Lecture Notes: 20

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20.1 Inductors analysis in time domain

When we we deal with circuits in real life, we need to do more than simply amplify signals. We need to reject signals that we do not want, such as DC signal, high frequency noise etc. To accomplish variety of signal filtering we need to use Inductors. Inductors are counterpart to a capacitor. Before we move directly to phasor domain, we need to understand the signal response of the inductors in real time.

Table 20.1: Long table caption.

Capacitor	Inductor
"Fights changes in voltage"	"Fights changes in current"
Stores energy in electric field	Stores energy in magnetic field
At DC, capacitor is OPEN	At DC, Inductors are SHORT

Physics behind inductors: Inductors are simply wires wrapped around in a coil. The current flowing through the looped wire creates a magnetic field. The current determines the change in magnetic field. The change in magnetic field creates electric field. The electric field creates voltage.

In the previous lectures we saw the following relationship for the capacitor.

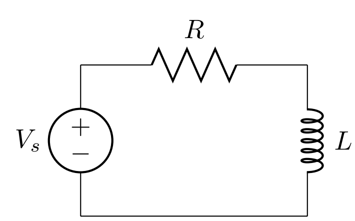
$$I(t) = C\frac{dv}{dt}$$

Inductor flips the roll of the current and the voltage.

$$V(t) = L\frac{dI}{dt}$$

The units of capacitance are in Ferads and Inductors are in Henrys

When we wanted to change the voltage fast in the RC circuit, we saw that the capacitor demanded high current. Similarly for Inductors, if we want to change the current, the inductor demands high voltage.



Observe the simple RL circuit above. If we replace the voltage source of an RL circuit by a short after a long period of time, what happens? Remember that after a long period of time the inductor behaves like a short.

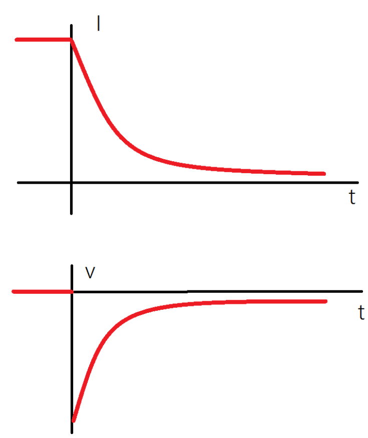
$$V_s = 1V \rightarrow 0V$$
$$V_L(t) = L\frac{dI}{dt}$$
$$V_R(t) = RI(t)$$

The voltage has to sum up to zero.

$$V_L(t) + V_R(t) = 0$$
$$L\frac{dI}{dt} = V_L(t) = -V_R(t) = -RI(t)$$
$$\frac{dI}{dt} = -\frac{R}{L}I(t)$$
$$I(t) = I_0 e^{-\frac{R}{L}t}$$

We see that the time constant for an RL cirucit is $\frac{L}{R}$.

Lets plot the respone



The jump we see in the voltage response is caused by the sharp discontinuity of the current.

20.2 Inductors analysis in phasor domain

What happens when we have an AC source? The relationship of an inductor to the current is given by

$$V_L(t) = L\frac{d}{dt}I(t)$$

Now, lets assume $I(t) = Ke^{j\omega t}$

$$= Lj\omega ke^{j\omega t}$$

We know that the general formula for impedance Z is given by

$$V = IZ$$

Solving for Z we find that impedance of an inductor is

$$Z_L = \frac{V}{I} = \frac{Lj\omega ke^{j\omega t}}{ke^{j\omega t}} = j\omega L$$

The equation tells us that inductors have a proportional relationship to the frequency.