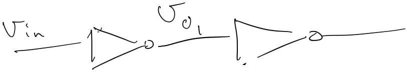
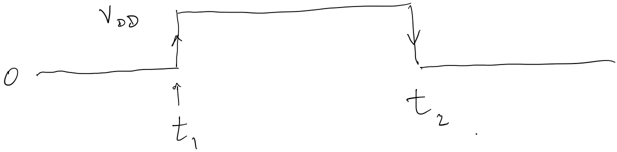
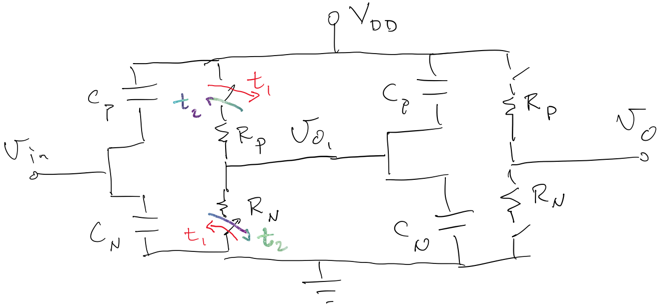
Delay analysis and more general first order circuits

Thursday, September 10, 2020 9:18 AM

Last Time

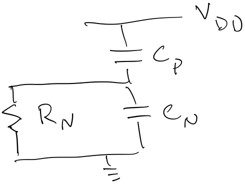




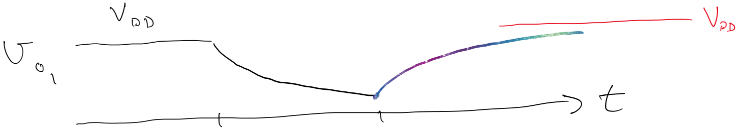


Last Time

$t_1 < t^x < t_2$



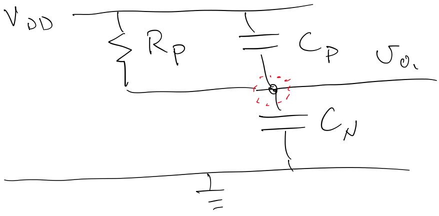
$v_{o_1}(t_1) = V_{DD}$



t_1 t_2

$t > t_2$: Draw reduced ckt model

PMOS of 1st device ON
NMOS of 1st " OFF



1 node eqn:

$$\frac{v_{o_1} - V_{DD}}{R_p} + C_N \frac{d}{dt} v_{o_1} + C_p \frac{d}{dt}\left(v_{o_1} - \cancelto{0}{V_{DD}}\right) = 0$$

$$(C_N + C_p) \frac{d}{dt} v_{o_1} = -\frac{1}{R_p} v_{o_1} + \frac{1}{R_p} V_{DD}$$

$$\boxed{\frac{d}{dt} v_{o_1} = -\frac{1}{(C_N + C_p) R_p} v_{o_1} + \underbrace{\frac{1}{(C_N + C_p) R_p}}_{\text{"new"}} V_{DD}}$$

$$\tau_p = (C_N + C_p) R_p$$

$$F \cdot \Omega = s$$

- Things to motivate guessing:

[· Like last class, this is a diff. equ.]

(i) Boundary condition:

$$v_{o_1}(t_2) = V_{DD}\, e^{-\frac{(t_2 - t_1)}{\tau_N}}$$

$$\tau_N = R_N (C_N + C_p)$$

(ii) Where do we end up or approach as $t \to \infty$?

can see that $v_{o_1} \to V_{DD}$ as $t \to \infty$

- motivation is we converge to an equilibrium where $\frac{d}{dt} v_{o_1} \to 0$, and thus cap. currents → zero, and current in $R_p \to 0$

- Guess $t \geq t_2$

$$v_{o_1}(t) = V_{DD} + \left(v_{o_1}(t_2) - V_{DD}\right) e^{-\frac{t - t_2}{\tau_p}}$$

check:

$$\frac{d}{dt} v_{o_1}(t) = -\frac{1}{\tau_p}\left(v_{o_1}(t_2) - V_{DD}\right) e^{-\frac{t - t_2}{\tau_p}}$$

(LHS)

$$-\frac{1}{\tau_p} v_{o_1} + \frac{1}{\tau_p} V_{DD}$$

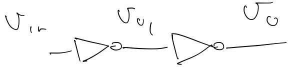
(RHS)

$$= -\frac{1}{\tau_p}\left[V_{DD} + \left(v_{o_1}(t_2) - V_{DD}\right) e^{-\frac{t-t_2}{\tau_p}}\right] + \frac{V_{DD}}{\tau_p}$$

little algebra, but looks good ✓

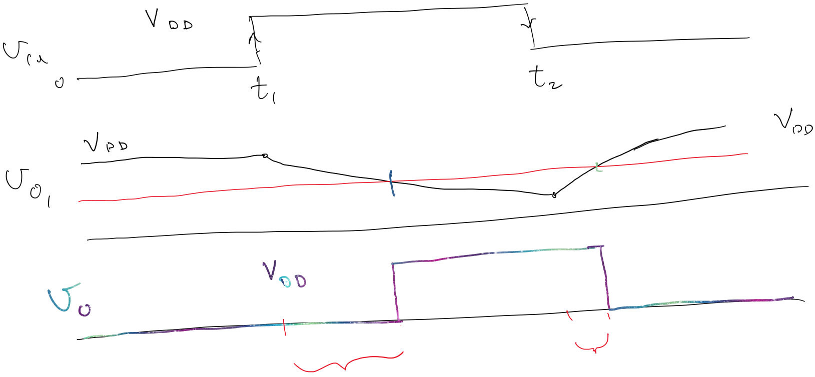
How do we know this solution is unique?

