This homework is due on Thursday, September 3, 2020, at 10:59PM.
Self-grades are due on Thursday, September 10, 2020, at 10:59PM.

1 16A Final Redo

a) Redo Fall 2019's EECS 16A final (attached).

2 Homework Process and Study Group

Citing sources and collaborators are an important part of life, including being a student!
We also want to understand what resources you find helpful and how much time homework is taking, so we can change things in the future if possible.

a) **What sources (if any) did you use as you worked through the homework?**

b) **If you worked with someone on this homework, who did you work with?**
List names and student ID's. (In case of homework party, you can also just describe the group.)

c) **Roughly how many total hours did you work on this homework?**

d) **Do you have any feedback on this homework assignment?**

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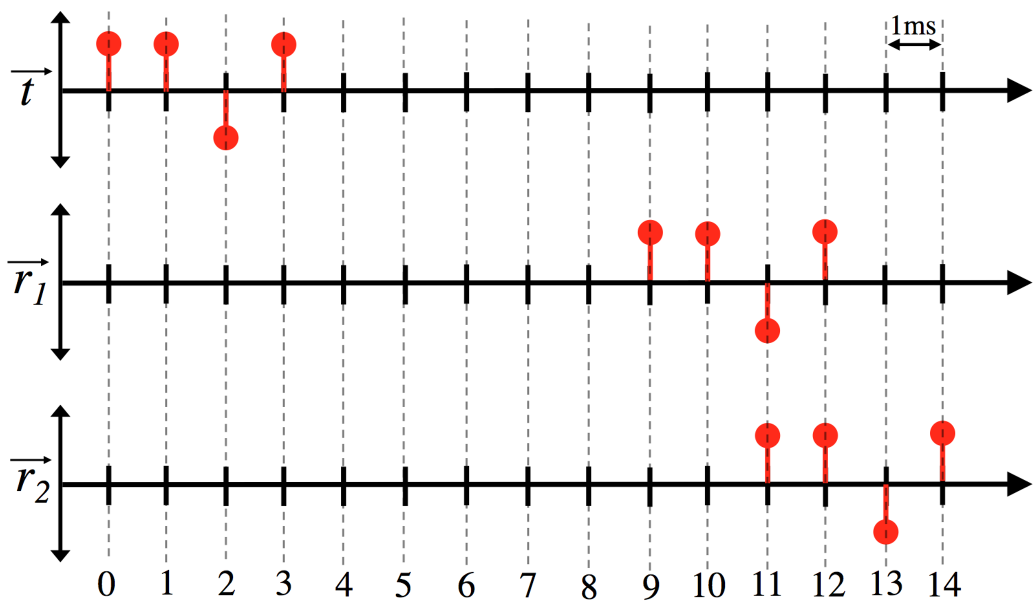
3. Where is the sound coming from? (14 points) (All subparts of this problem can be solved independently.)

In this problem we will use concepts from the class to determine the angle and position of an incoming audio signal recorded by microphones. All of the microphones and transmitting sources are in the same plane (i.e. the problem is in 2D).

(a) (4 points) A transmitter sends the signal $\vec{t}$. This signal is received by microphone 1 as $\vec{r}_1$ and by microphone 2 as $\vec{r}_2$. The microphones record 1 sample every millisecond ($1ms = 10^{-3}$ s) and the speed of sound is $v_s = 300m/s$. **How far are microphone 1 and microphone 2 from the transmitter?**

Hint: You do not have to do cross-correlation to solve this question.



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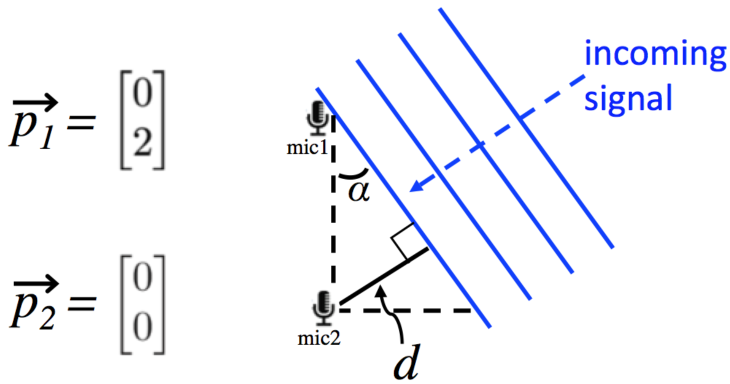
(b) (3 points) Microphone 1 and Microphone 2 receive signals $\vec{r}_1$ and $\vec{r}_2$ respectively from the transmitter. The time delay between $\vec{r}_1$ and $\vec{r}_2$ is related to the perpendicular distance d between Microphone 2 and the incoming signal $\vec{t}$. **You have measured that** $d = 1m$ (i.e. the distance between Microphone 2 and the incoming signal $\vec{t}$) in the figure below. The positions $\vec{p}_1 = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$ and $\vec{p}_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ of Microphones 1 and 2, respectively, are shown below. The units are in meters.

What is the angle of arrival α (see the figure below) between the incoming signal $\vec{t}$ and the line joining the microphones? You may leave your answer in terms of a trigonometric function.



