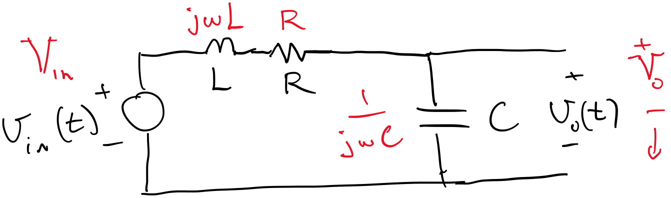
Bode Plots and LRC Resonance

Wednesday, September 30, 2020 8:54 PM

- Re-visit examples with an inductor
- Freq Response of LRC ckts - "Resonance"


$j\omega L$
R
V_{in}
L
R
$v_{in}(t)$
$+$
$-$
$\frac{1}{j\omega C}$
C
$+$
$v_o(t)$
$-$
$+$
V_o
$-$

$$v_{in}(t) = V_{in} \cos \omega t = \mathrm{Re}\left[V_{in} e^{j\omega t}\right]$$

$$\frac{V_o}{V_{in}} = H(j\omega) = \frac{1/j\omega C}{j\omega L + R + \frac{1}{j\omega C}}$$

$$= \frac{1}{(j\omega)^2 LC + (j\omega) CR + 1}$$

Inspection: (1) low-freq $H(j\omega) \simeq 1$

(2) High-freq $H(j\omega) = -\frac{1}{\omega^2 LC}$

This type of filter also really good at reducing high freq signals

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Recall: time-domain analysis

$$A = \begin{bmatrix} 0 & 1/C \\ -\frac{1}{L} & -\frac{R}{L} \end{bmatrix}$$

char eq: $\det(\lambda I - A) = 0$

$$\lambda^2 + \frac{R}{L}\lambda + \frac{1}{LC} = 0$$

roots: $\lambda_{1,2} = -\frac{1}{2}\frac{R}{L} \pm \sqrt{\left(\frac{1}{2}\frac{R}{L}\right)^2 - \frac{1}{LC}}$

- roots may be complex valued

$R = 0 \Rightarrow \lambda_{1,2} = \pm j\sqrt{\frac{1}{LC}}$

$\frac{1}{LC} > \left(\frac{1}{2}\frac{R}{L}\right)^2 \Rightarrow \lambda_{1,2}$ complex ; roots are conjugate

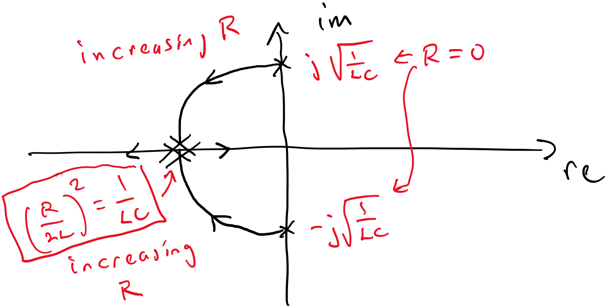
say R increases: $\frac{1}{LC} = \left(\frac{1}{2}\frac{R}{L}\right)^2$

$\Rightarrow$ repeated roots

$$\lambda_{1,2} = -\frac{1}{2}\frac{R}{L}$$

- Further increase in R results in

2 real distinct roots
[sort of like 2R + 2C ckt]



im
re
increasing R
$j\sqrt{\frac{1}{LC}}$ ← R = 0
$-j\sqrt{\frac{1}{LC}}$
$\boxed{\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}}$
increasing R

