EE16B: DESIGNING INFORMATION DEVICES AND SYSTEMS II

LECTURE NOTES

Controls Continued

Consider system:

$$\vec{x}(t+1) = A\vec{x}(t) + \vec{b}u(t) + \vec{\omega}(t)$$

where $\vec{\omega}(t)$ is the disturbance of the system.

Goal: Find a control law $u(t)$ such that $u(t) = [f_o f_1 \dots f_k]\vec{x}(t)$ where $\vec{x}(t+1) = (A+bk)\vec{x}(t) + \vec{\omega}(t)$ so that $(A+bk)$ has the desired eigenvalues.

The basis $[\vec{b}, A\vec{b}, \dots A^{n-1}\vec{b}]$ is full rank.

Subgoal: Find a transformation of coordinates so the system $\vec{z}(t) = T\vec{x}(t)$.

$$\vec{z}(t+1) = \begin{bmatrix} 0 & & & \\ 0 & & & \\ 0 & & I & \\ \vdots & & & \\ a_o & a_1 & \dots & a_{n-1} \end{bmatrix} \vec{z}(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} \vec{u}(t)$$

(I is the identity matrix.)
The eigenvalues are the roots of the characteristic polynomial

$$\sum_{i=0}^{n-1} a_i \lambda^i$$

They can also be found by setting $det(A - \lambda I)$ equal to 0.

The basis $[\vec{b}, A\vec{b}, \dots A^{n-1}\vec{b}]$ takes A, b to:

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$$\tilde{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & a_o \\ & & & & a_1 \\ & & I & & a_2 \\ & & & & \vdots \\ & & & & a_{n-1} \end{bmatrix}, \tilde{b} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$$

The eigenvalues of A are the same as the eigenvalues found by the above characteristic polynomial $\longrightarrow$ When you change coordinates, you don't change the eigenvalues!

Proof:

$$A = G\tilde{A}G^{-1}$$
$$det(A - \lambda I) = det(G\tilde{A}G^{-1} - \lambda I) = det(G(\tilde{A} - \lambda I)G^{-1})$$
$$= det(G)det(G^{-1})det(\tilde{A} - \lambda I)$$

$det(G)det(G^{-1}) = 1$ so,

$$det(A - \lambda I) = det(\tilde{A} - \lambda I)$$

$\longrightarrow$ A and $\tilde{A}$ have the same eigenvalues and the same characteristic polynomial.