Lecture Notes: 14

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1. Troll Revisited

Lets revisit the troll problem from EE16A and solve it using this semesters techniques. Recall that the Troll problem was formulated as follows:

There are 2 independent audio signal sources. One is a troll song, and the other is a speech. The signals are overlayed on top each other. The combined audio signal is then recorded from 2 separate microphones. How can we separate the audio source signals from what we have measured from 2 separate microphones?

In EE16A the microphone gains were given. Using the microphone gains, we set up a system of linear equations to recover the data. This time lets assume no information of the microphone gains are given. Given just the 2 recordings, can we separate these signals/sounds?

Link to the problem

Lets revisit our friend SVD! We can use PCA to isolate the sound.

- Group the data - The audio signal is simply an array of data. We can stack the audio signals.
- Subtract the mean - Calculate the mean by each time frame and subtract it to both signals.
- SVD - Observe the principal components

The intuition behind subtracting the mean is that the information we desire is stored in the variation of the data not the mean. We remove the mean from the data so the SVD does not "think" it is a contributing component to the overall variation.

Running the SVD, we see that the matrix U gives us the information on the direction we should project the data. The U contains our principal components! Recall from the lab that our principal components were from the matrix V not U. That is because the data samples were in rows. Here, unlike the lab, the data samples were positioned in columns. So our principal components are in the space U.

Listening to the first 2 principal components in U we see that we have the separated signals! Voila, it worked! ... some what.

What information is in V? Note that V should be a square matrix but *numpy* will not compute the rest of the matrix because it is the nullspace. Listening to components of V, we can still hear the separated audio in V. How So? This is because of the fundamentals relationship between U and V and the data matrix, A? The V was computed from U and A.

Using PCA is not the best tool for questions like this. The result is not perfect because it is not actually able to capture or take into account the complex audio features. We cannot also assume that orthogonal projection of the audio is necessarily separable in such way.

We try a simple approach to convince ourselves that PCA is not necessarily the best tool.

We subtract the mean from the data, normalize it and add them together.

$$naive = \frac{data_1 - \frac{1}{N}\sum_i^N data_1[i]}{||data_1 - \frac{1}{N}\sum_i^N data_1[i]||} + \frac{data_2 - \frac{1}{N}\sum_i^N data_2[i]}{||data_2 - \frac{1}{N}\sum_i^N data_2[i]||}$$

The $data_1$, $data_2$ corresponds to the recorded data. The summation is the mean. When we subtract the mean, remember that the subtraction is element wise. We divide the centered data by its norm to normalize it.

Listening to the *naive*, the sum of the mean normalized worked as well as PCA using SVD. A better choice of method we should have used is the Independent Principal Analysis which is discussed in upper division courses if interested.

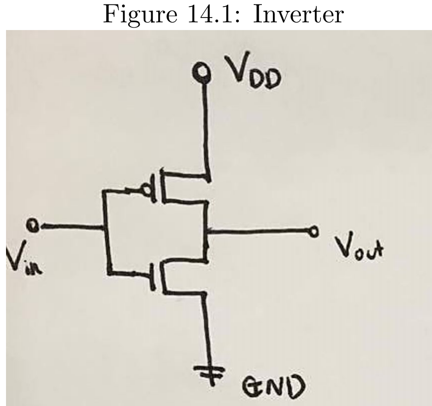
2. Introduction to Differential Equations

Last week we discussed about the inverter shown in figure 14.1. When the V_{in} is high, the gate on the bottom will turn on and the top will turn off so V_{out} will be connected to the ground. When the V_{in} is turned off, V_{out} will be turned on and set to V_{dd}.

When the inverters are connected, figure 14.2, it would be reasonable to assume that V_{in} and V_{out2} will be the same all the time, but it is not! There is a delay in the circuit induced the by its capacitive behavior. So how do we quantify this delay?