2nd order resonant system is universal

- circuit
- mechanical resonator [tuning fork]
- physics
-

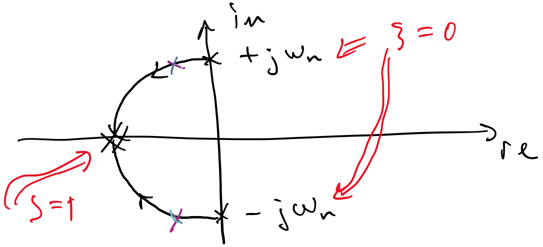
Parameters:

$$\omega_n = \sqrt{\frac{1}{LC}}$$

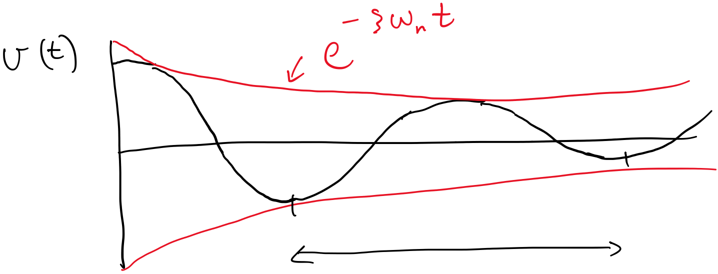
$$\zeta = \frac{1}{2}\frac{R}{\sqrt{L/C}}$$ dimensionless

"xi"

look at range of ζ: 0 to 1

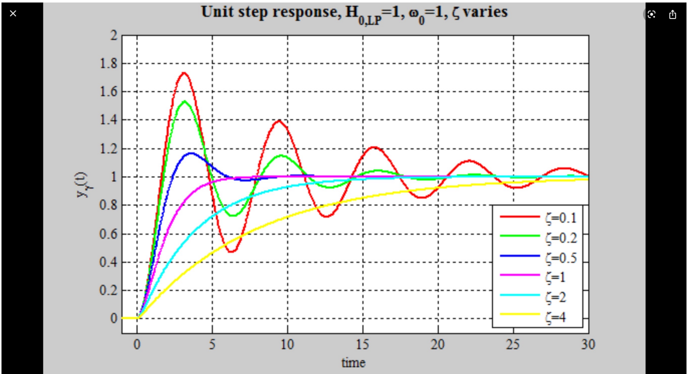

im
$+j\omega_n$
$\zeta=0$
re
$\zeta=1$
$-j\omega_n$

Time Domain Response : $\zeta = 0.1$


$v(t)$
$e^{-\zeta\omega_n t}$

$$T = \frac{2\pi}{\omega_n\sqrt{1-\zeta^2}}$$

$$\lambda_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$



Char poly. $\det[\lambda I - A] = 0$

$$\lambda^2 + 2\zeta\omega_n\lambda + \omega_n^2 = 0$$

get roots: apply quadratic formula

Freq Domain:

$$H(j\omega) = \frac{1}{LC(j\omega)^2 + RC(j\omega) + 1}$$

replace L, C with ζ, ω_n

$$H(j\omega) = \frac{1}{\frac{(j\omega)^2}{\omega_n^2} + \frac{(j\omega)2\zeta}{\omega_n} + 1}$$

As noted: ["interesting" examples $0 < \zeta < 1$]

Low freq: $|j\omega| << \omega_n$

$H(j\omega) \approx 1$; Phase ≈ 0

High freq: $|j\omega| \gg \omega_n$

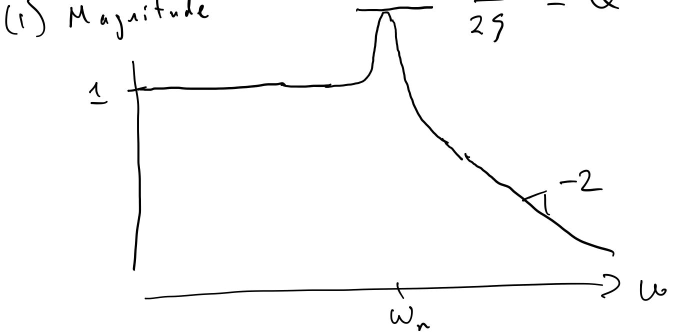
$$H(j\omega) = -\frac{\omega_n^2}{\omega^2}$$