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(c) (7 points) **Now you are considering a different setup of the microphones from earlier parts.** You have placed four microphones at $\vec{p}_1$, $\vec{p}_2$, $\vec{p}_3$, and $\vec{p}_4$ in the plane (see figure). You want to determine the position of the transmitting source. You used cross correlation and determined that the distances d_i between the microphones and the transmitting source as follows:

$$\vec{p}_1 = \begin{bmatrix} 0 \\ 4 \end{bmatrix}, \quad d_1 = 1\text{m}$$

$$\vec{p}_2 = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \quad d_2 = \sqrt{5}\text{m}$$

$$\vec{p}_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad d_3 = \sqrt{10}\text{m}$$

$$\vec{p}_4 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad d_4 = \sqrt{17}\text{m}$$

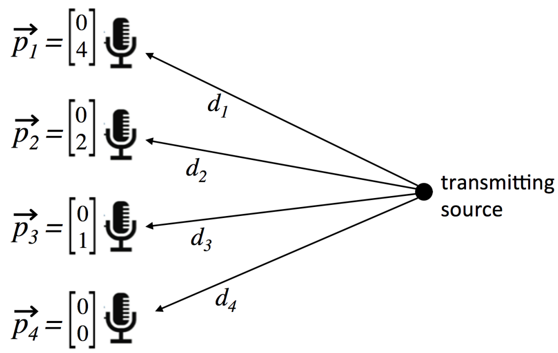

Figure 3.1: Note the microphone schematic is not drawn to scale.

Set up system of linear equations to compute the location of the transmitting source. If you can solve the system to identify the location of the source, solve it. If you cannot identify the location of the source, explain why. Then, propose a design/setup that would make the problem solvable, and explain why your design works.

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4. Building a classifier (22 points)

We used least squares to classify data in the "Labeling Patients Using Gene Expression Data" homework problem. We would like to now develop a classifier to classify points based on their distance from the origin.

You are presented with the following data. Each data point $\vec{d_i}^T = [x_i \, y_i]^T$ has the corresponding label $l_i \in \{-1, 1\}$.

x_i	y_i	l_i
-2	1	-1
-1	1	1
1	1	1
2	1	-1

Labels for data you are classifying

(a) (6 points) You want to build a model to understand the data. You first consider a linear model, i.e. you want to find $\alpha, \beta, \gamma \in \mathbb{R}$ such that $l_i \approx \alpha x_i + \beta y_i + \gamma$.
Set up a least squares problem to solve for α, β and γ. If this problem is solvable, solve it, i.e. find the best values for α, β, γ. If it is not solvable, justify why.

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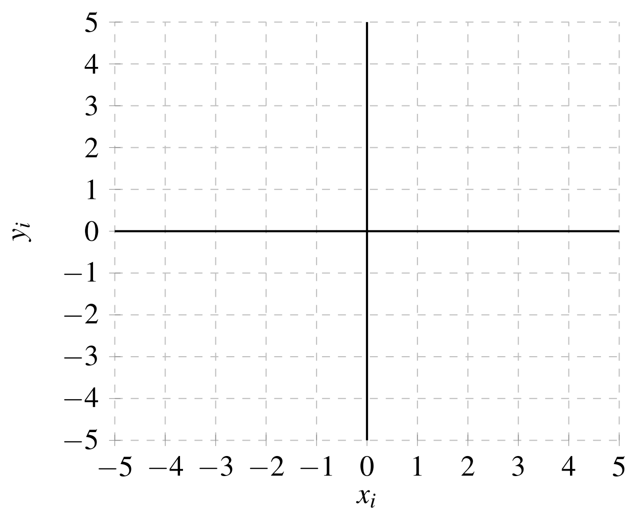
(b) (3 points) **Plot** the data points in the plot below with axes (x_i, y_i). **Is there a straight line such that the data points with a $+1$ label are on one side and data points with a -1 label are on the other side? Answer yes or no, and if yes, draw the line.**

x_i	y_i	l_i
-2	1	-1
-1	1	1
1	1	1
2	1	-1

Table repeated for your convenience: Labels for data you are classifying



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(c) (6 points) You now consider a model with a quadratic term: $l_i \approx \alpha x_i + \beta x_i^2$ with $\alpha, \beta \in \mathbb{R}$. *Read the equation carefully!*

Set up a least squares problem to fit the model to the data. If this problem is solvable, solve it, i.e, find the best values for α, β. If it is not solvable, justify why.

x_i	y_i	l_i
-2	1	-1
-1	1	1
1	1	1
2	1	-1

Table repeated for your convenience: Labels for data you are classifying

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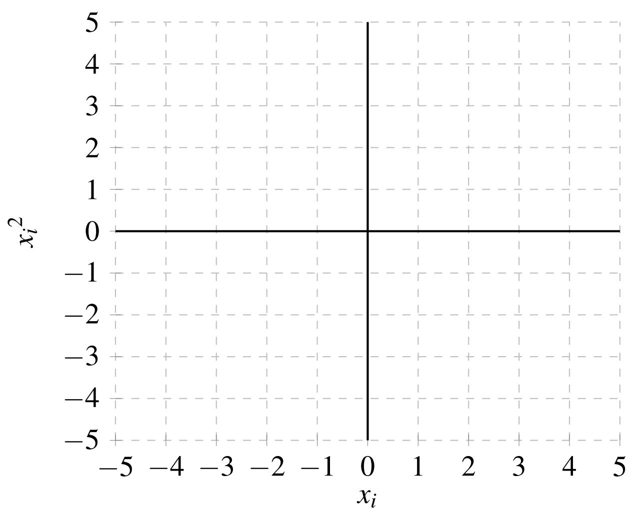
(d) (3 points) **Plot** the data points in the plot below with axes (x_i, x_i^2). **Is there a straight line such that the data points with a $+1$ label are on one side and data points with a -1 label are on the other side? Answer yes or no, and if yes, draw the line.**

