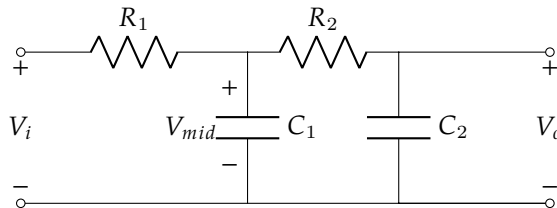**This homework is due on Thursday, October 8, 2020, at 10:59PM.**

Self-grades are due on Thursday, October 15, 2020, at 10:59PM.

1   Transfer functions

Consider the circuit below.

The circuit has an input phasor voltage V_i at frequency ω rad/sec applied at the input terminals shown in the illustration above, causing an output phasor voltage V_o at output terminals.

We are going to construct the transfer function $H(\omega) = \frac{V_o}{V_i}$ in two steps. We will compute two intermediate transfer functions, $H_1(\omega) = \frac{V_{mid}}{V_i}$ and $H_2(\omega) = \frac{V_o}{V_{mid}}$. Then, we will find the overall transfer function as the product of these two intermediate transfer functions, i.e. $H(\omega) = \frac{V_o}{V_i} = H_1(\omega)H_2(\omega)$.

This approach is valid since the V_{mid} cancel.

a) For the first step, **find the intermediate transfer function $H_2(\omega) = \frac{V_o}{V_{mid}}$.** Have your expression be in terms of Z_{R2} and Z_{C2}, that is the impedances of R_2 and C_2.

b) Now, **compute the other intermediate transfer function $H_1(\omega) = \frac{V_{mid}}{V_i}$.** Have your expression be in terms of Z_{R1}, Z_{R2}, Z_{C1}, and Z_{C2}. (i.e. Don't forget to consider the impact of loading by R_2 and C_2 in this transfer function.) *hint: Applying KCL at the V_{mid} node would be a good place to start. You should try to find an expression for H_1 that has factors that H_2 can cancel out.*

c) Then, **use these two intermediate transfer functions to calculate the overall transfer function** as $H(\omega) = \frac{V_o}{V_i} = H_1(\omega)H_2(\omega)$.

d) Sometimes it is useful to collect all the frequency dependence into one place and to figure out how to think about what scale might be somewhat natural for the frequency. **Obtain an expression for $H(\omega) = V_o/V_i$ in the form**

$$H(\omega) = \frac{V_o}{V_i} = \frac{1}{1 + 2\xi\dfrac{j\omega}{\omega_c} + \dfrac{(j\omega)^2}{\omega_c^2}},$$

given that $R_1 = 2\,\Omega$, $R_2 = 4\,\Omega$, $C_1 = \frac{9}{2}\,\mathrm{F}$, and $C_2 = 1\,\mathrm{F}$. What are the values of ξ and ω_c?

e) Use a computer to **draw Bode Plots of $|H(\omega)|$ and $\angle H(\omega)$.** What kind of filter (low-pass, high-pass, band-pass) is this? Comment on the slope of the log-log magnitude plot at low and high frequencies. Why would you use this instead of a simple RC filter?

2   Phasor-Domain Circuit Analysis

The analysis techniques you learned previously for resistive circuits are equally applicable for analyzing AC circuits (circuits driven by sinusoidal inputs) in the phasor domain. In this problem, we will walk you through the steps with a concrete example. Consider the circuit below.
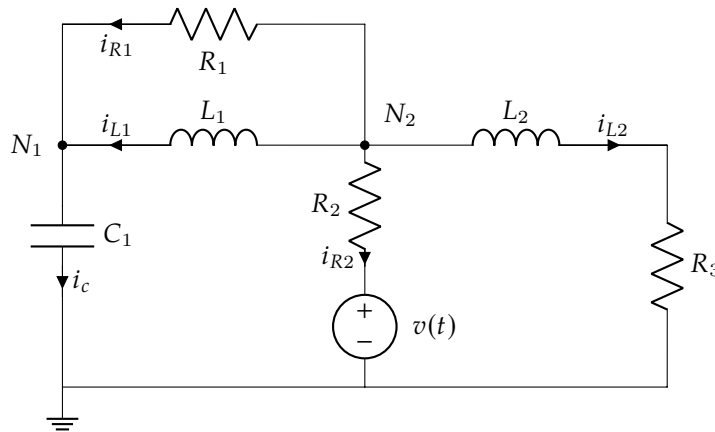
The components in this circuit are given by:
Voltage source:

$$v(t) = 10\sqrt{2} \cos\left(100t - \frac{\pi}{4}\right)$$

Resistors:

$$R_1 = 5\,\Omega, \quad R_2 = 5\,\Omega, \quad R_3 = 1\,\Omega$$

Inductors:

$$L_1 = 50\,\text{mH}, \quad L_2 = 20\,\text{mH}$$

Capacitor:

$$C_1 = 2\,\text{mF}$$

a) Transform the given circuit to the phasor domain (components and sources).

b) Write out KCL for node N_1 and N_2 in the phasor domain in terms of the currents provided.

c) Write the equations derived above in terms of the node voltages and the impedances. The node voltages V_1 and V_2 are the voltage drops from N_1 and N_2 to the ground.

d) Write the equations you derived in part (c) in a matrix form, i.e., $\mathbf{A} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \vec{b}$. Write out $\mathbf{A}$ and $\vec{b}$ numerically.

e) Solve the systems of linear equations you derived in part (d) with any method you prefer and then find $i_c(t)$.

3 RLC Circuit

In this question, we will take a look at an electrical system described by a second order differential equations and analyze it using the phasor domain. Consider the circuit below, where $R = 8\,\mathrm{k\Omega}$, $L = 1\,\mathrm{mH}$, $C = 200\,\mathrm{nF}$, and $v_\mathrm{s} = 2\cos\left(2000t + \frac{\pi}{4}\right)$.
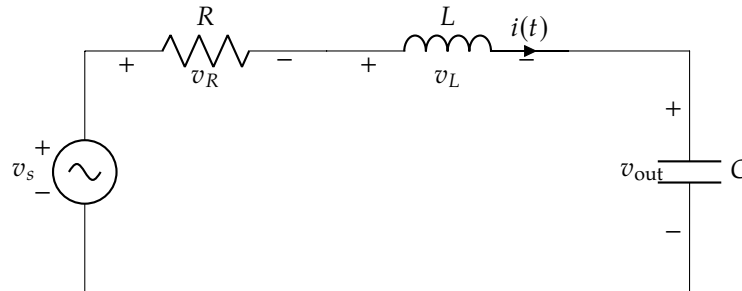
a) What are the impedances of the resistor Z_R, inductor Z_L, and capacitor Z_C?

b) Solve for V_out in phasor form.

c) What is V_out in the time domain?

d) Solve for the current $i(t)$.

e) Solve for the transfer function $H(\omega) = \frac{V_\mathrm{out}}{V_\mathrm{s}}$

   Leave your answer in terms of R, L, C, and ω.

f) Now assume $v_\mathrm{s}(t) = 2\cos\left(2000t + \frac{\pi}{4}\right) + \cos\left(1000t + \frac{\pi}{2}\right)$ and find the time domain output voltage, $v_{out}(t)$.

