

**This homework is due on Monday, September 30, 2019, at 11:59PM.
 Self-grades are due on Wednesday, October 2, 2019, at 11:59PM.**

1. Phasors

- (a) Consider a resistor ($R = 1.5\Omega$), a capacitor ($C = 1\text{F}$), and an inductor ($L = 1\text{H}$) connected in series. Give expressions for the impedances of Z_R, Z_C, Z_L for each of these elements as a function of the angular frequency ω .
- (b) Draw the individual impedances as “vectors” on the same complex plane for the case $\omega = \frac{1}{2}$ rad/sec. Also draw the combined impedance Z_{total} of their series combination. Give the magnitude and phase of Z_{total} . A logically sound graphical argument is sufficient justification.

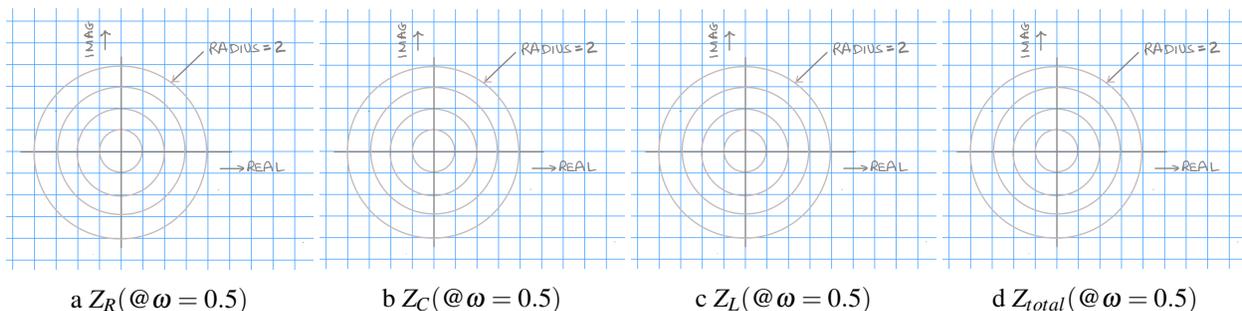


Figure 1: Impedances at $\omega = 0.5$.

- (c) Draw the individual impedances as “vectors” on the same complex plane for the case $\omega = 1$ rad/sec. Also draw the combined impedance Z_{total} of their series combination. Give the magnitude and phase of Z_{total} . A logically sound graphical argument is sufficient justification.

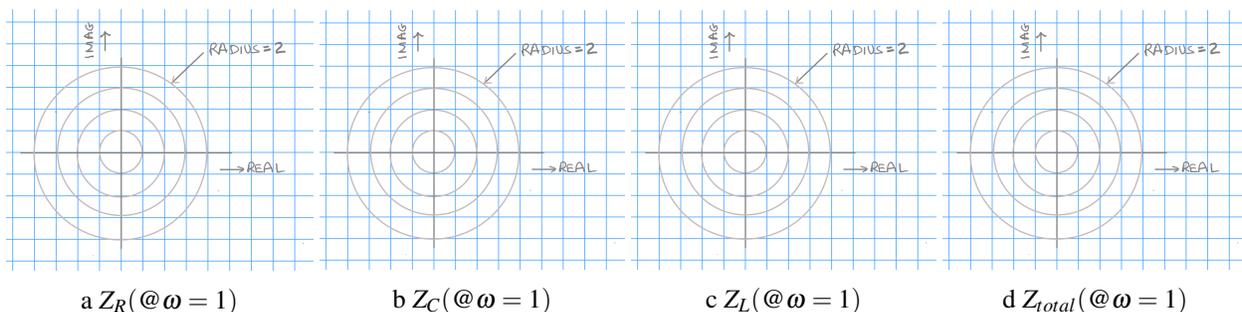


Figure 2: Impedances at $\omega = 1$.