x_i	y_i	l_i
-2	1	-1
-1	1	1
1	1	1
2	1	-1

Table repeated for your convenience: Labels for data you are classifying



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(e) (4 points) Finally you consider the model: $l_i \approx \alpha x_i + \beta x_i^2 + \gamma$, where $\alpha, \beta, \gamma \in \mathbb{R}$. Independent of the work you have done so far, **would you expect this model or the model in part (c) (i.e. $l_i \approx \alpha x_i + \beta x_i^2$) to have a smaller error in fitting the data? Explain why.**

x_i	y_i	l_i
-2	1	-1
-1	1	1
1	1	1
2	1	-1

Table repeated for your convenience: Labels for data you are classifying

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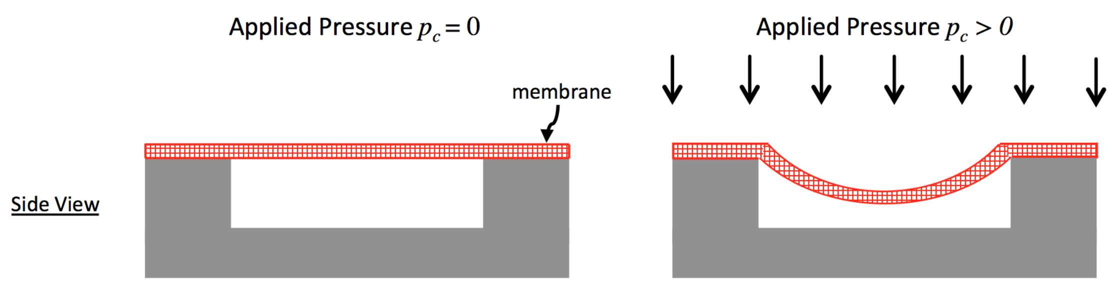
5. Putting on the Pressure: Build your own InstantPot (27 points) (All subparts of this problem can be solved independently.)

Prof. Ranade tells Prof. Boser about her great experience with her automatic pressure cooker, and they decide to try and build one together. The design of the pressure cooker uses a pressure sensor and heating element. Whenever the pressure is below a set target value, an electronic circuit turns on the heating element.

Pressure Sensor Resistance

The first step is designing a pressure sensor; the figure below shows your design. As pressure p_c is applied, the flexible membrane stretches.



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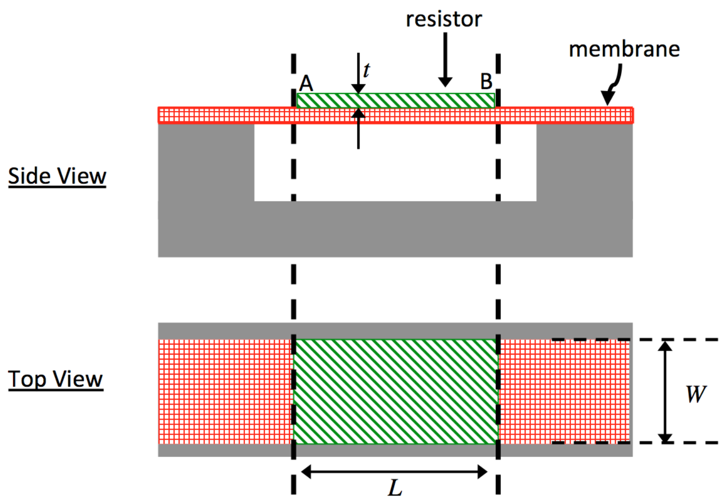
(a) (4 points) Now you attach a resistor layer R_p with resistivity $\rho = 0.1\ \Omega\text{m}$, width W, length L, and thickness t to the pressure sensor membrane, as illustrated in the figure below. When the pressure $p_c = 0kPa$ (i.e. there is no applied pressure), $W = 1mm$, $L = 1cm$, $t = 100\mu m = 100 \times 10^{-6}m$. **Calculate the value of R_p when there is no applied pressure.** Note that direction of current flow in the resistor is from A to B as marked in the diagram. Show all your work.



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(b) (5 points) When pressure is applied, the length L of the resistor changes and is a function of applied pressure p_c, and is given by

$$L(p_c) = L_0 + \beta p_c,$$

where L_0 is the length of the resistor with no pressure applied, and β is a constant. As a result, the value of resistance R_p changes from its nominal value R_{p0} (the value of R_p with no pressure applied) **Derive an expression for $R_p(p_c)$ as a function of resistivity ρ, width W, thickness t, nominal length L_0, constant β, and applied pressure p_c.**

Note: The width and thickness of the resistor will also change with applied pressure. However, we ignore this to keep the math simple.

Extra space for scratch work.

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(c) **Pressure Sensor Circuit** (6 points)

