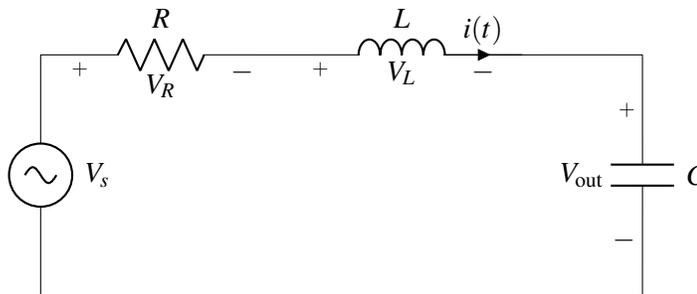


**This homework is due September 19, 2017, at Noon.  
Please use radians for all angles in phasor notation**

**1. RLC circuit #1**

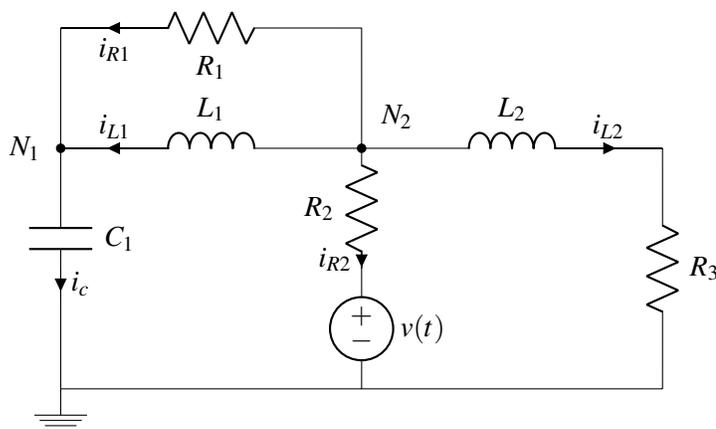
In this question, we will take a look at an electrical systems described by second order differential equations and analyze it using the phasor domain. Consider the circuit below where  $R = 3\text{k}\Omega$ ,  $L = 1\text{mH}$ ,  $C = 100\text{nF}$ , and  $V_s = 5 \cos(1000t + \frac{\pi}{4})$ :



- (a) What are the impedances of the resistor, inductor and capacitor,  $Z_R$ ,  $Z_L$ , and  $Z_C$ ?
- (b) Solve for  $\tilde{V}_{\text{out}}$  in phasor form.
- (c) What is  $V_{\text{out}}$  in the time domain?
- (d) Solve for the current  $i(t)$
- (e) Solve for the transfer function  $H(\omega) = \frac{\tilde{V}_{\text{out}}}{\tilde{V}_s}$   
Leave your answer in terms of  $R$ ,  $L$ ,  $C$ , and  $\omega$ .

**2. Phasor-domain circuit analysis**

The analysis techniques you learned previously for resistive circuits are equally applicable for analyzing AC circuits (circuits driven by sinusoidal inputs) in the phasor domain. In this problem, we will walk you through the steps with a concrete example. Consider the circuit below.



The components in this circuit are given by:

Voltage source:

$$v(t) = 20 \cos(50t - \frac{\pi}{3})$$

Resistors:

$$R_1 = 8\Omega, \quad R_2 = 8\Omega, \quad R_3 = 8\Omega$$

Inductors:

$$L_1 = 40 \text{ mH}, \quad L_2 = 40 \text{ mH}$$

Capacitor:

$$C_1 = 5 \text{ mF}$$

- (a) To begin with, transform the given circuit to the phasor domain.
- (b) Write out KCL for node  $N_1$  and  $N_2$  in the phasor domain.
- (c) Use KVL to express the currents in terms of node voltages in the phasor domain. The node voltages  $\tilde{V}_1$  and  $\tilde{V}_2$  are the voltage drops from  $N_1$  and  $N_2$  to the ground.
- (d) Write the equations you derived in part (b) and (c) in a matrix form, i.e.,  $A \begin{bmatrix} \tilde{V}_1 \\ \tilde{V}_2 \end{bmatrix} = b$  Solve for  $A$  and  $b$  in numerical form.
- (e) Solve the systems of linear equations you derived in part (d) with any method you prefer, and then find  $i_c(t)$ .

### 3. Op. Amp. Circuits

In this question, we'll be looking at op. amp. circuits using the phasor domain, and how different components in the feedback path affect the output. For this question, assume  $V_s$  is a sinusoidal input.

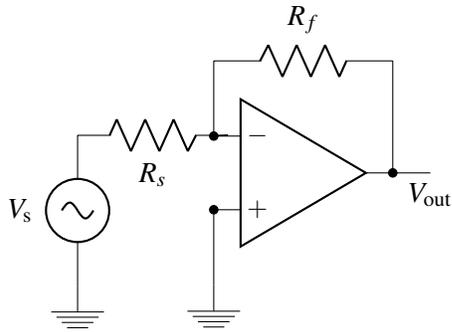


Figure 1: Circuit for part (a)

(a) Find the transfer function  $H(\omega) = \frac{\tilde{V}_{out}}{\tilde{V}_s}$  for Figure 1.

