EECS 16B Designing Information Devices and Systems II
Spring 2016 Anant Sahai and Michel Maharbiz **Discussion 8B**

1. Linearization of functions

(a) Linearize $\sin(\theta)$ for $|\theta| < 5^\circ$

(b) Linearize $\cos(\theta)$ for $|\theta| < 5^\circ$

(c) Linearize $\sqrt{x}$ for $0 \le x < 0.1$

2. Linear Temperature Control System

Imagine you are designing a room temperature controller. As a designer, you have the following tools: (1) a digital thermometer that gives you a temperature measurement in digital form and (2) a heat pump (air conditioner) which can warm up or cool down the room or do nothing.

Now you would like to keep the room temperature around $T^\circ F$.

(a) Describe how you will intuitively design the controller using English. Use a combination of English and math where x is the temperature and u is the setting on the heat pump (positive u pumps heat into the room and negative u pumps heat out of the room at the rate of u $Watt$.)

(b) Now assume the heat pump and controller together set the u so that the room temperatures evolve according to the following differential equation: $\frac{dx}{dt} = k_1(T - x)$, where $k_1 > 0$. Assume $x(0) = 0.8T^\circ F$, what is $x(t)$ for $t > 0$?

(c) What was the feedback control law $u(t) = g(T, x(t))$ used to get the above behavior, assuming there is no heat loss or gain elsewhere? (Let c be the heat capacity of the room. So c Joules of added heat are required to raise the room temperature by $1^\circ F$)

(d) In the real world, the heat pump will not change its behavior instantaneously, and your temperature sensor will report the temperature once a minute. Let's assume the heat pump is synchronized to the temperature reader and also modifies its behavior once a minute. We can write down a difference equation (discrete-time model) for the controlled air conditioner as $x[t+1] = x[t] + \triangle x = x[t] + k_2(T - x[t])$. Assume $x[0] = 0.8T^\circ F$, write down $x[t]$ for $t > 0$.

(e) (optional, for now) What is the relationship between k_2 and k_1?

(f) Consider the above difference equation, does bigger k_2 imply faster convergence? How about k_1 in the differential equation case?

3. Linearizing a Nonlinear System

Consider the following two-dimensional system. There are two states x_0 and x_1 and we can apply two inputs u_0 and u_1. The system evolves according to the following coupled differential equations:

$$\frac{d}{dt}x_0(t) = x_0^2(t) + 2x_1(t) + 2u_0(t) + x_1(t)u_1(t) \tag{1}$$

$$\frac{d}{dt}x_1(t) = x_1^2(t) + 2x_0(t) + u_1(t) + x_0(t)u_0(t) \tag{2}$$

(a) Write the above into the standard form $\frac{d}{dt}\vec{x}(t) = \vec{f}(\vec{x}(t), \vec{u}(t))$.

(b) For the above system, assume that $\vec{u} = 0$ for all time. For what values of $\vec{x}$ is $\vec{f}(\vec{x}, \vec{0}) = \vec{0}$? These are potential operating points where the control could be zero.

(c) For the above system, linearize the dynamics around both of those potential operating points.

(d) For the above linearized systems, what are the eigenvalues of the resulting A matrices for both of those operating points?

(e) Can you linearize around $\vec{x} = [-1, 0]^T$? Is there a control $\vec{u}$ that will keep it there?