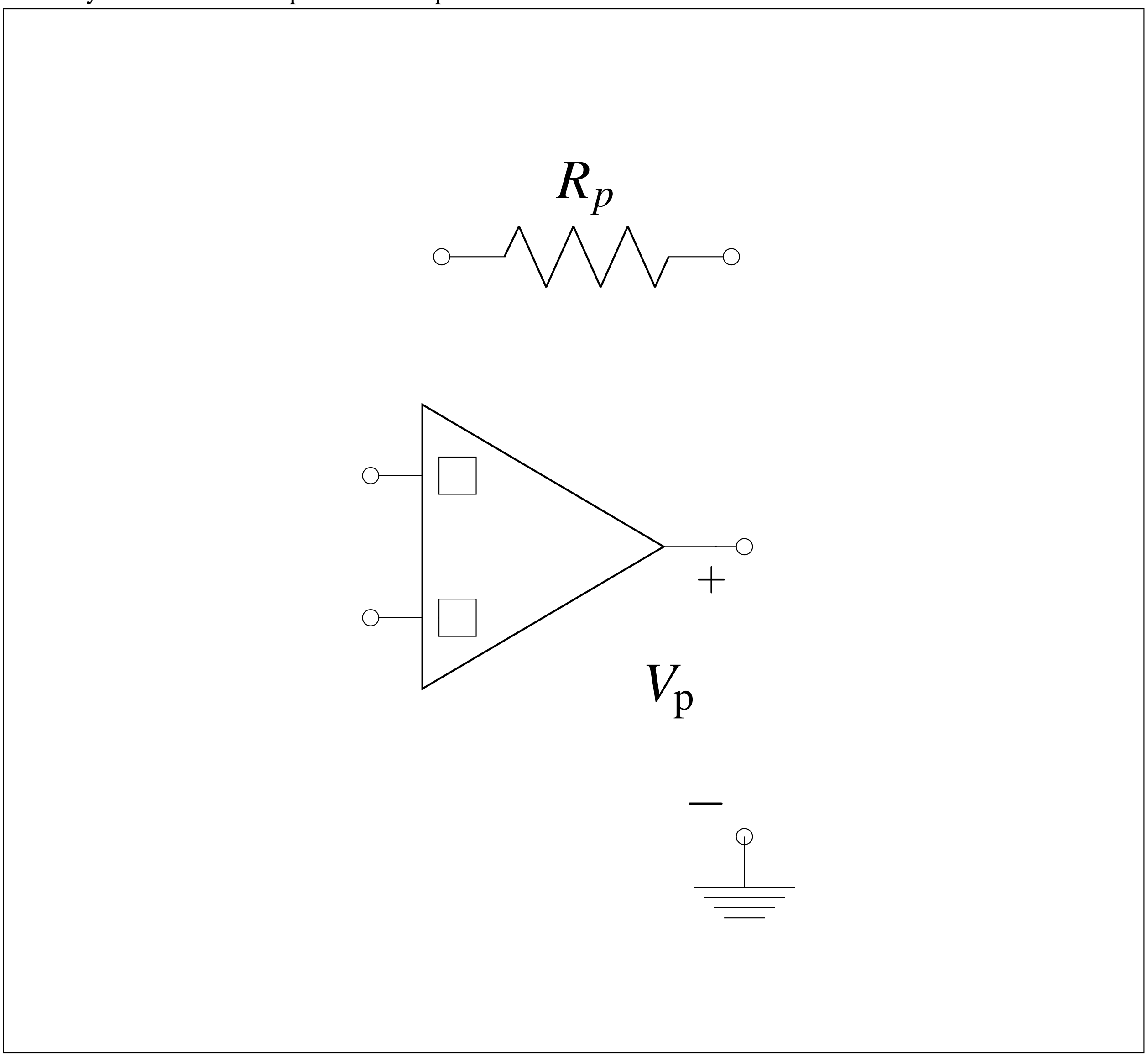
Now assume that the resistance R_p is a function of applied pressure p_c according to the relationship $R_p(p_c) = R_o \times \frac{p_c}{p_{ref}}$ where $R_o = 1\text{k}\Omega$ and $p_{ref} = 100\text{kPa}$.

To complete our sensor circuit, we would like to generate a voltage V_p that is a function of the pressure p_c. **Complete the circuit below so that the output voltage V_p depends on the pressure p_c as:**

$$V_p(p_c) = -V_o \times \frac{p_c}{p_{ref}}, \text{ where } V_o = 1V.$$

- **You may add at most one ideal voltage source and one additional resistor to the circuit, but you must calculate their values and mark them in the diagram.**
- **Mark the positive and negative inputs of the operational amplifier with "+" and "-" symbols, respectively, in the boxes provided.**

You may assume that the operational amplifier is ideal.



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(d) **Resistive Heating Element** (4 points)

To heat the pressure cooker, you use a heating element with resistance R_{heat}. **Calculate the value of R_{heat} such that the power dissipated is $P_{heat} = 1000W$ with $V_{heat} = 100V$ applied across the heating element.**

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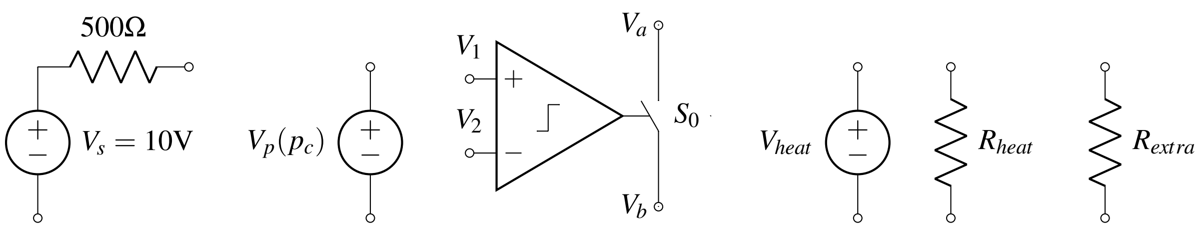
(e) **Pressure Regulation** (8 points)

You are finally ready to complete the design of your pressure cooker.

Using the circuit elements below, make a circuit that will turn the heater to on (i.e. current flowing through R_{heat}) when the pressure is less than 500 kPa, and off (i.e. no current flowing through R_{heat}) when the pressure is greater than 500 kPa.

