EE 16B Lecture Notes

Introduction to Controls

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Control is really about closing the loop. So far in this class, we've been talking about extracting information and processing this information. So, beyond "listening" we want to do something with this information, so we use"Controls" to "do" things. An example is a thermostat, which takes in information and adjusts its actions accordingly to "control" the temperature to a set range. Control ideas can be found in robotics, to computer science, to the body's own homeostatic mechanisms. The key idea in controls is "feedback". So you take in information, process, and act, and cycle through this loop.

So an outline of this discussion of controls will be:

1. Modeling
2. Controllability and Observability
3. Desired Closed-Loop Behavior

This lecture will focus on the Modeling aspect of controls.

Modeling:

Linear models for dynamics:

The models we will talk about in this course will be "State-Space Models". State-space models mean you have a sense of "state" of a system; the state of a system plays a key role in the model. State simply means that you know everything about a system to understand what will happen in the future. As an example, the "state" of a circuit is knowledge of explicit currents and voltages within a circuit. It's usually a sufficient state to know all the voltages on the capacitors and all the currents of the inductors.

Discrete time state-space model:

$$\vec{x}[t+1] = A\vec{x}[t] + B\vec{u}[t] + \vec{\omega}[t] \tag{1}$$

In which $A\vec{x}[t]$ is the current state, $\vec{x}[t+1]$ is the next state, B is the control that acts on some input $\vec{u}[t]$and $\vec{\omega}[t]$ is the disturbances. Within this equation A must be a square matrix and B and C do NOT have to be square matrices.

Sometimes we can't observe the entire state of a system untransformed, we can only see a transformed version of the state. So, we can model off of our observations:

$$\vec{Y}[t] = C\vec{x}[t] + \vec{v}[t] \tag{2}$$

In which $\vec{Y}[t]$ is the observation that is equal to some matrix C that transforms $\vec{x}[t]$, the current state, and there is some observed noise, $\vec{v}[t]$.

<u>Continuous time state-space model</u>:

$$\frac{d}{dt}\vec{x}(t) = A\vec{x}(t) + B\vec{u}(t) + \vec{\omega}(t) \tag{3}$$

$$\vec{y}(t) = C\vec{x}(t) + \vec{v}(t) \tag{4}$$

The usual flow of using these models is as follows:

Real World $\longrightarrow$ Write out differential equations

$\downarrow$

put it into standard form
$\frac{d}{dt}\vec{x}(t) = f(\vec{x}, \vec{u}, \vec{\omega})$

$\downarrow$

Pick Operating Point $\longrightarrow$ Linearize f around operating point

choose variables: $\vec{z} = \vec{x} - \vec{x}_{op}$, $\tilde{u} = \vec{u} - \vec{u}_{op}$
$\frac{d}{dt}\vec{z}(t) = f(\vec{x}_{op} + \vec{z}, \vec{u}_{op} + \vec{u}, \vec{\omega})$

How do we linearize?
Well in scalar form, we know:
Given scalar $y = g(v)$, we want to linearize about $y^*, v^*)$

$$y \approx y^* + g'(v^*)(v - v^*) \tag{5}$$

The $g'(v^*)$ value is really just a row vector of partial derivatives $[\frac{\partial g}{\partial v_0}, \frac{\partial g}{\partial v_1}, \ldots, \frac{\partial g}{\partial v_{n+1}}]$

So, when we need to linearize for matrices, given $\vec{y} = \vec{g}(\vec{v})$:

$$\vec{y} \approx \vec{y}^* + \Box(\vec{v} - v^*) \tag{6}$$

The $\Box$ value is really just a Jacobian Matrix of first of partial derivatives in which the value of the ith row and jth column is $\frac{\partial g_i}{\partial v_j}$