- (d) Draw the individual impedances as “vectors” on the same complex plane for the case $\omega = 2$ rad/sec. Also draw the combined impedance Z_{total} of their series combination. Give the magnitude and phase of Z_{total} . A logically sound graphical argument is sufficient justification.

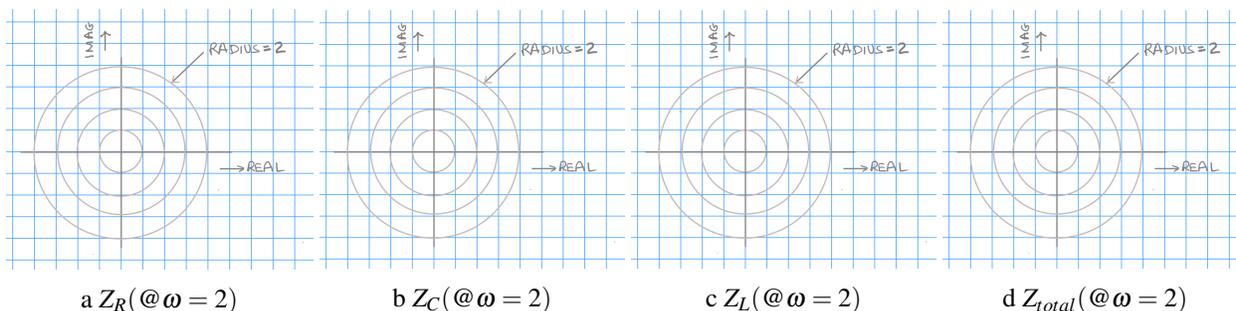


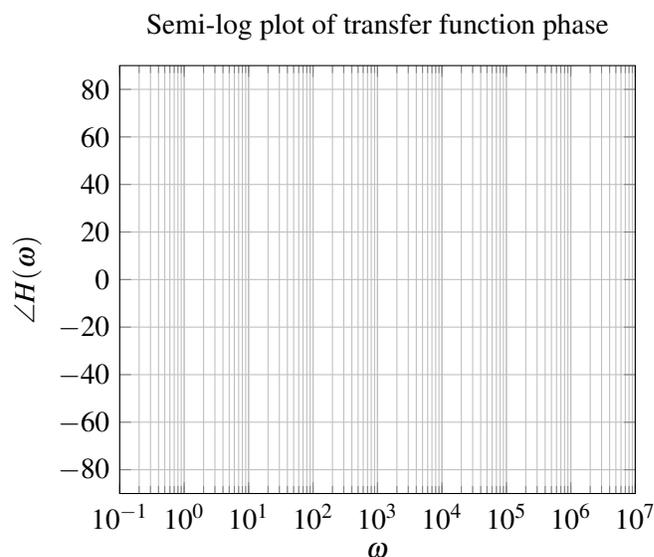
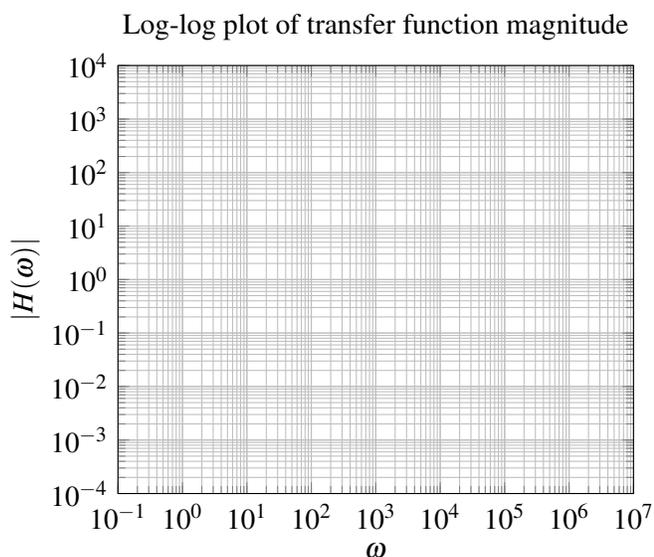
Figure 3: Impedances at $\omega = 2$.

