EE16B: DESIGNING INFORMATION DEVICES AND SYSTEMS II

LECTURE NOTES

Interpolation & Sampling

Light travels at a speed of 3x10$^8 m/s$. We want to sample the signal fast enough to track distances of 1 meter. This means we need 300x10$^6 samples/second$.

We use interpolation when we want to figure out what happens in between samples.

Consider the polynomial:

$$x(t) = \sum_{i=0}^{k-1} a_i t^i$$

which is at most a $k-1$ degree polynomial.
Given k samples (t_i, y_i) where $y_i = x(t_i)$, we can recover $\vec{\alpha}$ and hence $x(t)$ perfectly:

$$\begin{bmatrix} 1 & t_1 & t_1^2 & \dots \\ 1 & t_2 & t_2^2 & \\ 1 & t_3 & t_3^2 & \\ & \vdots & & \ddots \\ & & & \end{bmatrix} \begin{bmatrix} \vec{\alpha} \end{bmatrix} = \begin{bmatrix} \vec{y} \end{bmatrix}$$

So $\vec{\alpha} = V_s^{-1}\vec{y}$ where V_s is the first matrix in the above relation.

We can say that

$$\hat{x}(t) = \sum_{i=0}^{k-1} y_i l_i(t)$$

where

$$l_i(t) = \frac{\prod_{j \neq i} t - t_j}{\prod_{j \neq i} t_i - t_j}$$

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$l_i(t)$ is a degree $k-1$ polynomial and is zero at all $t = t_j$ for $j \neq i$.

By construction, $\hat{x}(t_i) = y_i = x(t_i)$ where $\hat{x}(t)$ is a degree at most k-1 polynomial.

We want to show that $x(t) = \hat{x}(t)$ everywhere. Consider $\hat{x}(t) - x(t)$. This is a degree at most $k-1$ polynomial.

Fact: A degree at most k polynomial is either zero everywhere or it has at most k roots.
Subfact: If $p(x) = 0$ at x, then $p(x) = q(x)(x - x_1)$ where q(x) has degree one less than p(x).

$x(t) - \hat{x}(t)$ has at least k roots, and $k > k - 1 \longrightarrow x(t) - \hat{x}(t) = 0$ everywhere.