- magnitude decreasing:
  Bode plot: slope $= -2$
- Phase function: real and neg.
  $\Rightarrow \approx -180^\circ$

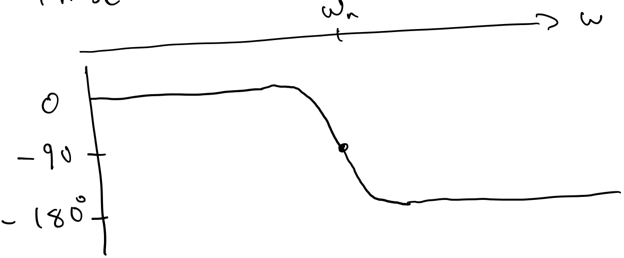
Bode Plot

(1) Magnitude

$\frac{1}{2\zeta} = Q$



(2) Phase

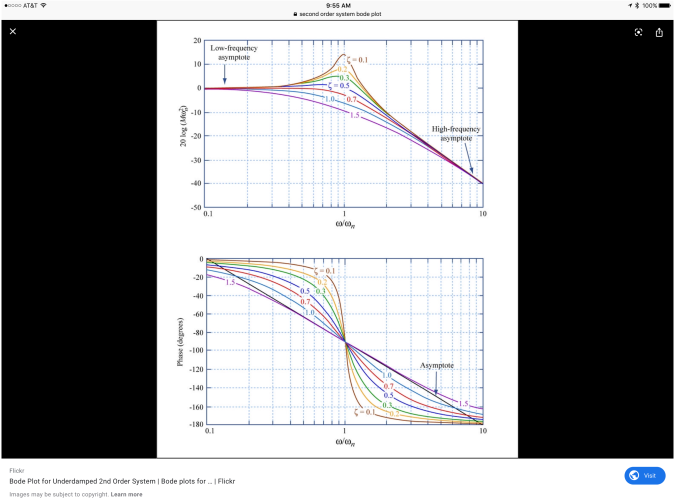


"Middle" Freq: $\omega = \omega_n$ or $\omega \approx \omega_n$

$$H(j\omega)\Big|_{\omega=\omega_n} = \frac{1}{2j\zeta} = -j\frac{1}{2\zeta}$$

$$\boxed{Q = \frac{1}{2\zeta}}$$ is the peak amplitude

Remarkably: peak amplitude is > 1



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second order system bode plot

20 log (Mω_n²)
20
10
0
-10
-20
-30
-40
-50
0.1
1
10
ω/ω_n
Low-frequency asymptote
High-frequency asymptote
ζ = 0.1
0.2
0.3
ζ = 0.5
0.7
1.0
1.5
Phase (degrees)
0
-20
-40
-60
-80
-100
-120
-140
-160
-180
0.1
1
10
ω/ω_n
Asymptote
ζ = 0.1
0.2
0.3
0.5
0.7
1.0
1.5
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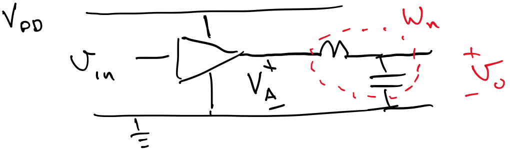
Basic applications:

1) Power Amp to drive antenna in cell phone

$V_{DD} = V_{BAT} = 3.6\text{ V}$


V_{DD}
v_{in}
V_A +
−
ω_n
+ v_o −

$|V_o| \approx \frac{1}{2\zeta}|V_A|$

$$v_{in} = a(t)\cos(\omega t + \phi(t))$$

$a(t)$; $\phi(t)$ slowly varying

Pick $\omega_n \approx \omega$

Q of 5 ⇒ voltage gain of 5

Battery: $V_{BAT} = 3.6$ ⇒ sine wave voltage only about ~ 1V

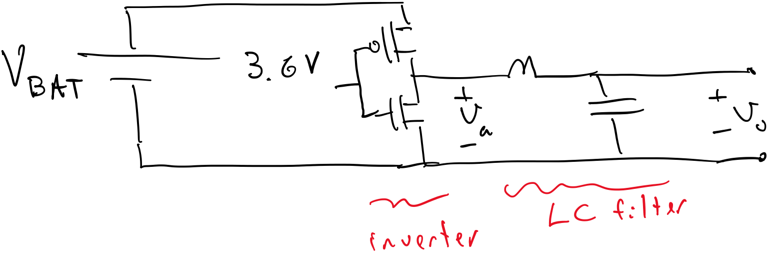
⇒ Boost voltage with "matching network" ⇒ drive more power into antenna