(e) For the previous series combination of RLC elements, what is the “natural frequency” ω_n where the series impedance is purely real?

2. Low-pass Filter

You have a 1 kΩ resistor and a 1 μF capacitor wired up as a low-pass filter.

- (a) Draw the filter circuit, labeling the input node, output node, and ground.
- (b) Write down the transfer function of the filter, $H(j\omega)$ that relates the output voltage phasor to the input voltage phasor. Be sure to use the given values for the components.
- (c) Write an exact expression for the *magnitude* of $H(j\omega = j10^6)$, and give an approximate numerical answer.
- (d) Write an exact expression for the *phase* of $H(j\omega = j1)$, and give an approximate numerical answer.
- (e) Write down an expression for the time-domain output waveform $V_{out}(t)$ of this filter if the input voltage is $V(t) = 1 \sin(1000t)$ V. You can assume that any transients have died out — we are interested in the steady-state waveform.
- (f) Use a computer or calculator to help you sketch the Bode plot (both magnitude and phase) of the filter on the graph paper below.



3. Color Organ Filter Design

In the fourth lab, we will design low-pass, band-pass, and high-pass filters for a color organ. There are red, green, and blue LEDs. Each color will correspond to a specified frequency range of the input audio signal. The intensity of the light emitted will correspond to the amplitude of the audio signal.

- (a) First, you remember that you saw in lecture that you can build simple filters using a resistor and a capacitor. **Design simple first-order passive low- and high-pass filters with the following frequency ranges for each filter using 1 μF capacitors.** (“Passive” means that the filter does not require any power supply to operate on the input signal. Passive components include resistors, capacitors, inductors, diodes, etc., while an example of an active component would be an op-amp).

- Low-pass filter: cut-off frequency $f_c = 2400\text{Hz}$, $\omega_c = 2\pi \cdot 2400 \frac{\text{rad}}{\text{sec}}$
- High-pass filter: cut-off frequency $f_c = 100\text{Hz}$, $\omega_c = 2\pi \cdot 100 \frac{\text{rad}}{\text{sec}}$

Recall that the cutoff-frequency of such filters is just where the magnitude of the filter is $\frac{1}{\sqrt{2}}$ of its peak value.

Show your work to find the resistor values that create these low- and high-pass filters. Draw the schematic-level representation of your designs. Please mark V_{in} , V_{out} , and the ground node(s) in your schematic. Round your results to two significant figures.

- (b) You can try to build a bandpass filter by cascading the first-order low-pass and high-pass filters you designed in part (a). To do this, you might be tempted to connect the V_{out} node of your low-pass filter directly to the V_{in} node of your high-pass filter. If you did this, just as you saw in 16A for voltage dividers, the purported high-pass filter would “load” the low-pass filter and you might get some potentially complicated mess instead of what you wanted.

Show how you can use an ideal op-amp configured as a buffer to eliminate this loading effect to cascade the low-pass and high-pass filter, and write the resulting transfer function of the combined circuit.

4. Phasors and Eigenvalues

Suppose that we have the two-dimensional system of differential equations expressed in matrix/vector form:

$$\frac{d}{dt}\vec{x}(t) = A\vec{x}(t) + \vec{b}u(t) \quad (1)$$

where for this problem, the matrix A and the vector \vec{b} are both real.

- (a) **Give a necessary condition on the eigenvalues λ_i of A such that any impact of an initial condition will eventually completely die out.** (i.e. the system will reach steady-state.)

You don’t have to prove this. (We will see a detailed proof later in the course.) The idea here is to make sure that you understand what kind of thing is required.

