Lecture Notes: 35

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35.1 Block Diagram

Assume an example of a system with position and velocity.

$$\frac{d}{dt}\vec{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \vec{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \vec{u}(t)$$

We can only observe

$$y(t) = [1 \quad 0]\vec{x}(t)$$

We check observability and see that it is full rank. We make an observer $\hat{x}(t) = \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix}$ and rewrite the relationship

$$\frac{d}{dt}\hat{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \hat{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \vec{u}(t) + \begin{bmatrix} l_1 \\ l_2 \end{bmatrix} (y(t) - \hat{y}(t))$$

$$\hat{y}(t) = [1 \quad 0]\hat{x}(t)$$

We want to design l_1, l_2 to control the error where we denote the error as

$$\vec{e}(t) = \vec{x}(t) - \hat{x}(t)$$

$$\frac{d}{dt} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \vec{e}(t) - \begin{bmatrix} l_1 \\ l_2 \end{bmatrix} [1 \quad 0]\vec{e}(t)$$

$$= \begin{bmatrix} -l_1 & 1 \\ -l_2 & 0 \end{bmatrix} \vec{e}(t)$$

We are interested in the characteristic polynomial

$$det(\lambda I - \begin{bmatrix} -l_1 & 1 \\ -l_2 & 0 \end{bmatrix}) = det(\begin{bmatrix} \lambda + l_1 & -1 \\ l_2 & \lambda \end{bmatrix}) = \lambda^2 + l_1\lambda + l_2$$

We want the system to be stable and want the eigenvalues to be in the left side of the complex plane.

For example, if we wanted our eigenvalues at -2, we would want our characteristic polynomial to be

$$\lambda^2 + 4\lambda + 4$$

Hence, we see that $l_1 = 4$ and $l_2 = 4$. Now rewriting our system with an observer

$$\frac{d}{dt}\hat{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \hat{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \vec{u}(t) + \begin{bmatrix} 4 \\ 4 \end{bmatrix} (\hat{y}(t) - \hat{x}_1(t))$$

Writing it out in scalars we see

$$\frac{d}{dt}\hat{x}_1(t) = \hat{x}_2(t) + 4y(t) - 4\hat{x}_1(t)$$

$$\frac{d}{dt}\hat{x}_2(t) = u(t) + 4y(t) - 4\hat{x}_1(t)$$

To enforce this relationship we would want an integrator. We can construct an integrator using a capacitor and an op amp. A block diagram representation allows us to represent complex circuits in a simple black-box blocks, where input comes in from one side and output goes out the other side.

We represent the above relationship using block diagram below.