EECS 16B Designing Information Devices and Systems II
Fall 2016 Murat Arcak and Michel Maharbiz **Homework 7**

This homework is due October 17, 2016, at Noon.

1. Homework process and study group

(a) Who else did you work with on this homework? List names and student ID's. (In case of homework party, you can also just describe the group.)

(b) How long did you spend working on this homework? How did you approach it?

2. Controllability and discretization

In this problem, we will use the car model

$$\frac{d}{dt}p(t) = v(t)$$
$$\frac{d}{dt}v(t) = u(t)$$

that was discussed in class.

(a) Assuming that the input $u(t)$ can be varied continuously, is this system controllable?

(b) Now assume that we can only change our control input every T seconds. Derive a discrete-time state space model for the state updates, assuming that the input is held constant between times t and $t+T$.

(c) Is the discrete-time system controllable?

3. Controllability in 2D

Consider the control of some two-dimensional linear discrete-time system

$$\vec{x}(k+1) = A\vec{x}(k) + Bu(k)$$

where A is a 2×2 real matrix and B is a 2×1 real vector.

(a) Let $A = \begin{bmatrix} a & c \\ 0 & d \end{bmatrix}$ with $a, c, d \neq 0$, and $B = \begin{bmatrix} f \\ g \end{bmatrix}$. Find a B such that the system is controllable no matter what nonzero values a, c, d take on, and a B for which it is not controllable no matter what nonzero values are given for a, c, d. You can use the controllability rank test, but please explain your intuition as well.

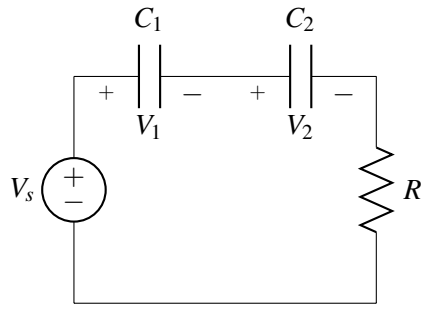
(b) Let $A = \begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix}$ with $a, d \neq 0$. and $B = \begin{bmatrix} f \\ g \end{bmatrix}$ with $f, g \neq 0$. Is this system always controllable? If not, find configurations of nonzero a, d, f, g that make the system uncontrollable.

(c) We want to see if controllability is preserved under changes of coordinates. To begin with, let $\vec{z}(k) = V^{-1}\vec{x}(k)$, please write out the system equation with respect to $\vec{z}$.

(d) Now show that controllability is preserved under change of coordinates. (Hint: use the fact that $rank(MA) = rank(A)$ for any invertible matrix M.)

4. Controllability in circuits

Consider the circuit below, where V_s is an input we can control:



(a) Write the state space model for this circuit.

(b) Show that this system is not controllable

(c) Explain, in terms of circuit currents and voltages, why this system isn't controllable. (Hint: think about what currents/voltages of the circuit we are controlling with V_s)

(d) Draw an equivalent circuit of this system that is controllable. What quantity can you control in this system?

Contributors:

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