EECS 16B Designing Information Devices and Systems II
Spring 2016 Anant Sahai and Michel Maharbiz **Discussion 5A**

1. RC Circuits

In this problem, we will be using differential equations to find the voltage across a capacitor V_C over time in an RC circuit. We set up our problem by first defining three functions over time: $I(t)$ is the current at time t, $V(t)$ is the voltage across the circuit at time t, and $V_C(t)$ is the voltage across the capacitor at time t.

Recall from 16A, that the voltage across a resistor is defined as $V_R = RI_R$ where I_R is the current across the resistor. Also, recall that the voltage across a capacitor is defined as $V_C = \frac{Q}{C}$ where Q is the charge across the capacitor.

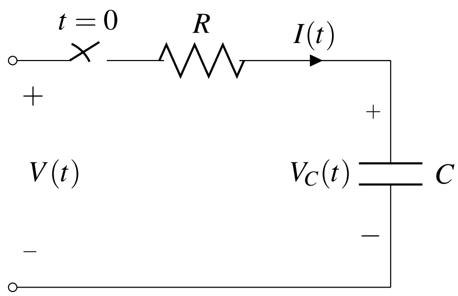

Figure 1: Example Circuit

(a) First, find an equation that relates the current across the capacitor $I(t)$ with the voltage across the capacitor $V_C(t)$.

(b) Using Kirchhoff's law, write an equation that relates the functions $I(t)$, $V_C(t)$, and $V(t)$.

(c) So far, we have three unknown functions and only one equation, but we can remove $I(t)$ from the equation using what we learned in part (a). Rewrite the previous equation in part (b) in the form of a differential equation.

(d) Let's suppose that for $t < 0$ the capacitor is precharged to a voltage V_{DD} and that $V(t) = 0\ \forall t \geq 0$, simply a short to ground. Assuming that we close the switch at $t = 0$, use the fact that $V_C(0) = V_{DD}$ to solve this differential equation for $V_C(t)$.

(e) Now, let's suppose that we start with an uncharged capacitor $V_C(0) = 0$. We apply some constant voltage $V(t) = V_{DD}$ across the circuit. Assuming the switch closes at $t = 0$, use your differential equation to solve for $V_C(t)$.

(f) Now that you know how the voltage across a capacitor acts over time in an RC circuit, how does the charge in the capacitor act over time? Write your answer as a function of $Q(t)$, and remember that $V_C = \frac{Q}{C}$.

2. RC Circuit of Inverter Input Let's now consider a slightly more complicated RC circuit.

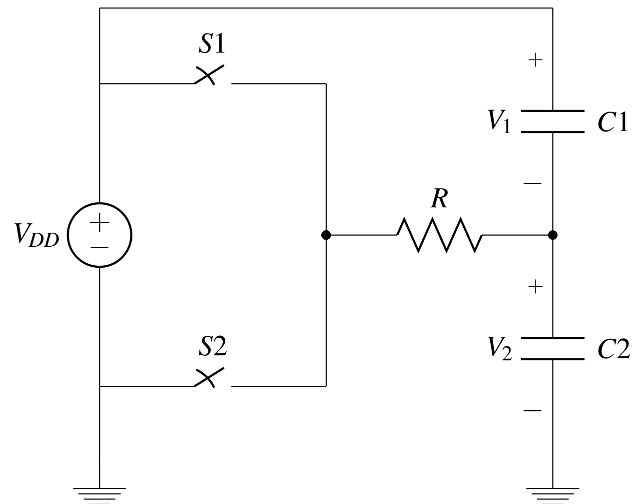

Figure 2: Inverter Input

In this problem, we will explore what happens when we change the voltage in between the capacitors.

(a) Suppose $S1$ has been closed and $S2$ has been open long enough that the voltage across $C1$ and $C2$ have settled to constant values. Suppose at $t = 0$, we open $S1$ and close $S2$. State the initial conditions of the differential equation (i.e. $V_1(0)$ and $V_2(0)$), and express the voltages $V_1(t)$ and $V_2(t)$ in both capacitors as a function of time using the equations you derived from the previous problem.

(b) Suppose $S2$ has been closed and $S1$ has been open long enough that the voltage across $C1$ and $C2$ have settled. Suppose at $t = 0$, we open $S2$ and close $S1$. State the initial conditions of the differential equation (i.e. $V_1(0)$ and $V_2(0)$), and express the voltage in both capacitors as a function of time using the equations you derived from the previous problem.

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