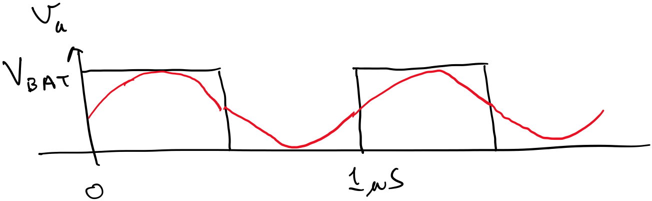
antenna has impedance like 50 − 75 Ω

* get more power

DC-DC Converter : convert V_{BAT} to ~ 1 V for CMOS

(PMIC)


V_{BAT}
3.6V
$+$
v_a
$-$
$+$
v_o
$-$
inverter
LC filter



Approx.: $v_a \approx \frac{1}{2} V_{BAT} \cos \omega t + \frac{1}{2} V_{BAT}$

Typical timing: period $\approx 1\ \mu S$
freq ≈ 1 MHz

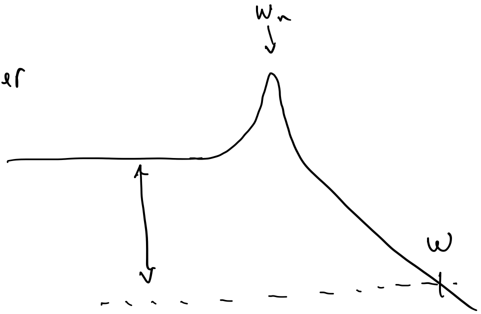
L-C filter:

- Output of LC filter has 2 components:

{ DC
attenuated AC signal

- superposition for analysis

Filter

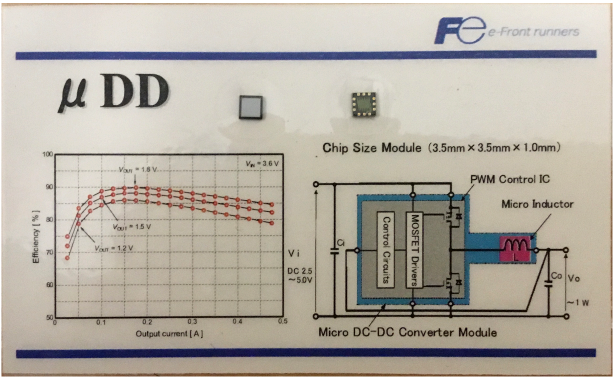


1000 × reduction

$$\left|\frac{\omega}{\omega_n}\right|^2 \simeq 1000$$

$$\omega = 1\,\text{MHz} \Rightarrow \omega_n \simeq \frac{1\,\text{MHz}}{\sqrt{1000}}$$

1 V AC in ⇒ 1 mV AC out ✓



e-Front runners

μDD

Chip Size Module (3.5mm × 3.5mm × 1.0mm)

Micro DC-DC Converter Module

Efficiency [%]
100
90
80
70
60
50
0
0.1
0.2
0.3
0.4
0.5
Output current [A]
V_{IN} = 3.6 V
V_{OUT} = 1.8 V
V_{OUT} = 1.5 V
V_{OUT} = 1.2 V
PWM Control IC
Micro Inductor
Control Circuits
MOSFET Drivers
L
Ci
Co
Vi
DC 2.5 ~5.0V
Vo
~1 W