EE16B: DESIGNING INFORMATION DEVICES AND SYSTEMS II

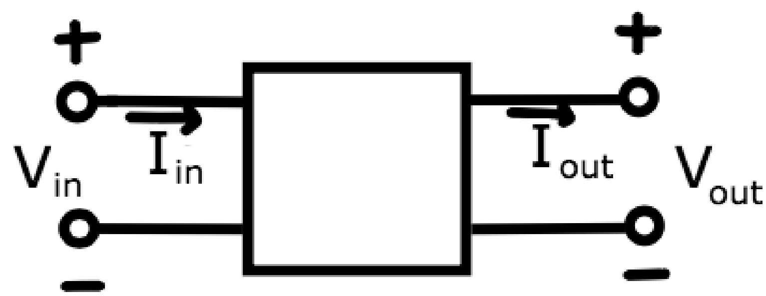
LECTURE NOTES

Introduction to Bode Plots

Bode plots are logarithmic plots of frequency response. Gain and phase are displayed in separate plots. The horizontal axis is frequency plotted on a log scale, and the vertical axis is gain, expressed in decibels (dB). The definition of a decibel is expressed in terms of power:

$$dB = 10 \log_{10} \left(\frac{P_{out}}{P_{in}} \right)$$

Consider the system below:



$$P = IV = \frac{V^2}{R}$$

Therefore,

$$dB = 10 \log_{10} \left(\frac{\frac{V_{out}^2}{R}}{\frac{V_{in}^2}{R}} \right)$$

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Canceling out the resistance values leads to :

$$dB = 10\log_{10}\left(\frac{V_{out}}{V_{in}}\right)^2 = 20\log_{10}\left(\frac{V_{out}}{V_{in}}\right)$$

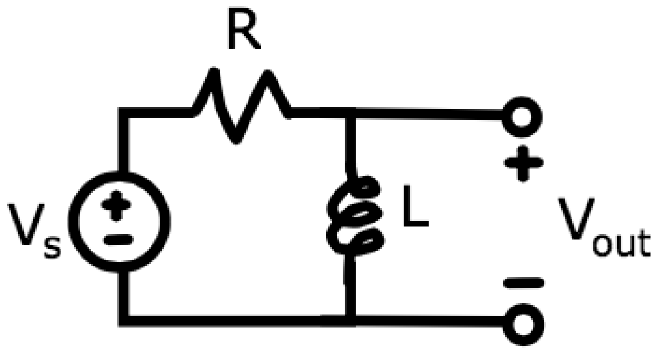
Decibels are useful because they cover an enormous range of gain values. For voltage, a decrease in 20 dB corresponds a reduction of the gain by 1 order of magnitude. A gain of 1 corresponds to 0 dB.

Other properties of decibels and transfer functions:

$$H = XY \rightarrow H(dB) = X[dB] + Y[dB]$$

$$H = \frac{X}{Y} \rightarrow H(dB) = X[dB] - Y[dB]$$

Consider the circuit below:



$$V_{out} = \left(\frac{j\omega L}{R + j\omega L}\right)V_s$$

$$H(\omega) = \frac{V_{out}}{V_s} = \frac{j\omega l}{R + j\omega L} = \frac{j\left(\frac{\omega}{\omega_c}\right)}{1 + j\left(\frac{\omega}{\omega_c}\right)}$$

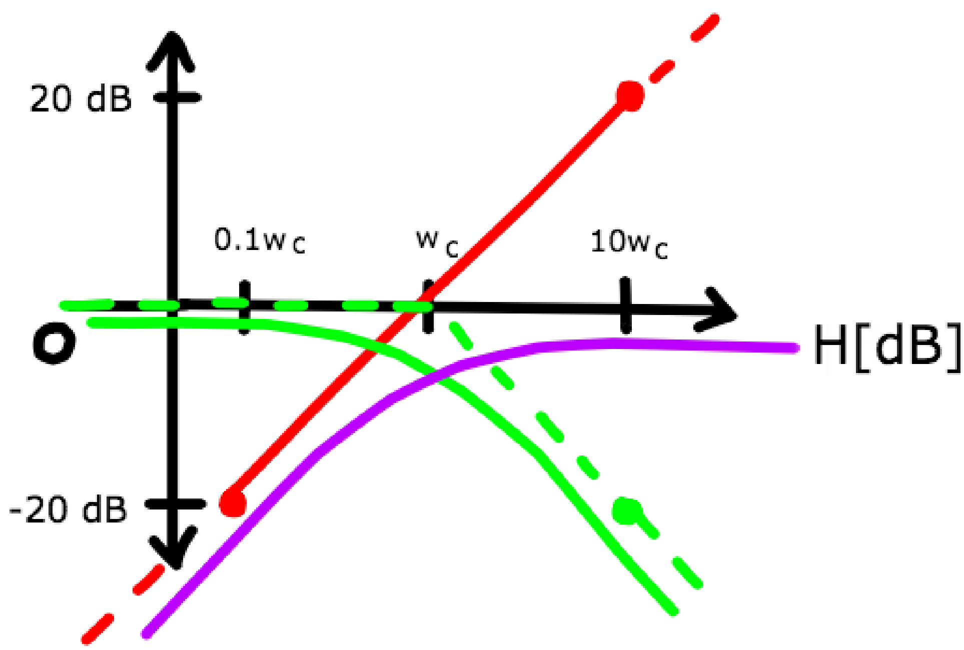
w_c is the *corner frequency* and in this case is defined as $\frac{R}{L}$

$$\rightarrow |H(\omega)| = \frac{\frac{\omega}{\omega_c}}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^2}}$$

$$H[dB] = 20\log_{10}|H| = 20\log\left(\frac{\omega}{\omega_c}\right) - 20\log\left[1 + (\frac{\omega}{\omega_c})^2\right]^{1/2}$$

$$\rightarrow H[dB] = 20\log_{10}|H| = 20\log\left(\frac{\omega}{\omega_c}\right) - 10\log\left[1 + (\frac{\omega}{\omega_c})^2\right]$$

The bode plot of the frequency response magnitude is (in purple):



The first term of the frequency response, $20\log\left(\frac{\omega}{\omega_c}\right)$, is shown in red. The second, $10\log\left[1 + (\frac{\omega}{\omega_c})^2\right]$, is in green. The asymptotes of the second term are shown with the dashed green line. Considering the second term as $\omega \rightarrow 0$, the term itself goes to 0. It then changes value at the corner frequency as $\omega \rightarrow \infty$. The bode plot of the total frequency response (shown in purple) can be determined by subtracting the second term from the first.

The phase response and plot are:

$$\Phi(w) = 90^o - \tan^{-1}\left(\frac{\omega}{\omega_c}\right)$$