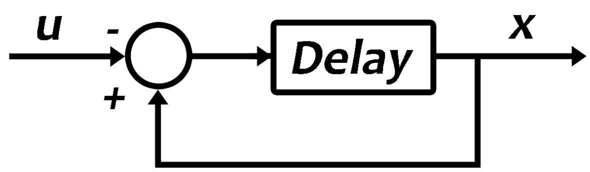
1. Block Diagrams for Difference Equations

In this problem, we will be looking at block diagrams for difference equations in discrete time. These diagrams will contain a "delay" block that indicates the input at the current time is delayed by one step. For example, the difference equation $x(t+1) = x(t) - u(t)$ can be represented by a block diagram with a delay element by:



For the following questions, we will consider only scalar difference equations.

(a) For our first diagram, let's consider the difference equation used to compute Fibonacci numbers: $x(t) = x(t-1) + x(t-2)$. By looking at this equation, we can see that the output at time t is the sum of the output of the previous two time steps. Therefore, we can write this equation as a block diagram using delay elements and a summation element. Draw the block diagram.

(b) Now, let's consider the difference equation

$$x(t+1) = 2x(t) - u(t) \tag{1}$$

Draw this equation as a block diagram.

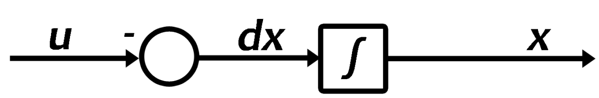
(c) Is this system stable?

(d) Find an expression for $u(t)$ so that $\lambda = 1/2$ for this system.

(e) Draw the closed-loop system with $\lambda = 1/2$ as a block diagram using the control you derived from the previous part.

2. Block Diagrams for Differential Equations

In this problem, we will consider using block diagrams to represent differential equations. The important component we will be using for these diagrams is the "integral" element. For example, the differential equation $\frac{dx}{dt} = -u(t)$ is drawn as a block daigram:



(a) Let's consider the differential equation

$$\frac{dx}{dt} = 2x(t) - u(t) \tag{2}$$

Draw this equation as a block diagram.

(b) Is this system stable?

(c) Find an expression for $u(t)$ so that $\lambda = -2$ for this system.

(d) Draw the closed-loop system with $\lambda = -2$ as a block diagram using the control you derived from the previous part.

(e) Now, let's extend block diagrams to the 2x2 case. We can represent matrix vector multiplication as a block with two inputs for the vector and two outputs. Similarly, we can represent a dot product with a fixed vector and an input vector as a block with two inputs and one output. Consider the continuous time system from discussion 11B:

$$\frac{d}{dt}\begin{bmatrix} x(t) \\ v(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix}\begin{bmatrix} x(t) \\ v(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} x(t) \\ v(t) \end{bmatrix}$$

Draw this system as a block diagram.

(f) Describe how to implement this system with several golden-rule op-amps for integrators, inverting and non-inverting amplifiers, and inverting voltage summers.

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