EECS 16B Designing Information Devices and Systems II Spring 2017 Murat Arcak and Michel Maharbiz Discussion 6A

System stability conditions

Discrete time systems

A discrete time system is of the form,

$$\vec{x}(t+1) = A\vec{x}(t) + B\vec{u}(t)$$

Let λ be any particular eigenvalue of *A*. This system is stable if $|\lambda| < 1$.

Continuous time systems

A continuous time system is of the form,

$$\frac{\mathrm{d}\vec{x}}{\mathrm{d}t}(t) = A\vec{x}(t) + B\vec{u}(t)$$

Let λ be any particular eigenvalue of *A*. This system is stable if Re{ λ } < 0.

Questions

1. Discrete-time Stability

Determine which values of α and β will make the following discrete-time state space models stable:

(a)

$$x[t+1] = \alpha x[t]$$

(b)

$$\vec{x}[t+1] = \begin{bmatrix} lpha & -eta \\ eta & lpha \end{bmatrix} \vec{x}$$

(c)

$$\vec{x}[t+1] = \begin{bmatrix} 1 & \alpha \\ 0 & 1 \end{bmatrix} \vec{x}$$

2. Continuous-time Stability

Consider the linearized state space model for an inverted pendulum in the up position, which we found in lecture:



Figure 1: Pendulum Free-body Diagram

$$\frac{d}{dt}\vec{x} = \begin{bmatrix} 0 & 1\\ \frac{g}{l} & \frac{-k}{l} \end{bmatrix} \vec{x}$$
$$\vec{x} = \begin{bmatrix} \theta\\ \frac{d\theta}{dt} \end{bmatrix}$$

Where

(a) Is this system stable?

Contributors:

- Siddharth Iyer.
- John Maidens.