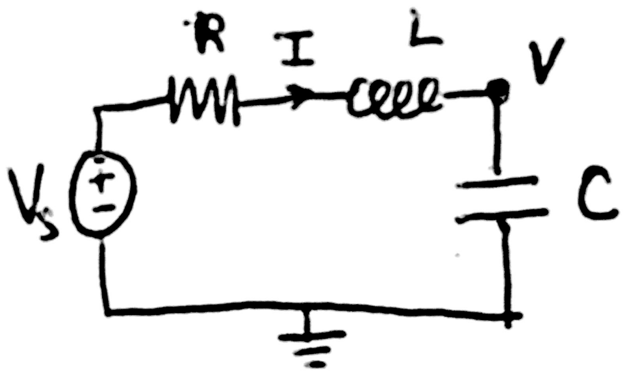
# EE 16B Designing Information Devices and Systems II
# Spring 2016 Lecture Note 3-9

## RLC Circuit

Recall the circuit from last lecture:




What would happen in a transient behavior like below:
$V_s$ = 1 for a long time then at time t=0, $V_s$ becomes 0 and stays at zero.

Unlike capacitors where we got two real eigenvalues, in this setup depending on the values of R, L and C, we can get complex values.

Especially, if R = 0, we are just left with L and C and it will be purely imaginary numbers and these imaginary eigenvalues will in general correspond to oscillations especially between the inductor and the capacitor.

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Recall that we always end up with equation as follows: $\frac{d}{dt}(x(t)) = s * x(t)$ X(0) = $X_0$ And it gets solved by x(t) = K$e^{st}$ where K = $X_0$. This is one solution.

How do we prove that this is the only solution?

Suppose there exists y(t) that solves the equation as well.

Consider $\frac{y(t)}{e^{st}} = y(t)e^{-st}$.

Check at t=0: $\frac{y(0)}{e^0} = X_0$ (from the condition "X(0) = $X_0$")

If we want to show that the value doesn't change over time, take derivative and see if it's zero:

Compute $\frac{d}{dt}(y(t)e^{-st}) = \frac{d}{dt}(y(t))e^{-st} + y(t)\frac{d}{dt}(e^{-st}) = s * y(t)e^{-st} - s * y(t)e^{-st} = 0$.

Thus, we know that this is the unique solution.

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Going back to the RLC problem, writing equations (through KVL and KCL), we get:

I(t) = $C\frac{dV}{dt}$

Knowing $V_s$ = 0, 0 - IR - V = $L\frac{dI}{dt}$

Writing these equations into a matrix form, $\frac{d}{dt}\begin{bmatrix} I(t) \\ V(t) \end{bmatrix} = \begin{bmatrix} \frac{R}{L} & \frac{-1}{L} \\ \frac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} I(t) \\ V(t) \end{bmatrix} = A \begin{bmatrix} I(t) \\ V(t) \end{bmatrix}$

Initial conditions are:
V(0) = 1
I(0) = 0

*TRICK: Usually, voltage through capacitor and current through inductor.*

Next step is to diagonalize this matrix by eigenvalues and eigenvectors: det(A - $\lambda I$) = 0
We get: $\lambda^2 + \frac{R}{L}\lambda + \frac{1}{LC} = 0$
Solve for $\lambda$:
$\lambda = \frac{-R}{2L} \pm \sqrt{(\frac{R}{2L})^2 - \frac{1}{LC}}$
Eigenvectors we choose to be: $\begin{bmatrix} 1 \\ \frac{1}{C\lambda} \end{bmatrix}$
Try: R= 0 and for simplicity, L=1, C=1 -> $\lambda_+ = \sqrt{-1} = +j$ and $\lambda_- = -\sqrt{-1} = -j$

Then for this example (after some algebra),
I(t) = $K_1 e^{jt} + K_2 e^{-jt}$ = -sin(t)
V(t) = cos(t)

Take note that if eigenvalues are purely imaginary, solutions in time domain are going to be sinusoids.

Try another example: R = 2, L=1, C=0.5 -> $\lambda = -1 \pm \sqrt{1-2} = -1 \pm j$

Then for this example (after some algebra),
I(t) = $K_1 e^{(-1+j)t} + K_2 e^{(-1-j)t} = e^{-t}(K_1 e^{jt} + K_2 e^{-jt})$ [phase shifted cosine, "damped oscillation" - looks like below graph]