- (b) Now assume that $u(t)$ has a phasor representation \tilde{U} . In other words, $u(t) = \tilde{U}e^{j\omega t} + \overline{\tilde{U}}e^{-j\omega t}$.

Assume that the vector solution $\vec{x}(t)$ to the system of differential equations (1) can also be written in phasor form as

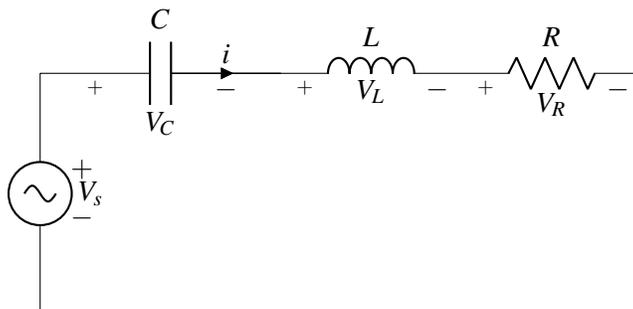
$$\vec{x}(t) = \tilde{X}e^{j\omega t} + \overline{\tilde{X}}e^{-j\omega t}. \quad (2)$$

Derive an expression for \tilde{X} involving $A, \vec{b}, j\omega, \tilde{U}$, and the identity matrix I .

(HINT: Plug (2) into (1) and simplify, using the rules of differentiation and grouping terms by which exponential $e^{\pm j\omega t}$ they multiply.)

5. RLC filter

Consider the following RLC circuit:



- (a) Write down the impedance of a series RLC circuit in the form $Z_{RLC}(\omega) = A(\omega) + jX(\omega)$, where $X(\omega)$ is a real valued function of ω .
- (b) Write the transfer function from V_S to V_R — the voltage drop across the resistor.
- (c) For the different specific values for R, L, C given by different cases (underdamped, overdamped, and critically damped) in the previous HW, use a computer to sketch plots of the magnitude of the transfer function above.
- (d) To see how the values of R, L, C impact the impedance at different frequencies, run the included Jupyter notebook **RLCSliders.ipynb**. The script will generate two plots, the transfer function of the circuit as a function of frequency and the location of the eigenvalues in the imaginary, real plane. **Explain what happens at the following sets of values, and why it is interesting:**

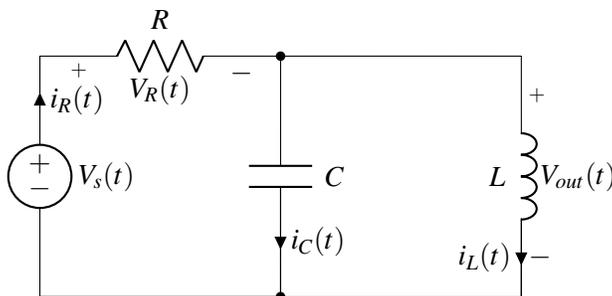
	R	L	C
I	1	2.5E-5	1E-8
II	10	2.5E-5	1E-8
III	10	2.5E-5	2E-9
IV	500	.0001	2E-8

Table 1: Values for RLC Bandwidth problem, part d

6. Phasor-Domain Circuit Analysis

The analysis techniques you learned previously in 16A for resistive circuits are equally applicable for analyzing circuits driven by sinusoidal inputs in the phasor domain. In this problem, we will walk you through the steps with a concrete example.

Consider the following circuit where the input voltage is sinusoidal. The end goal of our analysis is to find an equation for $V_{out}(t)$.