The elements are:

- A voltage source $V_s = 10$V with a Thevenin resistance of 500Ω.
- A voltage source $V_p(p_c) = V_o \times \frac{p_c}{p_{ref}}$, with V_o = 1V and $p_{ref} = 100$kPa. (This is a voltage source whose voltage is a function of pressure p_c, unrelated to any previous parts of the question.)
- A comparator that controls switch S_0. The switch is normally opened (i.e. an open circuit between nodes V_a and V_b), and is closed only when $V_1 > V_2$ (i.e. a short circuit between nodes V_a and V_b).
- The heater supply ($V_{heat} = 100$V).
- The heater resistor R_{heat}.
- One additional resistor R_{extra}. If you use this resistor you must calculate and note its value on your circuit diagram.
- You may assume you have access to a ground node.



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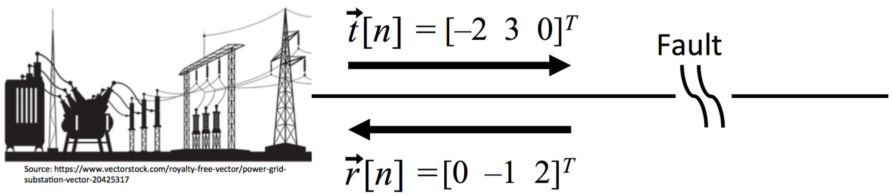
6. Finding faults with PG&E (16 points) (All subparts of this problem can be solved independently.)

PG&E has been having problems with the grid, and needs to more accurately locate faults in their transmission lines. EECS 16A students decide to use their new design skills to help.

All the transmission lines are connected to substations. To find a fault, a substation sends a signal $\vec{t}$ down a transmission line. If the line has a break in it, then a reflection occurs and the substation receives back a signal $\vec{r}$.

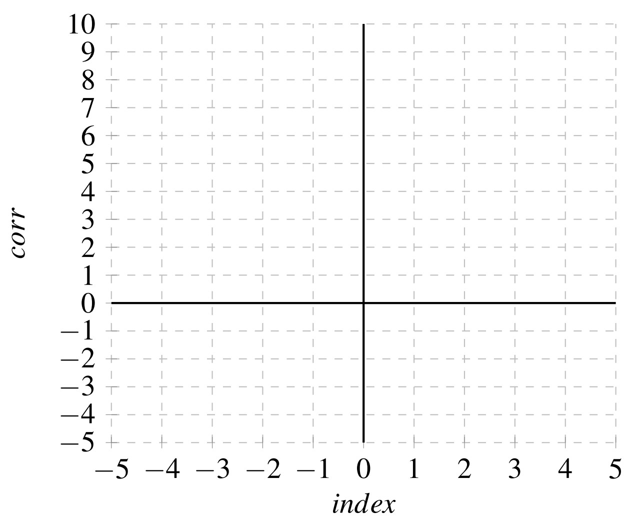


(a) (6 points) Assume that a substation sends the signal $\vec{t}$ and receives back a signal $\vec{r}$. The received signal $\vec{r}$ is a delayed, scaled version of $\vec{t}$ with added noise.

$$\vec{t}[n] = \begin{bmatrix} -2 & 3 & 0 \end{bmatrix}^T \tag{1}$$

$$\vec{r}[n] = \begin{bmatrix} 0 & -1 & 2 \end{bmatrix}^T \tag{2}$$

Use the axes below to plot the cross-correlation $corr_{\vec{r}}(\vec{t})$, and use this to identify the index corresponding to the peak (maximum magnitude) of the cross-correlation.



Index of maximum cross-correlation magnitude:

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(b) (10 points) Now the substations send signals along the transmission lines to send information about the state of the power grid. **Each substation has a unique code that it uses to transmit its information, $\vec{s}_1$, $\vec{s}_2$, $\vec{s}_3$, and $\vec{s}_4$. You receive the signal $\vec{r}[n] = \begin{bmatrix} 1 & 2 & 1 & 2 & 1 \end{bmatrix}^T$ and you know it contains signals from two different substations.**

Since the locations of the various substations are known, we can compute the delay with which the signals are received. The delays corresponding to the max correlation of $\vec{s}_1$, $\vec{s}_2$, $\vec{s}_3$, and $\vec{s}_4$ are: 1 unit, 2 units, 1 unit, and 2 units respectively. **We have provided shifted versions of the signals** $(\vec{u}_1[n], \vec{u}_2[n], \vec{u}_3[n], \vec{u}_4[n])$ that correspond to these distances, and included appropriate zeros to the signals **to make your calculations easier.**

$$\vec{s}_1[n] = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T \xrightarrow{\text{delayed by 1}} \vec{u}_1[n] = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \end{bmatrix}^T, \tag{3}$$

$$\vec{s}_2[n] = \begin{bmatrix} 1 & -1 & 1 \end{bmatrix}^T \xrightarrow{\text{delayed by 2}} \vec{u}_2[n] = \begin{bmatrix} 0 & 0 & 1 & -1 & 1 \end{bmatrix}^T, \tag{4}$$

$$\vec{s}_3[n] = \begin{bmatrix} 1 & 1 & -1 \end{bmatrix}^T \xrightarrow{\text{delayed by 1}} \vec{u}_3[n] = \begin{bmatrix} 0 & 1 & 1 & -1 & 0 \end{bmatrix}^T, \tag{5}$$

$$\vec{s}_4[n] = \begin{bmatrix} -1 & -1 & 1 \end{bmatrix}^T \xrightarrow{\text{delayed by 2}} \vec{u}_4[n] = \begin{bmatrix} 0 & 0 & -1 & -1 & 1 \end{bmatrix}^T. \tag{6}$$

Determine which two unique signals are contained in the received signal $\vec{r}[n]$. What are the weights on the two signals? Show all of your work.

