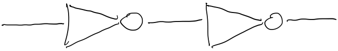
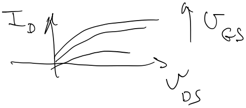
Inverter transient analysis - intro to differential eqns

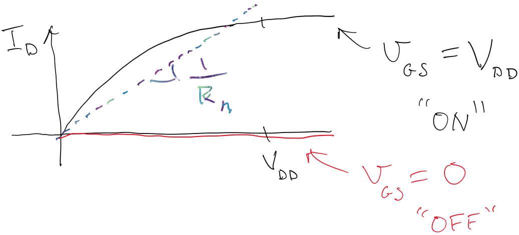
Tuesday, September 8, 2020 9:13 AM



- Last Time(s)



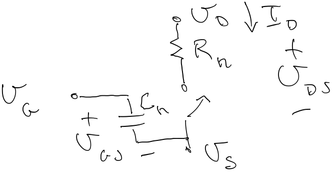
- For digital fn's: ON - OFF is important
- Simplify model



ON: $v_{GS} > V_{Tn}$

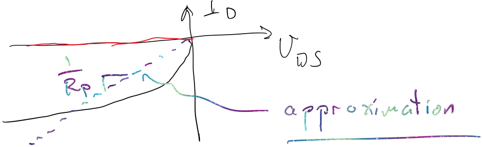
OFF: $v_{GS} < V_{Tn}$

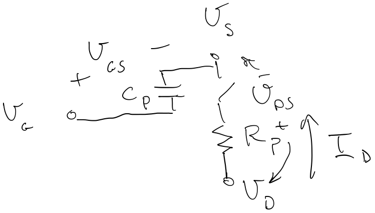
Approx. I_D vs V_{DS} as straight line in "ON" state. No longer distinguish between "linear" and "saturation" regions

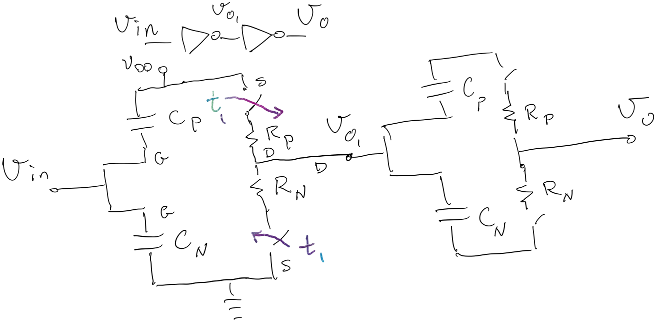


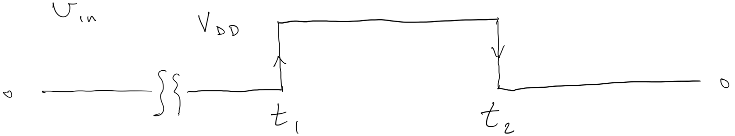
- NMOS has capacitance due to gate dielectrics: C_{GS}
- For today and unless otherwise specified: $V_{T_n} = \frac{1}{2} V_{DD}$

Analogous steps for PMOS







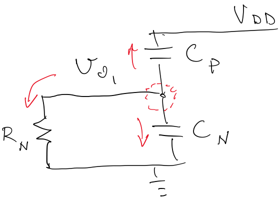


Before t_1: v_{in} low $(= 0\text{ V})$

v_{O_1} high $(= V_{DD})$

v_O low $(= 0\text{ V})$

Immediately after t_1



What to do?

- write node eqn
  - label node potential v_{O_1}
  - write 1 KCL eqn
  - incorporate data from other node potentials and branch characteristics

$$\frac{v_{O_1}}{R_N} + C_N \frac{d}{dt} v_{O_1} + C_P \frac{d}{dt}\left(v_{O_1} - V_{DD}\right) = 0$$

$$\frac{d}{dt} V_{DD} = 0$$ ✓

$$\boxed{(C_N + C_P)\frac{d}{dt} v_{o_1} + \frac{v_{o_1}}{R_N} = 0}$$

At t_1 : know $\boxed{v_{o_1}(t_1) = V_{DD}}$

- Simplest example of a differential equ

$$\frac{d}{dt} v_{o_1} = -\frac{v_{o_1}}{(C_N + C_P) R_N}$$

- product $(C_N + C_P) R_N$ has (units of cap) * (units of resistance)

$$F \cdot \Omega = s$$ ⟸ units of time

product of cap * res = time constant

- Micro electronic devices

kilo

femto $f F \cdot k\Omega$

$$10^{-15} F \cdot 10^{3} \Omega = 10^{-12} s$$

$$= ps$$ pico

- state of art $\sim 10 ps$

- How to solve diff equ ?

$$\frac{d}{dt} v_{o_1} = - \frac{v_{o_1}}{\tau}$$

$$\tau = \text{time const} = (C_N + C_P) R_N$$

$$\frac{d}{dt} v_{o_1} = \lambda v_{o_1} \qquad \lambda = -\frac{1}{\tau}$$

1st attempt

$$\frac{d v_{o_1}}{v_{o_1}} = \lambda \, dt \qquad ; \; t^* > t_1$$

$$\int_{t_1}^{t^*} \frac{d v_o}{v_{o_1}} = \int_{t_1}^{t^*} \lambda \, dt = \lambda (t^* - t_1)$$

$$\int_{v_{o_1}(t_1)}^{v_{o_1}(t^*)} \frac{d v_{o_1}}{v_{o_1}} = \ln v_{o_1} \Big|_{v_{o_1}(t_1)}^{v_{o_1}(t^*)}$$

$$\ln \frac{v_{o_1}(t^*)}{v_{o_1}(t_1)} = \lambda (t^* - t_1)$$

exp

$$v_{o_1}(t^*) = v_{o_1}(t_1) \, e^{\lambda (t^* - t_1)}$$

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well ok : This will not be adequate for all examples. Sep. of variables is limited.

