EE16B: DESIGNING INFORMATION DEVICES AND SYSTEMS II

LECTURE NOTES

Recap: DFT (Cosine Transform) and OMP in Imaging Context

DFT - Motivated by wireless (search for a basis that would be an eigenbasis for "channels with echoes") $\rightarrow$ modeled using circulant matrices.

Important Properties of DFTs:

1. The DFT basis is an eigenbasis for all circulant matrices of DFT.

2. DFT basis is orthonormal.

3. The p^{th} eigenvalue of $C_{\vec{h}}$ is just $\sqrt{n}\langle \vec{u_p}, \vec{h} \rangle = \sqrt{n}(\vec{u_p}^* \vec{h})$, where $C_{\vec{h}}$ is a circulant matrix in the time domain with $\vec{h}$ as its first vector.

The DFT complex basis can be transformed using Euler's formula ($e^{i\theta} = \cos(\theta) + i\sin(\theta)$) into a sine/cosine basis which is real.

V is basis:

$$\begin{bmatrix} \cos 0 \\ \cos 1 \\ \sin 1 \\ \cos 2 \\ \sin 2 \end{bmatrix}$$

$C_{\vec{h}}$ is circulant and real:

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$$C_{\vec{h}} = V \begin{bmatrix} \begin{bmatrix} \quad \end{bmatrix} & & & \mathbf{0} \\ & \begin{bmatrix} \quad \end{bmatrix} & & \mathbf{0} \\ & & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \begin{bmatrix} \quad \end{bmatrix} \end{bmatrix} V^T$$

Exercise: Look at the $2x2$ blocks on the diagonals. Show these are scaled "rotation" matrices.

Fact: We can view real vectors as sums of phase-shifted cosines.