The components in this circuit are given by:

$$V_s(t) = 10\sqrt{2}\cos\left(100t - \frac{\pi}{4}\right)$$

$$R = 5\ \Omega$$

$$L = 50\ \text{mH}$$

$$C = 2\ \text{mF}$$

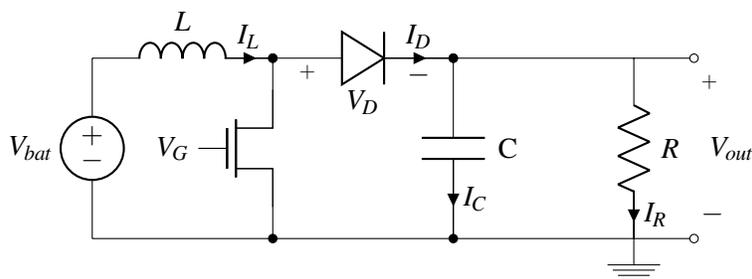
- Give the amplitude V_0 , oscillation frequency ω , and phase ϕ of the input voltage V_s .
- Transform the circuit into the phasor domain. What are the impedances of the resistor, capacitor, and inductor? What is the phasor \tilde{V}_s of the input voltage $V_s(t)$?
- Use the circuit equations to solve for \tilde{V}_{out} , the phasor representing the output voltage.
- Convert the phasor \tilde{V}_{out} back to get the time-domain signal $V_{out}(t)$.

7. Time-Domain: Boost Converter

Below is a circuit called a Boost Converter. The boost converter is a simple version of a circuit that is very common in battery-operated devices. While a single alkaline battery has a voltage that varies from 1.6 V when it is fresh to 0.8V at end of life, most electronic devices require a constant supply voltage that stays within $\pm 10\%$ of a certain nominal value. That nominal value is often greater than 1.6 V. So how do we power electronics with batteries? And in particular, how do we power electronics that wants 3 V or 5 V to function when our battery is providing significantly less than that.

The boost converter is a commonly used circuit to provide that stable supply voltage, even from a battery whose voltage is slowly decreasing as it discharges. With proper choice of components and proper control of the gate voltage V_G of the transistor, power from the battery can be converted to higher voltages with almost perfect efficiency.

Consider the following circuit:



with

$$V_{bat} = 1\ \text{V}$$

$$L = 1\ \mu\text{H}$$

$$C = 1\ \text{mF}$$

$$R = 1\ \text{k}\Omega$$

We model the electronic component that is consuming power as a resistor R with voltage V_{out} across it. Assume that the gate voltage at the NMOS is low (and thus the NMOS is an open circuit) for $t < 0$.

This circuit introduces a new nonlinear circuit element called a diode — this is also called a rectifier and its behavior is related to something that is very popular these days in the design of neural networks. The fact that the simple nonlinearity of the diode is powerful enough to do interesting computations is what allows neural networks based on ReLU units (basically diodes) to be useful.

An ideal diode allows current to flow when $V_D > 0$ and blocks current flows through an ideal diode when $V_D \leq 0$. A physical diode is a circuit element that allows current to flow when the voltage V_D is above a certain threshold voltage. (We approximate this threshold voltage as being zero for an ideal diode, but in reality, this threshold like behavior is actually the consequence of an underlying exponential behavior. Take EE130 to understand the very interesting physics that underlies this behavior.) That means, when the voltage is below the threshold, this circuit element behaves like an open circuit. This is very similar to how a transistor can behave as a switch, but the diode is directly switching current through it without having a separate gate controlling it.

In the robot car project, you will use the diode in a manner similar to this problem.

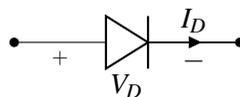


Figure 4: Diagram of a diode in isolation. The arrow denotes the direction that current is allowed to flow.

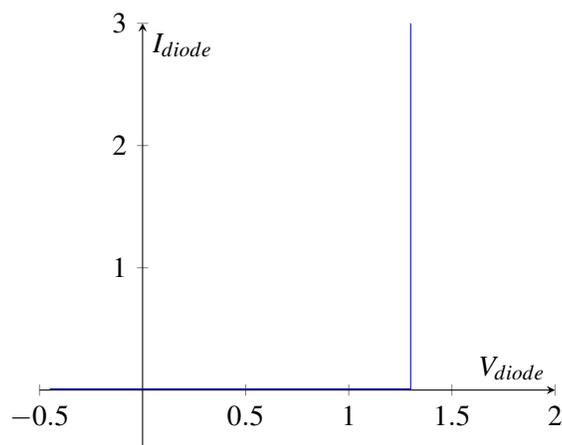


Figure 5: Basic current voltage response of a diode, which shows an impedance is 0 when on, and ∞ when off. (In this example, the threshold voltage is about 1.3V — most physical diodes are treated as having a threshold that is closer to 0.7V.)