More general method:

(0) Guess the solution structure

(1) get it right [check]

(2) match up boundary conditions

(3) Existence & uniqueness

$$\frac{d}{dt} V_{o_1} = \lambda V_{o_1} \quad ; \quad V_{o_1}(t_1) = V_{DD}$$

Guess exponential response: $V_{o_1} = A e^{\lambda t}$

$$\lambda = -\frac{1}{R_N(C_N+C_P)}$$

- Appeal to linear algebra background

$A\vec{x} = \lambda \vec{x}$ true if λ eigenvalue

$\vec{x}$ eig-vector

- In diff eq'n: A is replaced by $\frac{d}{dt}$

$e^{\lambda t}$ is eigenvector for this problem

- Guess & Check:

$$V_{o_1} = \underbrace{A e^{\lambda t}}_{\frac{d}{dt} \text{ to get LHS}}$$

$$\overbrace{\frac{d}{dt} V_{o_1}}^{LHS} = \overbrace{\lambda V_{o_1}}^{RHS}$$

$$V_{o_1}(t_1) = V_{DD}$$

check

$$\underbrace{\lambda A e^{\lambda t}}_{LHS} = \underbrace{\lambda A e^{\lambda t}}_{RHS} \quad \checkmark$$

- generalizes notion of eigenvalue – eigenvector pair

  λ = eig value

  $e^{\lambda t}$ = eig vector [eigenfunction]

- $v_{o_1} = A e^{\lambda t}$

$$\underbrace{v_{o_1}(t_1) = V_{DD}}$$

boundary cond. is determining of value of A

$$A e^{\lambda t_1} = V_{DD}$$

$$\Downarrow$$

$$A = V_{DD} e^{-\lambda t_1}$$

Broader Picture

scalar differential eqn

$$\frac{d}{dt} x = f(x,t) \quad ; \quad x(t_1) = x_0$$

(*) $\left\{ \begin{array}{l} \text{say } f(\,,t) \text{ is differentiable in } x \\ \text{and } \left|\frac{\partial f}{\partial x} f(x,t)\right| < M \\ \qquad \text{for all } x \text{ and all } t \end{array} \right.$

(**) $\left\{ \begin{array}{l} f(x,t) \text{ has only finite number} \\ \text{of discontinuities w.r.t. time } t \\ \text{in any unit interval (eg. 1 s)} \end{array} \right.$

If (*) and (**) hold, diff eqn has unique solution for all finite time t, with $x(t_1) = x_0$

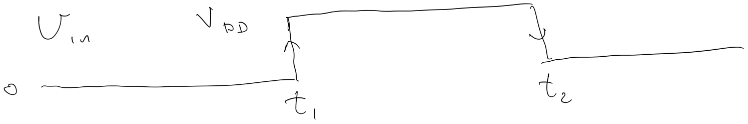
Our example

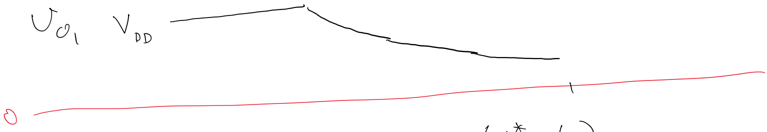
$$\frac{d}{dt} x = \underbrace{\lambda x}_{f(x,t)} \qquad \lambda = \frac{1}{R_N(C_N+C_p)}$$

- no t dependence in $f(x,t)$ ✓
- $\frac{\partial f}{\partial x} = \lambda = \text{const}$

$$\left|\frac{\partial f}{\partial x}\right| = |\lambda| < M \qquad ✓✓$$

Return to "picture" of solution v_{o_1}





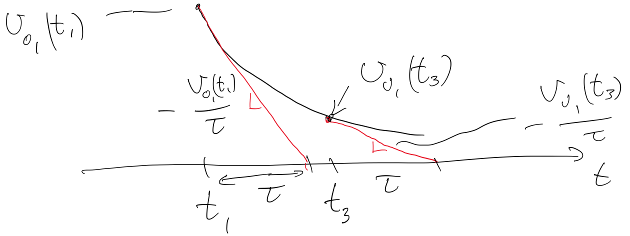
$$v_{o_1}(t^*) = v_{o_1}(t_1)\, e^{\lambda(t^*-t_1)}$$

$t_1 < t^* < t_2$

example only good till t_2 because ckt changes at t_2

$$\lambda = -\frac{1}{R_N(C_n+C_p)}$$

Nature of exponential fn



slope (or tangent) at time t_1

$$\frac{d}{dt} v_{o_1} = \lambda v_{o_1}$$

At t_1 $$\boxed{\left.\frac{d}{dt} v_{o_1}\right|_{t_1} = \lambda\, v_{o_1}(t_1)}$$

$$\lambda = -\frac{1}{\tau}$$