Some calculations that might be useful:

$<\vec{r}[n], \vec{u}_1[n]>= 5$ $<\vec{r}[n], \vec{u}_2[n]>= 0$ $<\vec{r}[n], \vec{u}_3[n]>= 1$ $<\vec{r}[n], \vec{u}_4[n]>= -2$	$<\vec{u}_1[n], \vec{u}_2[n]>= 0$ $<\vec{u}_1[n], \vec{u}_3[n]>= 1$ $<\vec{u}_1[n], \vec{u}_4[n]>= -2$	$<\vec{u}_2[n], \vec{u}_3[n]>= 2$ $<\vec{u}_2[n], \vec{u}_4[n]>= 1$	$<\vec{u}_3[n], \vec{u}_4[n]>= 0$

This might also help:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \tag{7}$$

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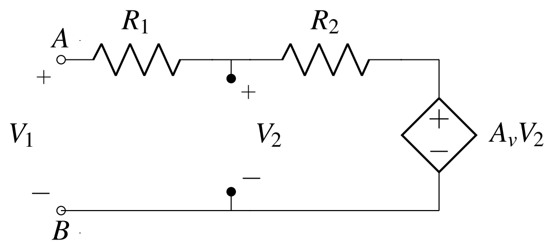
7. Fun With Circuits (14 points) (All subparts of this problem can be solved independently.)

In his spare time Professor Boser invents new circuits. The circuit schematic below shows his latest creation that uses a voltage controlled voltage source.

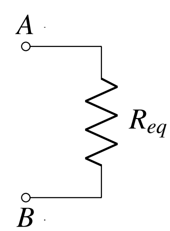


(a) **Equivalent Resistance** (8 points)

Analyze the circuit above and model it with an equivalent resistance R_{eq} as illustrated below. The I/V curves of the original and equivalent circuit should be identical. Use the following values: $A_v = 3$, $R_1 = 1\ k\Omega$, and $R_2 = 4\ k\Omega$. Show your calculations.

Recall that a resistor is just a model of the IV dependence of a circuit element; you may get a positive or a negative answer for your equivalent resistance.

Hint: Apply a test voltage (or current) and compute the resulting current (or voltage).



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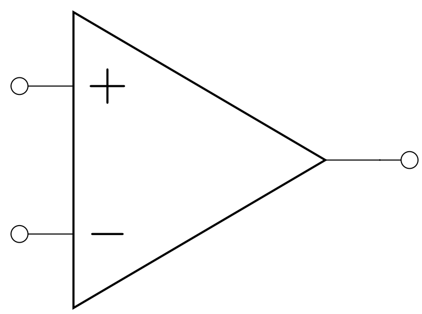
(b) **Amplifier Design** (6 points) This part is independent of the previous part.
Design a circuit which implements a voltage controlled voltage source with gain $A_v = 3$, i.e. $V_{out} = 3V_{in}$. Use the ideal operational amplifier shown below and up to three additional 1 kΩ resistors. Label V_{in} and V_{out}.



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8. Projections and eigenvectors (32 points) (All subparts of this problem can be solved independently.)

Consider two n-dimensional vectors $\vec{x} \in \mathbb{R}^n$ and $\vec{y} \in \mathbb{R}^n$. Consider the matrix $\mathbf{M} = \vec{x}\,\vec{y}^T$. **Note the order of the multiplication, this is distinct from** $<\vec{x}, \vec{y}> = \vec{x}^T\vec{y}$.

(a) (2 points) What are the dimensions of matrix $\mathbf{M}$?

(b) (6 points) Let $\vec{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\vec{y} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$. **Find the eigenvalues and eigenvectors of M.** Show your work.

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(c) (6 points) Now let $\vec{x} \in \mathbb{R}^n$ and $\vec{y} \in \mathbb{R}^n$. Let $\mathbf{M} = \vec{x}\,\vec{y}^T$. **Find the projection of $\vec{x}$ onto the columnspace of M.** Show all your work. *Hint: Write out the columns of* **M**.

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(d) (8 points) $\vec{x} \in \mathbb{R}^n$ and $\vec{y} \in \mathbb{R}^n$. Let $\mathbf{M} = \vec{x}\,\vec{y}^T$. Let $\vec{z} \in \mathbb{R}^n$ be a vector such that $< \vec{z}, \vec{y} >= 0$. **Prove that** $\vec{z} \in Null(\mathbf{M})$**.** You must prove this from first principles. No theorems can be used in the proof. Show all your work.

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(e) (10 points) $\vec{x} \in \mathbb{R}^n$ and $\vec{y} \in \mathbb{R}^n$. Let $\mathbf{M} = \vec{x}\,\vec{y}^T$. Further, assume that $< \vec{x}, \vec{y} > \neq 0$. **Find an eigenvector of M corresponding to a non-zero eigenvalue.** Show all your work and justify your answer.

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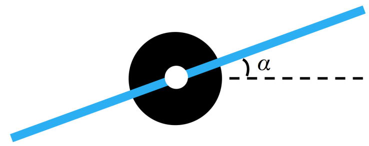
9. Electronic Level (24 points) (All subparts of this problem can be solved independently.)

A one-wheel scooter (as shown below) is fun to ride, but unfortunately quite expensive to buy. Inspired by the success of the pressure cooker, you decide to build your own.



One challenge of building a one-wheel scooter is designing a circuit that measures the angle α of the riding platform, as shown above. You are considering the design of an angle sensor consisting of two capacitors C_a and C_b whose values depend on the angle α (measured in degrees) as follows:

$$C_a(\alpha) = C_o\left(100 + \frac{\alpha}{\alpha_{ref}}\right), \tag{8}$$

$$C_b(\alpha) = C_o\left(100 - \frac{\alpha}{\alpha_{ref}}\right), \tag{9}$$

where $\alpha_{ref} \neq 0$ and C_o are properties of the sensor.