- Assume that there is no current flowing through the diode — treat it like an open circuit. **Calculate the time constant of the RC circuit. How long will it take for the output voltage to decay by 10% from any non-zero value?** If the voltage on the capacitor is 3 V at $t = 0$, **what is the current in the resistor at time $t = 0$? What is the voltage in the capacitor for $t > 0$?**
- Let's assume that at $t = 0$, the voltage on the capacitor is 3 V. At $t = 0$, the gate voltage on the NMOS V_G goes high, turning the transistor on (effectively shorting that circuit path). **What is the rate of change of current in the inductor? How long does it take for the current in the inductor to increase to 100mA?**

Don't be scared by the fact that the voltage across an inductor is permitted to change discontinuously.

- (c) Based on your two previous answers, **how much has the capacitor voltage decayed during this time?** Your answer should be such a small number that in the following parts, we will consider the output voltage constant.
- (d) When the current in the inductor hits 100 mA, the transistor gate goes low, turning the transistor off. The energy stored in the magnetic field in the inductor quickly causes the voltage on the positive side of the diode to increase to 3 V, turning the ideal diode on (to understand why, think about what a rapid decrease in current will do to the inductor voltage when the transistor turns off). **What is the rate of change of current in the inductor when the diode is on? How long does it take the current in the inductor to go to zero?**
- (e) **Sketch the following currents vs time: i_L , i_{NMOS} , i_{diode} for a single inductor charge/discharge cycle described above.**
If you like circuits like this, take EE113 Power Electronics.
- (f) To maintain V_{out} at 3V, we need the average current through the diode to bring enough charge to the capacitor. **Calculate the charge that flows through the diode in a single cycle. At what frequency does the transistor need to cycle through the on/off phases you analyzed above in order to supply the current consumed by the resistor, computed in part (a)?**

8. Write Your Own Question And Provide a Thorough Solution.

Writing your own problems is a very important way to really learn material. The famous “Bloom’s Taxonomy” that lists the levels of learning is: Remember, Understand, Apply, Analyze, Evaluate, and Create. Using what you know to create is the top level. We rarely ask you any homework questions about the lowest level of straight-up remembering, expecting you to be able to do that yourself (e.g. making flashcards). But we don’t want the same to be true about the highest level. As a practical matter, having some practice at trying to create problems helps you study for exams much better than simply counting on solving existing practice problems. This is because thinking about how to create an interesting problem forces you to really look at the material from the perspective of those who are going to create the exams. Besides, this is fun. If you want to make a boring problem, go ahead. That is your prerogative. But it is more fun to really engage with the material, discover something interesting, and then come up with a problem that walks others down a journey that lets them share your discovery. You don’t have to achieve this every week. But unless you try every week, it probably won’t ever happen.

9. Homework Process and Study Group

Citing sources and collaborators are an important part of life, including being a student! We also want to understand what resources you find helpful and how much time homework is taking, so we can change things in the future if possible.

- (a) **What sources (if any) did you use as you worked through the homework?**
- (b) **Who did you work on this homework with?** List names and student ID’s. (In case of homework party, you can also just describe the group.)
- (c) **How did you work on this homework?** (For example, *I first worked by myself for 2 hours, but got stuck on problem 3, so I went to office hours. Then I went to homework party for a few hours, where I finished the homework.*)
- (d) **Roughly how many total hours did you work on this homework?**

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