EE16B Designing Information Devices and Systems II **Spring 2016**

Lecture Notes: 35

Lecturer: Anant Sahai, Michel Maharbiz
Scribe: Minyoung Huh

We want to figure out the amount of samples required to understand a continuous signal in discrete space. We want to discretize a signal without losing information. Given object x, we can make measurements at discrete points in time $x(t)$. What does it mean to extract all information? If we are given the measurement of x we can perfectly reconstruct x.

Suppose $x \in R^n$. We want to make m measurements of x to to perfectly reconstruct x. If nothing is known about x, then $m = n$. If there is a prior information $m \leq n$. We want a structure in x.

Take an example of x with known structure

$$\vec{x} = \begin{bmatrix} a \\ b \\ c \\ 0 \\ 0 \end{bmatrix}$$

We do not need 5 measurement to know x, and only need 3. We only need to measure a, b, c since we know last 2 measurements are 0.

Take a specific example of a line where we know how to draw the line with just 2 points

$$\vec{x} = a \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + b \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}$$

Given 2 unique measurements of

$$x[1] = y_1, x[3] = y_3$$

How can we figure out a, b ? We can construct the system of linear equations

$$y_1 = a + b$$
$$y_3 = a + 3b$$

Now lets generalize the above example

$$\vec{x} = a\vec{1} + b\vec{t}$$

For general $k-1$-polynomial we have

$$\vec{x} = \sum_{i=0}^{k-1} a_i \vec{t}^{\,i}$$

This polynomial is defined by k parameters a_i. The dimension of this function is k.

<u>Theorem</u>
Given k samples, we can reconstruct a polynomial of at most $k-1$ degree.

Explicit algortihm to reconstruct $\vec{x}$

We know that $\vec{x}$ is expressible in the a_i basis. We know k pairs of measurements $(t_{ji}, x(t_{ji}))$. We are going to use Lagrange Interpolation.

Goal
We want to find appropriate basis $\vec{l_i}$ to represent $\vec{x}$

$$\vec{x} = \sum_{i=0}^{k-1} x(t_{ji}) \vec{l_i}$$

Sub Goal
We want l_i to have the following property

$$\vec{l_i}(t_{ji}) = 1$$

for $m \neq i$

$$\vec{l_m} t_{ji} = 0$$

If this property holds for all $\vec{l_i}$ then we can reconstruct back the measurements.

We also want the degree of polynomial l_i to be $k-1$.

$$l_i(t) = \frac{\prod_{j \neq i}(t - t_j)}{\prod_{j \neq i}(t_i - t_j)}$$

We will continue this proof in next lecture