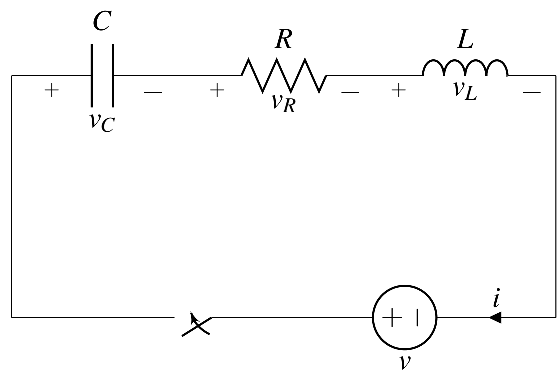
1. **RLC circuit** In this problem, we study the differential equations governing a series RLC circuit, which we solve to get the transient behavior. We consider the simple RLC circuit below. Suppose that the switch is closed at time $t = 0$.



(a) Write the voltages v_C, v_R, v_L in terms of the current i, with respect to time t.

(b) Write down a second order differential equation for the current in the circuit with respect to time, in terms of the constants R, L, C.

(c) Rewrite your second order differential equation in the form

$$\begin{pmatrix} \frac{di}{dt} \\ \frac{dv_L}{dt} \end{pmatrix} = A \begin{pmatrix} i \\ v_L \end{pmatrix} \tag{1}$$

where A is a 2×2 matrix with coefficients that depend only on R, L, C.

(d) Find the eigenvalues and corresponding eigenvectors of your matrix A from the previous part.

(e) For the case where the two eigenvalues are real, we claim that the solution to this system of differential equations is of the form

$$\begin{pmatrix} i \\ v_L \end{pmatrix} = c_0 e^{\lambda_0 t} \vec{v}_0 + c_1 e^{\lambda_1 t} \vec{v}_1 \,,$$

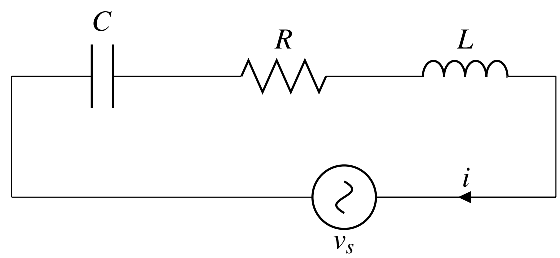
where c_0, c_1 are constants, and λ_0, λ_1 are the eigenvalues of A with eigenvectors $\vec{v}_0, \vec{v}_1$ respectively. Solve for the constants c_0, c_1, with the initial conditions $i = 0$, $v_L = 1$ at $t = 0$. Write your solution for i as a function of t.

(f) For the case where the eigenvalues are complex, the solution to the system has the same form as in the previous part. Find i as a function of t in this case.

2. **RLC circuit in AC**

We study a simple RLC circuit with an AC voltage source given by

$$v_s = B\cos(\omega t - \phi)$$



(a) Write out the phasor representation of v_s, R, C, L.

(b) Use Kirchhoff's laws to write down a loop equation relating the phasors in the previous part.

(c) Solve the equation in the previous step for the current I. What is the polar form of I?

(d) Compute the polar form of V_R, V_L, V_C.

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