EECS 16B Designing Information Devices and Systems II
Fall 2016 Murat Arcak and Michel Maharbiz **Homework 2**

This homework is due September 12, 2016, at Noon.

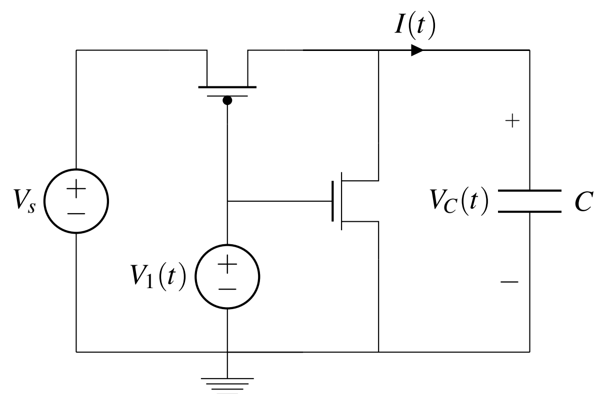
1. Homework process and study group

(a) Who else did you work with on this homework? List names and student ID's. (In case of homework party, you can also just describe the group.)

(b) How long did you spend working on this homework? How did you approach it?

2. Transistors as switches

Consider the transistor and capacitor switch circuit below:



In this circuit, the source of the PMOS transistor is connected to the positive node of V_s and the source of the NMOS transistor is connected to ground. $|V_{th}| = 2$V for both transistors. $R_p = R_n = 1k\Omega$ for the transistor resistances and $V_s = 5$V. $V_1(t)$ is defined as follows:

$$V_1(t) = \begin{cases} 5\text{V for } t \leq 0 \\ 0\text{V for } t > 0 \end{cases}$$

(a) For $t \leq 0$, Redraw the circuit with the transistors replaced with their resistor switch models. Assuming this circuit has reached steady state, what is $V_c(0)$?

(b) For $t > 0$, Redraw the circuit with the transistors replaced with their resistor switch models.

(c) Find the differential equation that models this circuit for $t > 0$.

(d) Solve for $V_C(t)$. Starting from $t = 0$, how long does it take for $V_C(t)$ to be within 1% of its final value?

3. Matrix differential equations

In this problem, we consider ordinary differential equations which can be written in the following form

$$\begin{pmatrix} x'(t) \\ y'(t) \end{pmatrix} = A \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}, \tag{1}$$

where x, y are variables depending on t, $x' = \frac{dx}{dt}$, $y' = \frac{dy}{dt}$, and A is a 2×2 matrix with constant coefficients. We call (1) a matrix differential equation.

(a) Suppose we have a system of ordinary differential equations

$$x(t)' = 8x(t) + 7y(t) \tag{2}$$

$$y(t)' = -4x(t) - 3y(t) \tag{3}$$

Write this in the form of (1).

(b) Compute the eigenvalues and eigenvectors of the matrix A from the previous part.

(c) We claim that the solution for $x(t), y(t)$ is of the form

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = c_0 e^{\lambda_0 t} \vec{v}_0 + c_1 e^{\lambda_1 t} \vec{v}_1 ,$$

where c_0, c_1 are constants, and λ_0, λ_1 are the eigenvalues of A with eigenvectors $\vec{v}_0, \vec{v}_1$ respectively. Suppose that the initial conditions are $x(0) = 1, y(0) = 1$. Solve for the constants c_0, c_1.

(d) Verify that the solution for $x(t), y(t)$ found in the previous part satisfies the original system of differential equations (2), (3).

(e) We now apply the method above to solve a second order ordinary differential equation. Suppose we have the system

$$z''(t) - 5z'(t) + 6z(t) = 0 \tag{4}$$

Write this in the form of (1), by using the change of variables $x(t) = z(t)$, $y(t) = z'(t)$.

(f) Solve the system in (4) with the initial conditions $z(0) = 1$, $z'(0) = 1$, using the method developed in parts (b) and (c).

4. Solving second-order differential equations

Solve the following second-order differential equation:

$$\frac{d^2}{dt^2} y(t) - 2\frac{d}{dt} y(t) - 3y(t) = 0$$

where the initial conditions are $y(0) = 0, \frac{d}{dt} y(0) = 1$.

(a) Firstly, please write the above equations into the matrix form $\frac{d}{dt}\vec{x}(t) = A\vec{x}(t)$. How do you define $\vec{x}(t)$, A, and the initial conditions $\vec{x}(0)$?

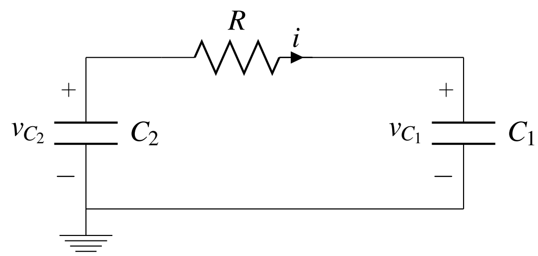
(b) What are the eigenvalues and eigenvectors of A?

(c) Based on the above results, what is the solution to the second-order equation?

5. Charge sharing, revisited

Consider the following circuit which features two capacitors connected together with a resistor. With the exception of the resistor in between, you should recognize this as the idea of charge sharing from EE 16A which you used in your capacitive touchscreen lab. In this problem, we will take a more in-depth look at this charge sharing circuit to make it more rigorous and intuitive on a circuit level.

We will analyze this circuit for $t \geq 0$. The initial voltage across C_2 is given as V_2, and the initial voltage across C_1 is given as V_1. In other words:



$$v_{C_1}(t \leq 0) = V_1$$
$$v_{C_2}(t \leq 0) = V_2$$

(a) Determine a formula for $v_{C_1}(t)$ for $t \geq 0$.

(b) Determine a formula for $v_{C_2}(t)$ for $t \geq 0$. (Hint: use the previous part of this problem.)

(c) Determine a formula for the current $i(t)$ through the resistor for $t \geq 0$.

(d) Check your work by showing that at $t = 0$, the initial conditions given at the start of this problem are satisfied. (In other words, show that $v_{C_1}(t = 0) = V_1$ and that $v_{C_2}(t = 0) = V_2$.

(e) At steady-state ($t \to \infty$), what happens to the voltages across the two capacitors? Is this consistent with what you learned in EE 16A?

(f) Sketch the two voltages over time on the same chart.

(g) Is there any current flowing at steady-state? Why or why not?

Contributors:

- Lynn Chua.
- Ming Jin.
- Edward Wang.
- Baruch Sterin.