EECS 16B Designing Information Devices and Systems II
Fall 2016 Murat Arcak and Michel Maharbiz **Homework 3**

This homework is due September 19, 2016, at Noon.

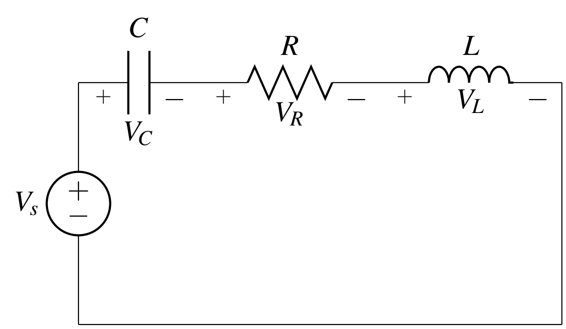
1. Homework process and study group

(a) Who else did you work with on this homework? List names and student ID's. (In case of homework party, you can also just describe the group.)

(b) How long did you spend working on this homework? How did you approach it?

2. RLC circuit #1

In this question, we will take a look at an electrical systems described by second order differential equations and analyze their transfer function. Consider the circuit below where $R = 200\Omega$, $L = 1$mH, and $C = 20$nF:



(a) Write the KVL equation for this circuit.

(b) Using the current-voltage relationships of capacitors and inductors, to write the previous equation in terms of V_C and V_S only.

(c) Suppose the voltage source was shorted at $t \geq 0$, i.e, $V_C = V_s = V$ for $t < 0$ and $V_s = 0$ at $t \geq 0$. Write down the differential equations in the form of the matrix

$$\frac{d}{dt}\begin{bmatrix} V_C \\ \frac{dV_C}{dt} \end{bmatrix} = A \begin{bmatrix} V_C \\ \frac{dV_C}{dt} \end{bmatrix} \tag{1}$$

(d) Find the eigenvalues and eigenvectors of the matrix A

(e) Diagonalize matrix A so $A = PDP^{-1}$, where D is the diagonal matrix containing the eigenvalues of A and P is the matrix whose columns are the eigenvectors of A. Note: you do not need to actually compute P^{-1}.

(f) Given $\vec{x} = P^{-1}\begin{bmatrix} V_C \\ \frac{dV_C}{dt} \end{bmatrix}$, write the differential equation in the following form:

$$\frac{d}{dt}(\vec{x}) = D\vec{x}$$

(g) Solve the above differential equation for $\vec{x}$

(h) In order to find our final solution, transform $\vec{x}$ back to our original basis by doing the following operation:

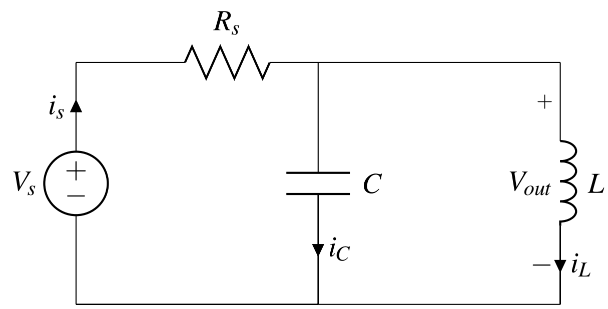
$$\begin{bmatrix} V_C \\ \frac{dV_C}{dt} \end{bmatrix} = P\vec{x}$$

(i) What are the initial conditions $V_C(0)$ and $\frac{dV_C}{dt}(0)$? Use them to find the constants in the solution from the last par.

(j) Find $V_C(t)$.

3. RLC circuit #2

Now consider the circuit shown below:



(a) Write down the KCL equation; express the total current through the source as a sum of currents in the capacitor and inductor.

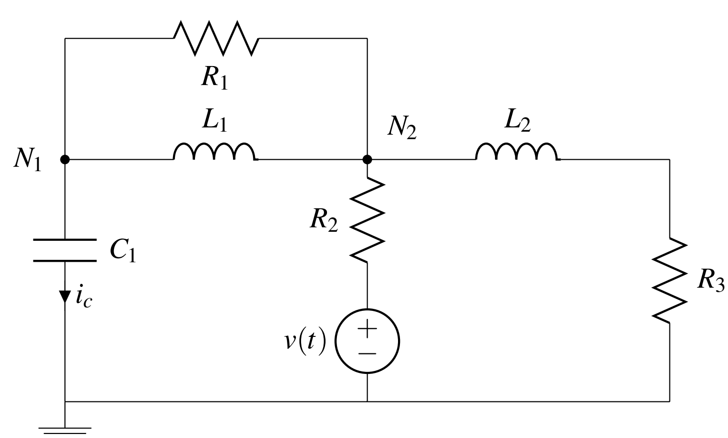
(b) Determine the equivalent impedance of the parallel circuit in the phasor domain.

