This homework is due September 26, 2016, at Noon.

1. Homework process and study group

(a) Who else did you work with on this homework? List names and student ID's. (In case of homework party, you can also just describe the group.)

(b) How long did you spend working on this homework? How did you approach it?

2. Ring oscillator

Figure 1 shows a ring oscillator circuit with three inverters. These inverters are modeled as non-ideal op-amps using the general op-amp model we saw earlier in the semester. Remember, our golden rules don't apply for the models below. Each op-amp acts as an inverter with gain. The voltage inputs terminals are considered open circuits. $R_{out} = 10\text{k}\Omega$, $C_{o1} = C_{o2} = C_{o3} = 1pF$, and $K_1 = K_2 = K_3 = 2$

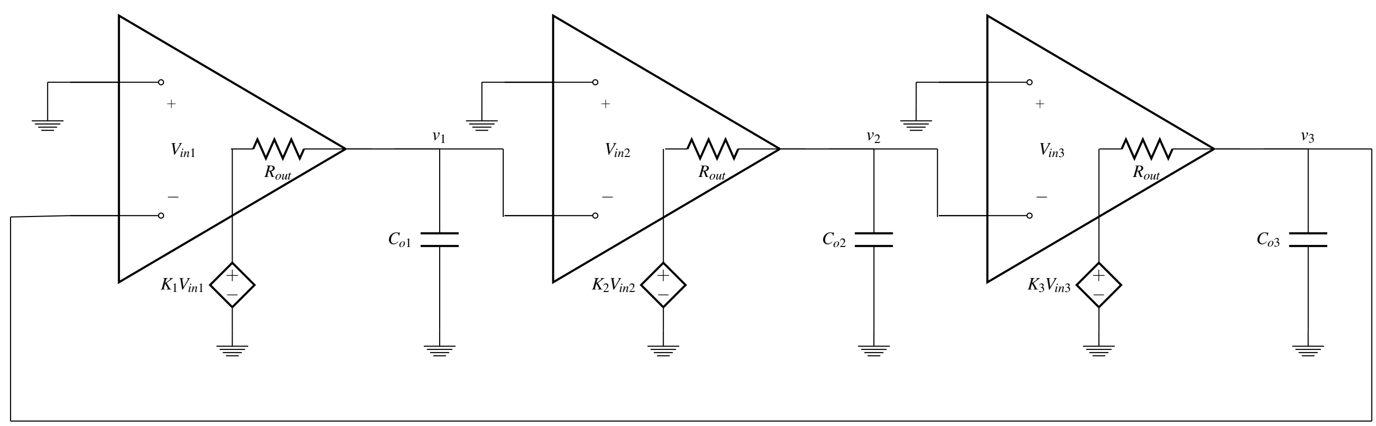

Figure 1: Ring Oscillator Modeled with Non-Ideal Op-Amps

(a) First, lets look at the first op-amp in the chain. For the circuit in figure 2, find the transfer function for $\frac{v_1}{v_0}$.

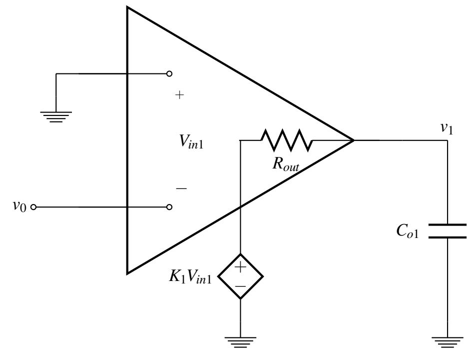

Figure 2: First Op-Amp in Ring Oscillator

(b) Now, let's look at three of these op-amps cascaded together as seen in figure 3. What is the transfer function for $\frac{v_3}{v_0}$? (Hint: since the input of each op-amp is an open circuit, the overall transfer function can be represented as the individual transfer functions of each amplifier cascaded together.)

