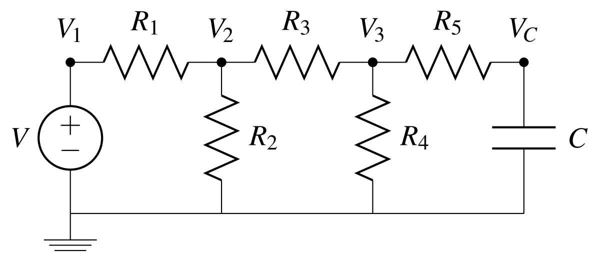


Figure 6: Example Circuit

(a) Find a system of linear equations that could be solved to find the node voltages.

(b) Given that the component values are $R_1 = 500\,\Omega$, $R_2 = 3\,\text{k}\Omega$, $R_3 = 1\,\text{k}\Omega$, $R_4 = 2\,\text{k}\Omega$, and $R_5 = 4\,\text{k}\Omega$, Solve the circuit equations using Gaussian elimination.

(c) What's the voltage V_C across the capacitor?

(d) How would you check your work? Do so.

6. Solving Recurrence Relations

For this problem, we'll work with a sequence defined by the following recurrence relation, where $S[n]$ is the nth number in the sequence:

$$S[n+1] = 3S[n] - 2S[n-1]$$
$$S[0] = 0$$
$$S[1] = 1$$

You can probably see how this could be computed recursively or iteratively, but let's try a linear-algebraic approach and see where it takes us.

(a) Starting from the definition of the sequence, determine the matrix A to calculate $S[n+1]$ such that:

$$A \begin{pmatrix} S[n] \\ S[n-1] \end{pmatrix} = \begin{pmatrix} S[n+1] \\ S[n] \end{pmatrix}$$

(b) Find the eigenvalues λ_+ and λ_- of the matrix A.

(c) Find the eigenvectors associated with λ_+ and λ_- from above.

(d) How would you check your work? Do so. (Hint: Based on the eigenvalues you determined, how should this sequence behave for different initial conditions? Does it?)

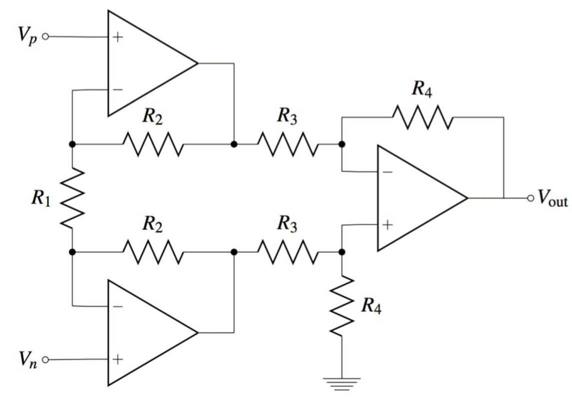
(e) Using your previous results, diagonalize A.

(f) How can you use this new information to more efficiently compute any arbitrary $S[n]$ *without using any iteration or recursion*? (Hint: if a matrix $M = PDP^{-1}$ where D is a diagonal matrix, then think about $M^2 = MM$, M^3, and even M^n.)

(g) Finally, using your results from above, derive a closed-form expression with no summations, no recursion, and no matrix multiplications for $S[n]$.

7. Op-Amp Review

Now let's look at the circuit below:



(a) Write down all the branch and node equations using the golden rules of Op-Amps.

(b) Notice that there exists a symmetry between the two op-amps at the first stage of this circuit. What are the directions of the currents going through the two R_2s? How do the currents of R_2s influence the current through R_1?

(c) What is the current through R_1?

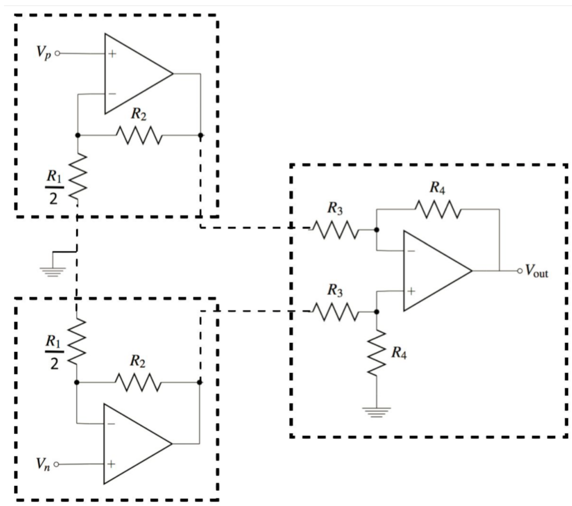
(d) What are the output voltages of the two op-amps at the first stage?

(e) Compute the voltage at the + terminal of the second-stage op-amp.

(f) What is V_{out}?

(g) What is the voltage in the middle of the resistor R_1?

(h) Based on the above analysis, we could introduce a "fake ground" in the middle of the resistor R_1 and come up with the following circuit:



Now, each of the first two op-amps is being used in a form that resembles building blocks that you have seen before. What are the gains of those blocks?

What is $V_{out}/(V_p - V_n)$ for this revised circuit?

Contributors:

- Nathan Mailoa.
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