4 LC Tank

The chip below was designed in EE194 in Spring 2017. It is 1.1 mm on a side, and contains a 32 bit RISC-V microprocessor, 64kB RAM, and a 2.4 GHz Bluetooth Low Energy radio for communicating with cell phones.
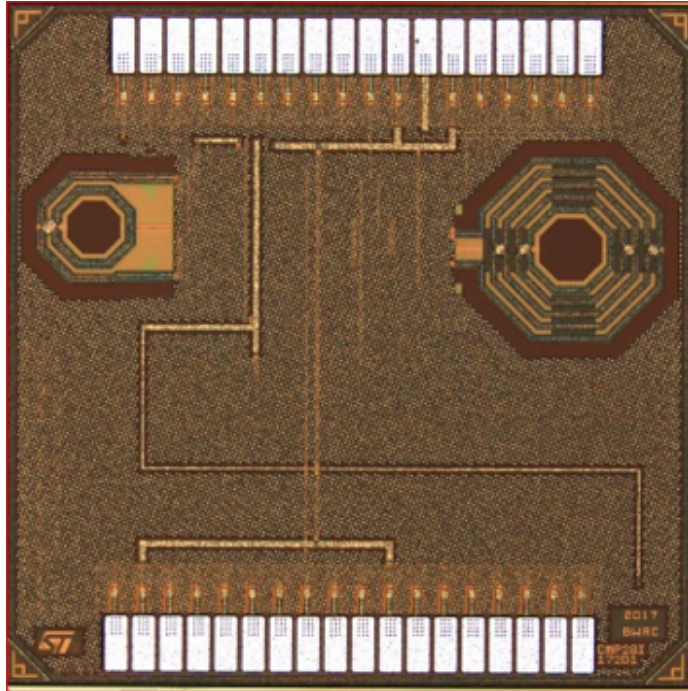
There are two inductors on the chip. The inductor on the right is part of an impedance-matching circuit for the antenna interface. We will refer to this as the *antenna interface inductor*. Its inductance is 9 nH and the filter it is a part of has a Q of 8 at 2.4 GHz.

The inductor on the left has an inductance of 1nH, and participates in an LC filter with a Q of 15 at 4.8 GHz, and this LC filter is the "local oscillator" at the core of the radio's transmit- and receive-circuits. We will call this the *local oscillator inductor*.

There is a capacitor connected to each of the two inductors, which make resonators.
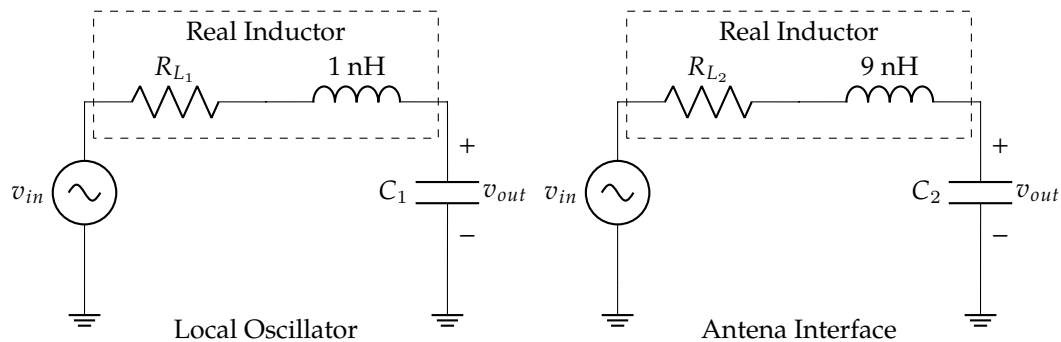
We are including the resistance that is present in real inductors in our model because our Q is not infinite. These circuit diagrams are meant to be representative of the two resonators described, which are a small part of the overall circuit seen in the chip above.

a) **For the antenna interface inductor, what is the capacitance (to one significant figure) needed to make it resonant at 2.4 GHz?**

b) **For the local oscillator inductor on the left, what capacitance (to four significant figures) is needed to make it resonant at 4.8 GHz? How about at 5 GHz?**

   c) **For the local oscillator inductor on the left with resonance at 4.8 GHz, what is the inductor resistance, R_{L_1} such that the $Q = 8$?**

   What happens to Q if we have a higher resistance?

5 Homework Process and Study Group

Citing sources and collaborators are an important part of life, including being a student!
We also want to understand what resources you find helpful and how much time homework is taking, so we can change things in the future if possible.

   a) **What sources (if any) did you use as you worked through the homework?**

   b) **If you worked with someone on this homework, who did you work with?**
     List names and student ID's. (In case of homework party, you can also just describe the group.)

   c) **Roughly how many total hours did you work on this homework?**

   d) **Do you have any feedback on this homework assignment?**