The circuit in Fig. 9.1 produces a voltage $V_{out}(k)$ at time k that depends on α through the capacitances $C_a(\alpha), C_b(\alpha)$. The timing diagram (Fig. 9.2) shows the state of the switches as a function of time k. **At time** $k = 0$ **all of the capacitors are completely discharged, meaning that** $\mathbf{V_{out}(0) = 0V}$**.**

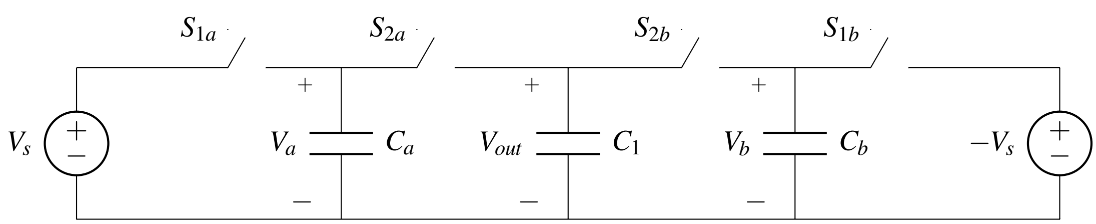


Figure 9.1: The circuit for the electronic level

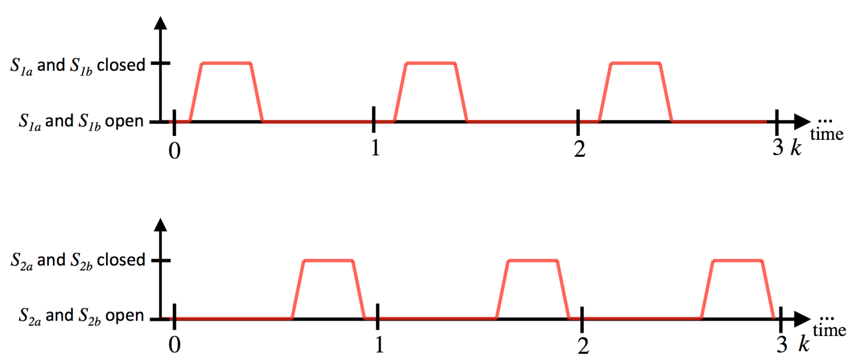


Figure 9.2: The timing diagram for the circuit

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(a) (5 points) We define $Q_a = C_a V_a$, $Q_b = C_b V_b$ and $Q_1 = C_1 V_{out}$, per the labels in Figure 9.1. **Find an expression for the total charge on the capacitors $Q_{tot}(k) = Q_a + Q_b + Q_1$ at time $k = 1$ as a function of α, α_{ref} C_o, C_1, and V_s.** At time $k = 1$, all switches have been closed and opened once and are now in the open state. Show all your work.

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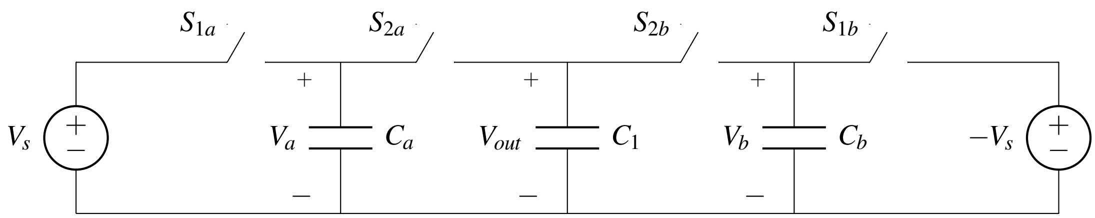

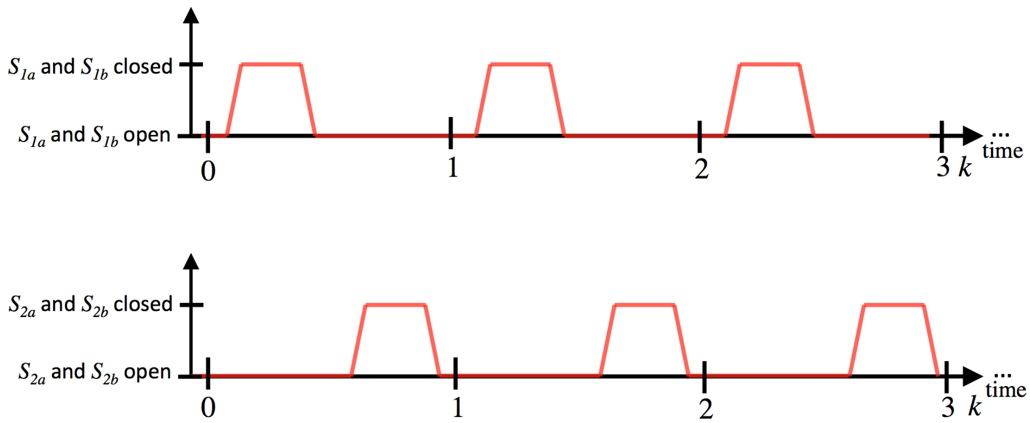

The same circuit and timing diagram from earlier, reproduced for your convenience.

(b) (5 points) **Calculate** $V_{out}(1)$ **as a function of** $Q_{tot}(1)$**,** C_o**, and** C_1**.** Show your work.