answer: This diff equ again satisfies the cond. for existence and uniqueness



NMOS, PMOS have $V_{T_n} = \frac{1}{2} V_{DD}$

$V_{T_p} = -\frac{1}{2} V_{DD}$



* Begins to illustrate limit on highest possible clock rate in a processor

* Overclocking: increase V_{DD} ⇒ effectively reduce R_P & R_N

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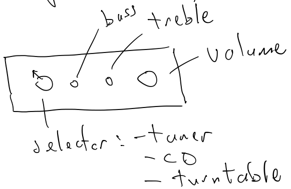
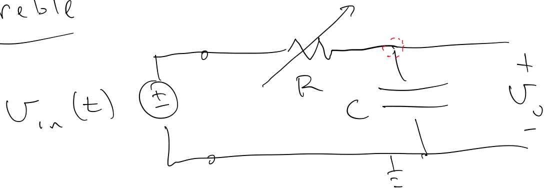
* Ckt just analyzed is termed an R-C ckt

Pre-Amp



Treble



Do analysis

1 KCL eqn:

$$\frac{v_o - v_{in}}{R} + C\frac{d}{dt}v_o = 0$$

$$\frac{d}{dt}v_o = -\frac{1}{RC}v_o + \frac{1}{RC}v_{in}$$

New types of inputs

(i) Try $v_{in}(t) = V_{in} e^{st}$

[Concept: this waveform may be an eigen function for input - output behavior]

⇒ guess: output looks like input

$$v_o = V_o e^{st}$$

plug in:

$$s V_o e^{st} = -\frac{1}{RC} V_o e^{st} + \frac{1}{RC} V_{in} e^{st}$$

factor out e^{st} & solve for V_o:

$$s V_o + \frac{1}{RC} V_o = \frac{1}{RC} V_{in}$$

$$V_o = \frac{1}{RC} \frac{1}{s + \frac{1}{RC}} V_{in}$$

$$v_o(t) = \frac{1}{RC} \frac{1}{s + \frac{1}{RC}} V_{in} e^{st}$$

scaling function
- multiplies input to get output
- "System function" ≈ this works because e^{st} is an eigenfunction for this linear system

- What's unspecified?
  - we did not identify any specific init. cond.

- say $v_0\big|_{t=0} = V_1$

- Complete solution [Guess & Check]

$$v_o(t) = A e^{-\frac{t}{RC}} + \frac{1}{RC}\frac{1}{s+\frac{1}{RC}} V_{in} e^{st}$$

How to find A?

$$A \underbrace{e^{-\frac{0}{RC}}}_{1} + \frac{1}{RC}\frac{1}{s+\frac{1}{RC}} V_{in} \underbrace{e^{(0)}}_{1} = V_1$$

$$A = V_1 - \frac{1}{RC}\frac{1}{s+\frac{1}{RC}} V_{in}$$

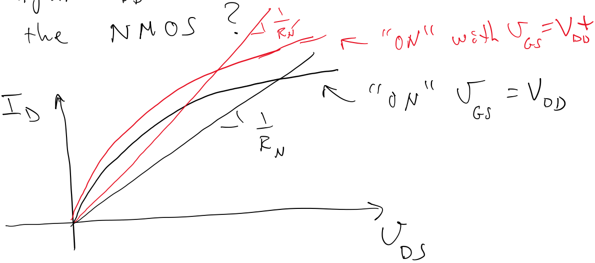
- Will leave the "check" part to you!

why did I add $\underbrace{Ae^{-t/RC}}_{\text{homogeneous part of solution}}$?

Linear diff eqn : $\frac{d}{dt} x = \lambda x + u$

$$x = \underbrace{x_{\text{homogeneous}}}_{Ae^{\lambda t}} + \underbrace{x_{\text{part.}}}_{\text{"just works"}}$$

Why does higher V_{DD} reduce effective resistance of the NMOS ?



"General" input case

$$\frac{d}{dt} x = \lambda x + u \qquad x(t_0) = X_0$$

$$x(t) = \underbrace{e^{\lambda(t-t_0)} x_0} + \int_{t_0}^{t} e^{\lambda(t-\tau)} u(\tau)\, d\tau$$

homog.
resp.

part. part of
response

How did I know?

already known ⇒ use guess & check

someone
told me

Check: $x(t_0) = x_0$? ✓

$$\frac{d}{dt}x(t) = \lambda e^{\lambda(t-t_0)} x_0 + \int_{t_0}^{t} \lambda e^{\lambda(t-\tau)} u(\tau)\,d\tau + \underbrace{e^{\lambda(t-\overset{\downarrow}{t})}}_{1} u(t)$$

$$= \lambda x t + u(t)$$ ✓

Hard part: need to know derivative of an integral: Leibnitz rule