(c) Write down the transfer function of this circuit (take V_s as V_{in}, and the voltage across the inductor as V_{out} as indicated in the circuit.)

(d) What type of filter is this?

4. Phasor-domain circuit analysis

The analysis techniques you learned previously for resistive circuits are equally applicable for analyzing AC circuits (circuits driven by sinusoidal inputs) in the phasor domain. In this problem, we will walk you through the steps with a concrete example. Consider the circuit below.



The components in this circuit are given by:

Voltage source:

$$v(t) = 12\cos(400t - 30^\circ)$$

Resistors:

$$R_1 = 5\Omega, \quad R_2 = 5\Omega, \quad R_3 = 5\Omega$$

Inductors:

$$L_1 = 20 \text{ mH}, \quad L_2 = 20 \text{ mH}$$

Capacitor:

$$C_1 = \frac{1}{1.6} \text{ mF}$$

(a) To begin with, transform the given circuit to the phasor domain.

(b) Write out KCL for node N_1 and N_2 in the phasor domain.

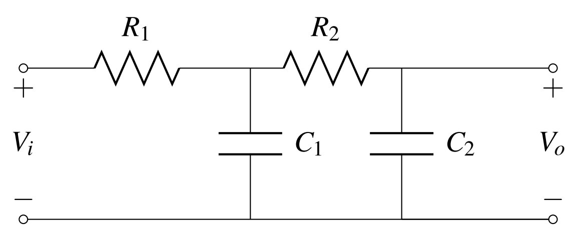
(c) Use KVL to express the currents in terms of node voltages in the phasor domain. The node voltages V_1 and V_2 are the voltage drops from N_1 and N_2 to the ground.

(d) Write the equations you derived in part (b) and (c) in a matrix form, i.e., $A \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = b$

(e) Solve the systems of linear equations you derived in part (d) with any method you prefer, and then find $i_c(t)$.

5. Transfer functions

Consider the circuit below.



The circuit has an input phasor voltage V_i at frequency ω rad/sec applied at the input terminals shown in the illustration above, causing an output phasor voltage V_o at output terminals.

(a) Obtain an expression for $H = V_o/V_i$ in terms of $Z_{R1}, Z_{R2}, Z_{C1}, Z_{C2}$ (they are the impedance of R_1, R_2, C_1, C_2, respectively).

(b) Obtain an expression for $H(\omega) = V_o/V_i$ in the form of

$$H(\omega) = \frac{V_o}{V_i} = \frac{1}{1 + j2\xi(\omega/\omega_c) + (j\omega/\omega_c)^2},$$

given that $R_1 = 1\Omega$, $R_2 = 2\Omega$, $C_1 = 1F$, and $C_2 = 2F$. What are the values of ξ and ω_c?

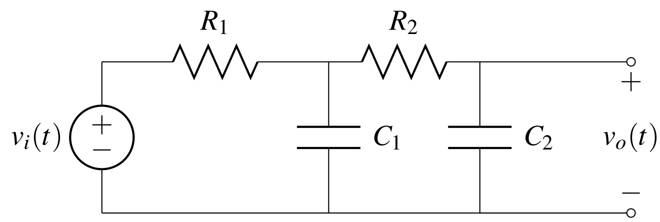
(c) We can express the transfer function $H(\omega)$ in the polar form. That is,

$$H(\omega) = M(\omega)e^{j\phi(\omega)}$$

The functions $M(\omega)$ and $\phi(\omega)$ are the magnitude and the phase angle of $H(\omega)$, respectively. Write down $M(\omega)$ and $\phi(\omega)$ using the transfer function you derived in part (b).

(d) Compute the phasors of $H(0)$, $H(\omega_c)$, and $H(\infty)$ using the results in part (b) and (c).

(e) Consider the circuit below.



The voltage source is given by

$$v_i(t) = 12\sin(\frac{1}{2}t - \frac{\pi}{4})$$

The values of R_1, R_2, C_1, and C_2 are the ones given in part (b). Obtain an expression for $v_o(t)$ in the form of $\alpha\cos(\frac{1}{2}t + \theta)$.

6. Bass-booster

RC circuits and filters are very useful for altering the frequency content of signals. For example, audio equalization equipment can use these filters to adjust the pitch content of audio signals. Suppose we want to boost the bass of our favorite music, we can use the active filter circuit below to tune the frequency content of our favorite jams.