Lets think of a simpler RC Circuit given by figure 14.3. Assume the capacitor was fully charged to V_{dd}. The voltage starts off high but without the voltage source, the current starts to flow out of capacitor due to its voltage difference with the ground. As the current flows out, the voltage of the capacitor falls. The current is proportional to the voltage, hence we see a curve, figure 14.4. How can we show this mathematically?

Figure 14.1: Inverter



Real life is continuous so we need to use differential calculus.

$$Q = CV$$

What is current? Current is simply the change of the charge over time!

$$I = \frac{dQ}{dt}$$

Taking the derivative of the both side we get

$$\frac{dQ}{dt} = C\frac{dV}{dt} = I$$

We know

$$V = IR$$

So we subsitute in for the I

$$C\frac{dV}{dt} = -\frac{V}{R}$$

We know when the voltage is high the current flows the opposite of the way we drew the diagram hence we have put a $-\frac{V}{R}$. This equation is relating the varible V to its differential counterpart. We call this the differential equation.

Rearranging the term we have

$$\frac{dV}{dt} = -\frac{1}{RC}V$$

$$\frac{d}{dt}V = -\frac{1}{RC}V$$

Where V can be abstractly seen as a vector. Is the differentiator linear? Yes
Think of $\frac{d}{dt}$ as some kind of a matrix. Then the equation looks similar to the eigenvalues / eigenvectors relationship.

The Eigenfunction of $\frac{d}{dt}$ is Ke^{st}.

$$\frac{d}{dt}Ke^{st} = Kse^{st} = sKe^{st}$$

and s is the corresponding eigenvalue.

Figure 14.2: Cascadded Inverter

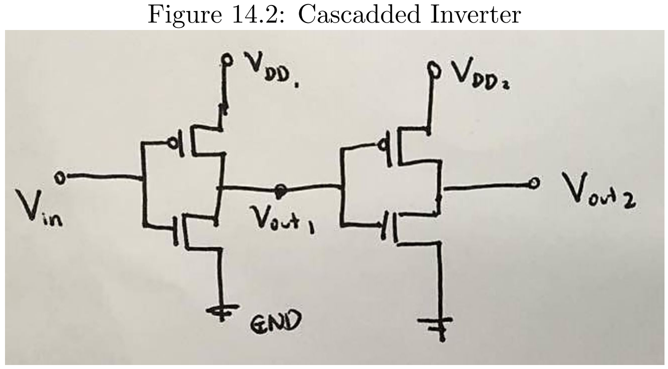


Figure 14.3: Simple RC Circuit

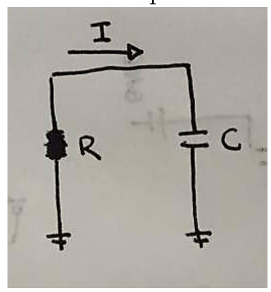
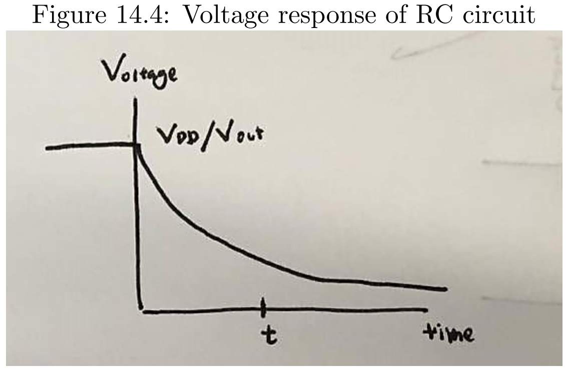


Figure 14.4: Voltage response of RC circuit



Our goal is to solve the differential equation in the representation of eigenvalues and eigenvectors.

$$V(t) = Ke^{\frac{t}{RC}}$$

The constant K is defined by the boundary condition. The initial condition before the voltage started to drop. In this case V_{out}

Now we can evaluate how long it took. How long does it take to reach the half life of the voltage?

$$t_{\text{half life}} = \ln 2RC$$

The equation is derived by setting $V(t) = \frac{1}{2}$ and solving for t. We see that bigger the values of R and C the longer it takes for the voltage to drop.