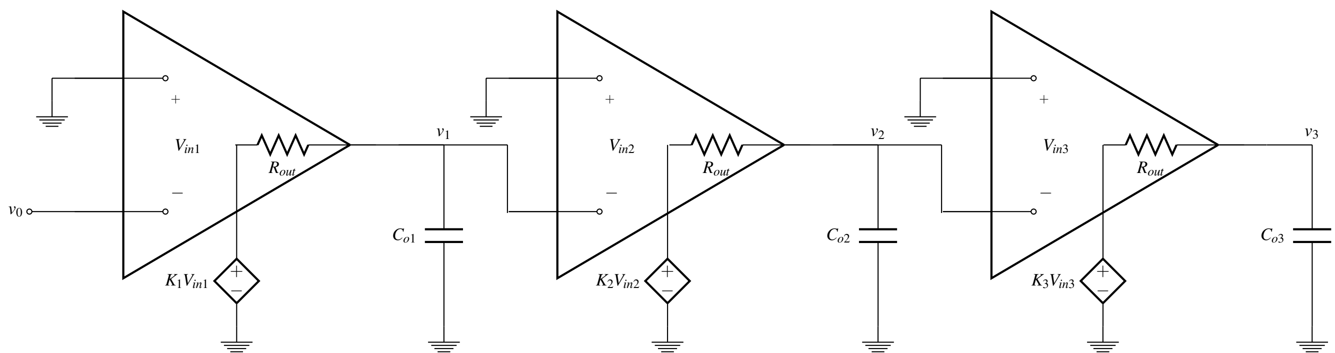

Figure 3: Ring Oscillator without Feedback

(c) Draw the bode plots for the magnitude and phase of $\frac{v_3}{v_0}$.

(d) At what frequency is the phase of $\frac{v_3}{V_0}$ equal to $-\pi$? What is the magnitude of $\frac{v_3}{V_0}$ at that frequency? How does v_3 compare to v_0 at this frequency? An interesting consequence of this result is that this system will have a sustained oscillation when placed in feedback.

3. Warm-up: matrix powers

For real numbers, a "power" of that number is computed by recursive multiplication. For example, for a real number x

$$\begin{aligned} x^1 &= x \\ x^2 &= xx^1 = xx \\ x^3 &= xx^2 = xxx \\ &\vdots \end{aligned}$$

Powers of a matrix A are defined similarly as

$$A^1 = A$$
$$A^{k+1} = AA^k.$$

(a) Compute A^2 and A^3 for the following two matrices

i.
$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

ii.
$$A = \begin{bmatrix} 1.1 & 1 & 0 \\ 0 & 0.9 & 1 \\ 0 & 0 & 0.8 \end{bmatrix}$$

(b) If A is diagonalizable as $A = PDP^{-1}$, how would you compute a general form for A^k?

4. Transistor review

(a) Draw a CMOS circuit (one which employs a pull-up network of PMOS transistors and pull-down network of NMOS transistors) that gives the Boolean formula $Y = A \oplus B$. For inputs, you may use A, B, $\bar{A}$, and $\bar{B}$.

(b) Write the differential equations of the circuit below in the form $\frac{d}{dt}\vec{x}(t) = A\vec{x}(t) + \vec{b}$. Model the PMOS as a switch with an on-resistance of R_p and threshold voltage of $|V_{th}| = 0.4V$. The voltage source $v_p(t)$ behaves as follows:

$$v_p(t) = \begin{cases} 1\text{V for } t \leq 0 \\ 0\text{V for } t > 0 \end{cases}$$

(Hint: $\vec{x}(t)$ should contain $i_L(t)$ and $v_C(t)$.)

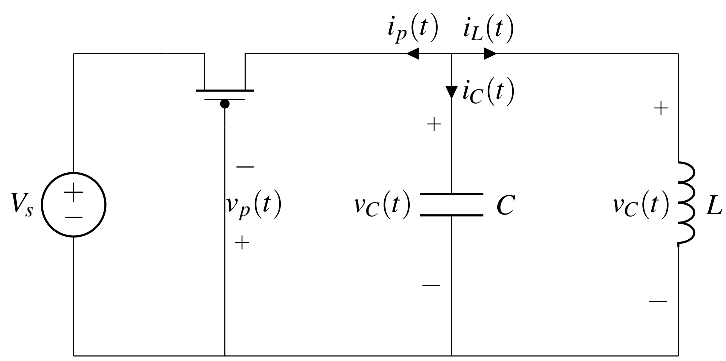

Figure 4: Second-order circuit

Contributors:

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