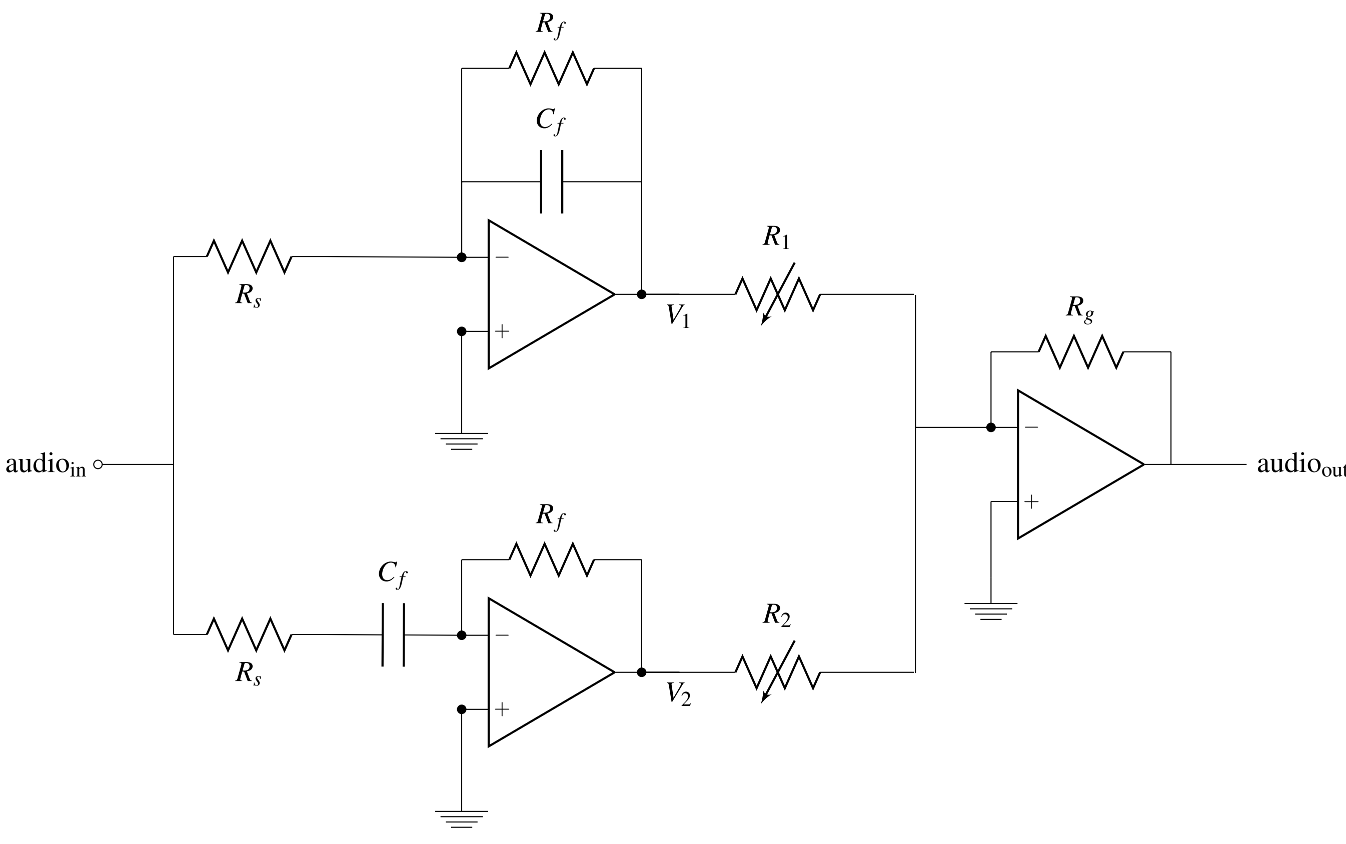


Figure 1: Audio Equalizer Circuit

$C_f = 400$ nF, $R_f = 1\text{k}\Omega$, $R_g = 100\Omega$, and $R_s = 1\text{k}\Omega$. R_1 and R_2 are both variable capacitors that can be used to tune the frequency balance of our output signal. Assume $\text{audio}_{\text{in}} = A\cos(\omega t)$.

(a) Find transfer functions for $H_1(\omega) = \frac{V_1}{\text{audio}_{\text{in}}}$ and $H_2(\omega) = \frac{V_2}{\text{audio}_{\text{in}}}$. Sketch the bode plots for the magnitude of these transfer functions. What kind of filter is each transfer function?

(b) Find a function that describes $\text{audio}_{\text{out}}$ in terms of V_1 and V_2

(c) Combine the results of the last two parts to find an overall transfer function for $H_{ov}(\omega) = \frac{\text{audio}_{\text{out}}}{\text{audio}_{\text{in}}}$.

(d) Using this circuit, how could we set R_1 and R_2 to boost our bass frequency signals ($f < 400$ Hz) without affecting mid and treble range signals? Sketch a bode plot of the magnitude of your overall bass-boosting function.

Contributors:

- Varun Mishra.
- Yuxun Zhou.
- Yen-Sheng Ho.