This homework is due October 3, 2016, at Noon.

1. **Homework process and study group**

   (a) Who else did you work with on this homework? List names and student ID's. (In case of homework party, you can also just describe the group.)

   (b) How long did you spend working on this homework? How did you approach it?

2. **Non-linear ring oscillator**

   Figure 1 shows a ring oscillator circuit with three non-linear inverters. Suppose we have new inverters that have more realistic non-linear dependent voltage sources, as seen in figure 1. For these dependent voltage sources

$$g(v) = V_{\text{sat}} \tanh\left(\frac{v}{V_0}\right)$$

   where V_0 is a constant and $\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$. The voltage input terminals are considered open circuits. $V_{sat} = 2$V, $V_0 = 1$V, $R_{out} = 10\text{k}\Omega$, $C_{o1} = C_{o2} = C_{o3} = 1pF$.

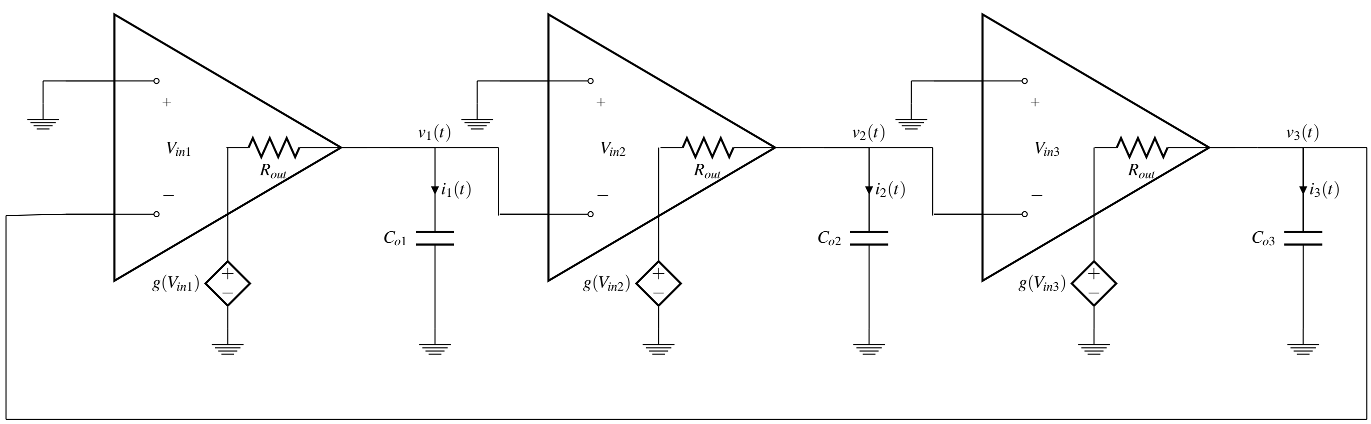

Figure 1: Ring Oscillator Modeled with Non-Linear Inverters

(a) Find expressions $\frac{dv_1(t)}{dt}$, $\frac{dv_2(t)}{dt}$, and $\frac{dv_3(t)}{dt}$ that only depend on $v_1(t)$, $v_2(t)$, and $v_3(t)$.

(b) Find a state space model for $\vec{x}$ and write it in the form

$$\frac{d}{dt}\vec{x} = f(\vec{x})$$

(c) What is a convenient equilibrium point for your state space model from (b)?

(d) Find $\nabla f(\vec{x})$ and evaluate it at the equilibrium point. As we saw in lecture, $\nabla f(x)$, where x is a vector, is described below:

$$\nabla f(x) = \begin{bmatrix} \frac{\partial f_1(x_1,\ldots,x_n)}{\partial x_1} & \frac{\partial f_1(x_1,\ldots,x_n)}{\partial x_2} & \cdots & \frac{\partial f_1(x_1,\ldots,x_n)}{\partial x_n} \\ \frac{\partial f_2(x_1,\ldots,x_n)}{\partial x_1} & \frac{\partial f_2(x_1,\ldots,x_n)}{\partial x_2} & \cdots & \frac{\partial f_2(x_1,\ldots,x_n)}{\partial x_n} \\ \vdots & \vdots & & \vdots \\ \frac{\partial f_n(x_1,\ldots,x_n)}{\partial x_1} & \frac{\partial f_n(x_1,\ldots,x_n)}{\partial x_2} & \cdots & \frac{\partial f_n(x_1,\ldots,x_n)}{\partial x_n} \end{bmatrix}$$

(e) Find a linearized state space model for $\vec{x}$ about the equilibrium point and write it in the form

$$\frac{d}{dt}\vec{x} = A\vec{x}$$

(f) What are the eigenvalues of A?

3. Redo Problem 1 on the midterm

(a)

(b)

(c)

4. Redo Problem 2 on the midterm

(a)

(b)

(c)

(d)

5. Redo Problem 3 on the midterm

(a)

(b)

(c)

(d)

(e)

6. Redo Problem 4 on the midterm

(a)

(b)

7. Redo Problem 5 on the midterm

(a)

(b)

(c)

Contributors:

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