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(c) (8 points) **Find an expression for $V_{out}(k)$ as a function of $V_{out}(k-1)$, α, α_{ref}, C_o, C_1, and V_s.** It is important to have an exact formula. Vague answers will receive no credit. Show your work. *Hint: You might try to first find $V_{out}(2)$ as a function of $V_{out}(1)$.*

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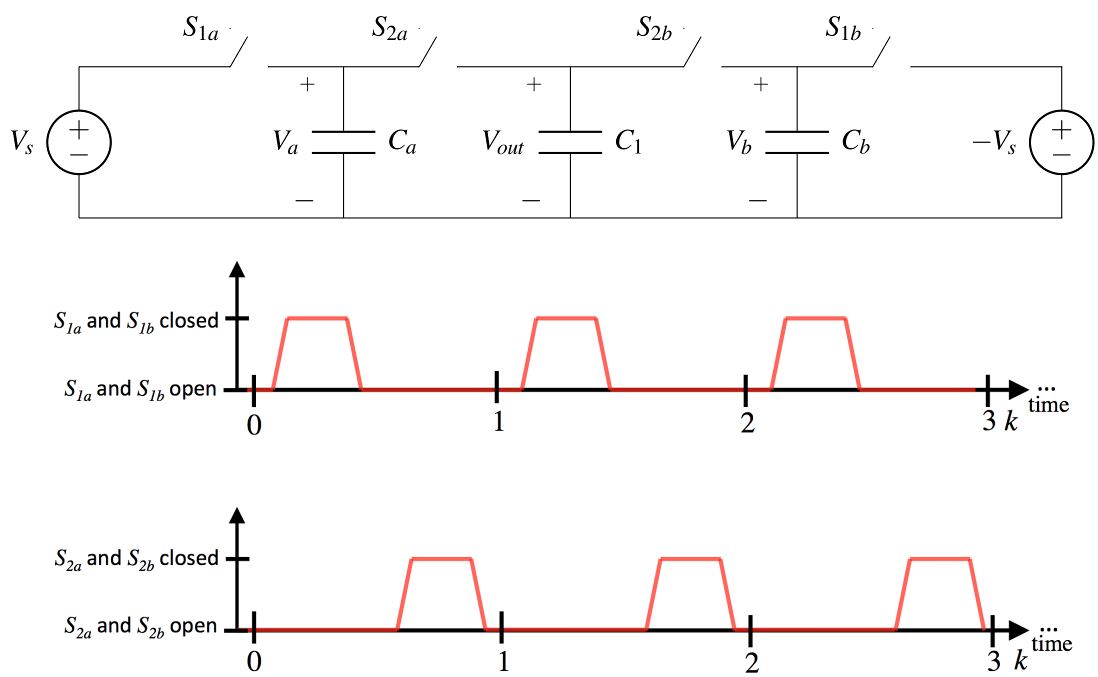

The same circuit and timing diagram from earlier, reproduced for your convenience.

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(d) (6 points) It turns out that the behavior of this circuit is exactly like the linear control systems you modeled in Module 1, and can be analyzed similarly.

Assume

$$V_{out}(k) = \gamma V_{out}(k-1) + \beta.$$

Both $\gamma, \beta \in \mathbb{R}$, and $0 < \gamma < 1$. Recall $V_{out}(0) = 0$. **What does V_{out} converge to as $k \to \infty$, i.e., what is $V_{out}(\infty)$?** Your answer should be in terms of β, γ and numbers. Show your work. (*Hint: The infinite series* $\sum_{i=0}^{\infty} x^i = \frac{1}{1-x}$ *if* $0 < x < 1$.)

PRINT your student ID: ____

Extra page for scratchwork.
Work on this page will NOT be graded.

PRINT your student ID: ____

Extra page for scratchwork.
Work on this page will NOT be graded.

PRINT your student ID: ____

Doodle page!
Draw us something if you want or give us suggestions, compliments, or complaints.
You can also use this page to report anything suspicious that you might have noticed.

EECS 16A Designing Information Devices and Systems I
Fall 2019 Final Exam Instructions

Read the following instructions before the exam.

There are 9 problems of varying numbers of points. Not all subparts of a question are related to each other. You have 180 minutes for the exam. The problems are of varying difficulty, so pace yourself accordingly and avoid spending too much time on any one question until you have gotten all of the other points you can.

There are 40 pages on the exam, so there should be 20 sheets of paper in the exam. The exam is printed double-sided. Do not forget the problems on the back sides of the pages! Notify a proctor immediately if a page is missing. **Do not tear out or remove any of the pages. Do not remove the exam from the exam room.**

No collaboration is allowed, and do not attempt to cheat in any way. Cheating will not be tolerated.

Write your student ID on each page before time is called. If a page is found without a student ID, we are not responsible for identifying the student who wrote that page.

You may consult ONE handwritten $8.5'' \times 11''$ note sheet (front and back). No phones, calculators, tablets, computers, other electronic devices, or scratch paper are allowed.

Please write your answers legibly in the boxed spaces provided on the exam. The space provided should be adequate. **If you still run out of space, please use a blank page and clearly tell us in the original problem space where to look for your solution.**

Show all of your work in order to receive full credit. Partial credit will be given for substantial progress on each problem.

If you need to use the restrooms during the exam, bring your student ID card, your phone, and your exam to a proctor. You can collect them once you return from the restrooms.

Our advice to you: if you can't solve the problem, state and solve a simpler one that captures at least some of its essence. You might get some partial credit, and more importantly, you will perhaps find yourself on a path to the solution.

Good luck!

Do not turn this page until the proctor tells you to do so.