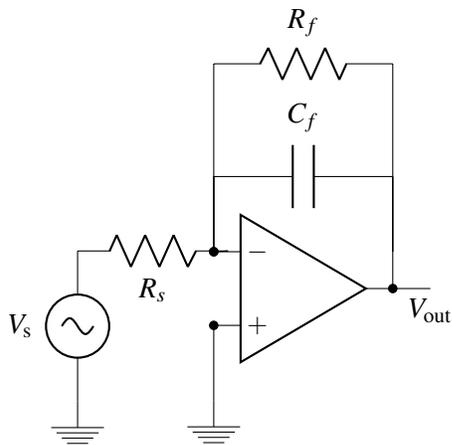


Figure 2: Circuit for part (b) and (c)

(b) Now we add a capacitor in parallel with  $R_f$ , seen in Figure 2. Find the new transfer function for this circuit.

(c) What type of filter is the circuit in Figure 2?

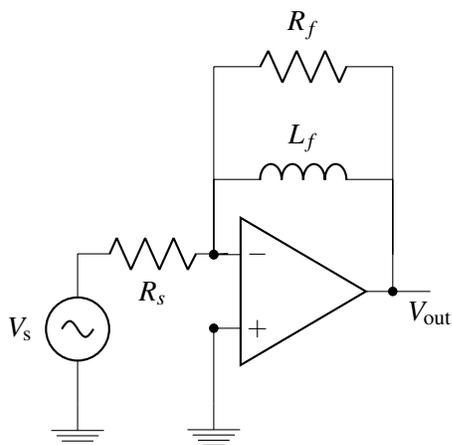


Figure 3: Circuit for part (d) and (e)

- (d) Now instead of a capacitor, we add an inductor in parallel with  $R_f$ , seen in Figure 3. What is the new transfer function?
- (e) What type of filter is the circuit in Figure 3?

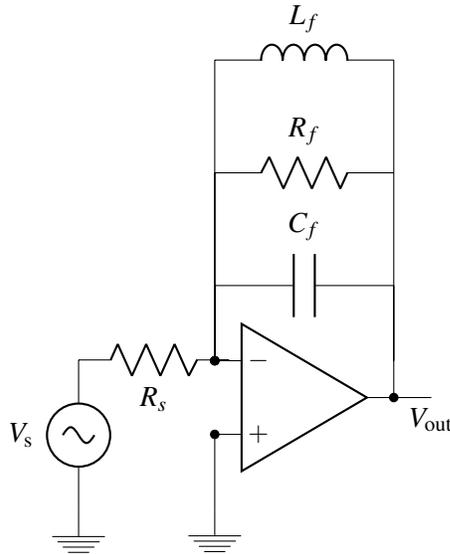


Figure 4: Circuit for part (f) and (g)

- (f) Now we have both a capacitor and inductor in parallel with  $R_f$ , seen in Figure 4. Find the new transfer function.
- (g) What type of filter is the circuit in Figure 4?

#### 4. Mystery microphone

You are working for APPLE Microphone Corporation<sup>1</sup> when your manager asks you to test a batch of the company's new microphones. You grab one of the new microphones off the shelf, play a uniform tone with varying frequencies, and measure the resultant peak-to-peak voltages using an oscilloscope. In order to play a uniform tone, you used a tone generator which outputs an audio wave of uniform amplitude for all frequencies involved. Below is the data obtained from your experiments:

<sup>1</sup>Advanced Powerful Pleasant Lovely Experiences Microphone Corporation

Input frequency (Hz)	Output peak-to-peak (V)
10	0.4
20	0.5
40	0.5
60	0.6
100	2.2
160	2.3
320	2.4
640	2.5
1200	5
2500	5
5000	5
10000	4.9
12000	1.6
16000	1.4
20000	1.5

- (a) Plot the output peak-to-peak voltage against the input frequency in log scale.
- (b) What do you notice? To what frequencies is the microphone most sensitive, and to what frequencies is the microphone least sensitive?

You report these findings to your manager, who thanks you for the preliminary data and proceeds to co-ordinate some human listener tests. In the meantime, your manager asks you to predict the effects of the microphone recordings on human listeners, and encourages you to start thinking more deeply about the relationships.

- (c) For testing purposes, you have a song with sub-bass (150 Hz or less), mid-range (1 KHz), and some high frequency electronic parts (> 12 KHz). Which frequency ranges of the song would you be able to hear easily, and which parts would you have trouble hearing? Why?

After a few weeks, your manager reports back to you on the findings. Apparently, this microphone causes some people's voices to sound really weird, resulting in users threatening to switch to products from a competing microphone company.

- (d) It turns out that we can design some filters to "fix" the frequency response so that the different frequencies can be recorded more equally, thus avoiding distortion. Imagine that you have a few (say up to 5 or so) blocks. Each of these blocks detects a set range of frequencies, and if the signal is within this range, it will switch on a op-amp circuit of your choice. For example, it can be configured to switch on an op-amp filter to double the voltage for signals between 100 Hz and 200 Hz.

What ranges of signals would require such a block, and what gain would you apply to each block such that the resulting peak-to-peak voltage is about 5V for all frequencies?

Next week in EE16B, you will learn more about frequency response and filter design which will enable you to realize these kinds of filters, so stay tuned!

### Contributors:

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