EE16B: DESIGNING INFORMATION DEVICES AND SYSTEMS II

LECTURE NOTES

Linear Models for Dynamics

A *discrete-time dynamical system* has the form:

$$\vec{x}(t+1) = a\vec{x}(t) + \vec{u}(t) \tag{1}$$

If $\vec{x}(0) = x_o$, then
$\vec{x}(1) = ax_o + u(0)$
$\vec{x}(2) = a^2x_o + au(0) + u(1)$
$\vec{x}(3) = a^3x_o + a^2u(0) + au(1) + u(2)$

The system $\vec{x}$ can be represented in matrix form:

$$\vec{x} = \begin{bmatrix} 1 & 0 & 0 & \dots \\ a & 1 & 0 & \dots \\ a^2 & a & 1 & \dots \\ a^3 & a^2 & a & \dots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} x_o \\ u(0) \\ u(1) \\ u(2) \\ \vdots \end{bmatrix}$$

By inspection, we can write the solution to the discrete case as:

$$\vec{x}(t) = a^t x_o + \sum_{\tau=0}^{t-1} a^{t-1-\tau}\vec{u}(\tau)$$

This solution can be verified by plugging in $t+1$

$$\vec{x}(t+1) = a^{t+1}x_o + \sum_{\tau=0}^{t} a^{t-\tau}\vec{u}(\tau)$$

so,

$$\vec{x}(t+1) = a\vec{x}(t) + \vec{u}(t)$$

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This is the same as equation **??**, which means our solution is correct!

Consider system

$$\vec{x}(t+1) = A\vec{x}(t) + \vec{u}(t) \tag{2}$$

Where A is a diagonalizable matrix $A = V\Lambda V^{-1}$.

Let

$$\vec{z}(t) = V^{-1}\vec{x}(t) \tag{3}$$

So then,

$$\vec{z}(t+1) = V^{-1}\vec{x}(t+1)$$

Plugging in equation **??** leads to

$$\vec{z}(t+1) = V^{-1}(A\vec{x}(t) + \vec{u}(t)) = V^{-1}A\vec{x}(t) + V^{-1}\vec{u}(t)$$

By definition of matrix A and plugging in equation **??**,

$$\vec{z}(t+1) = \Lambda z(t) + V^{-1}\vec{u}(t)$$

By inspection again, $\vec{z}(t)$ can be written as:

$$\vec{z}(t) = \Lambda^t \vec{z}_o + \sum_{\tau=0}^{t-1} \Lambda^{t-1-\tau} V^{-1}\vec{u}(\tau)$$

Solving for $\vec{x}(t)$:

$$\vec{x}(t) = V\vec{z}(t) = V\Lambda^t V^{-1}\vec{x}_o + \sum_{\tau=0}^{t-1} V\Lambda^{t-1-\tau} V^{-1}\vec{u}(\tau)$$

$$\implies \vec{x}(t) = A^t\vec{x}_o + \sum_{\tau=0}^{t-1} A^{t-1-\tau}\vec{u}(\tau)$$

We can check if this solution is correct by finding $\vec{x}(t+1)$,

$$\vec{x}(t+1) = A(A^t\vec{x}_o + \sum_{\tau=0}^{t-1} A^{t-1-\tau}\vec{u}(\tau)) + \vec{u}(t)$$

$$\vec{x}(t+1) = A\vec{x}(t) + \vec{u}(t)$$

This is the same as equation **??**, so this is the correct solution.