EECS 16B Designing Information Devices and Systems II
Spring 2017 Murat Arcak and Michel Maharbiz **Homework 11**

This homework is due April 19, 2017, at 17:00.

1. Interpolation

Samples from the sinusoid $f(x) = \sin(0.2\pi x)$ are shown in Figure 1. Draw the results of interpolation using each of the following three methods:

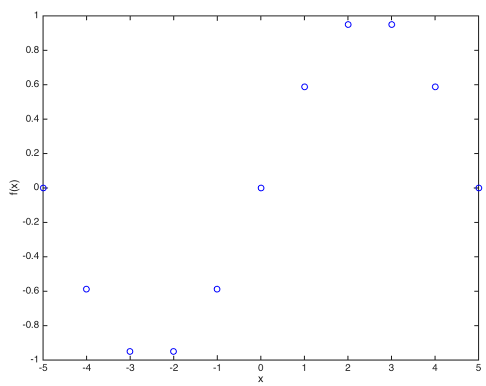

Figure 1: Samples of $f(x)$.

.

(a) Zero order hold interpolation.

(b) Linear interpolation.

(c) Sinc interpolation assuming the Nyquist limit has been satisfied.

2. Linear interpolation

Consider a piecewise-linear real valued function $f(x)$ such that,

(a) In the interval $[k, k+1]$ where k is an integer, f is a line straight line.

(b) $f(x)$ is zero for $x < k_1$ and $f(k_1) = 0$.

(c) $f(x)$ is zero for $x > k_2$ and $f(k_2) = 0$.

Consider the function $\phi(x)$ defined as,

$$\phi(x) = \begin{cases} 1 - |x|, & x \in [-1, 1] \\ 0, & \text{otherwise} \end{cases}$$

(a) Sketch $\phi(x-k)$ for some arbitrary integer k.

(b) Write the basis function and coefficient that captures the line of $f(x)$ from $x = k_1$ to $x = k_1 + 1$. That is to say, find real number α and integer p such that,

$$f(x) = \alpha\phi(x - p) \text{ for } x \in [k_1, k_1 + 1]$$

(c) What is the equation of the line from k to $k+1$, where $[k, k+1]$ is within $[k_1, k_2]$? That is to say, find an equation of the form,

$$y = mx + c,$$

that represents the line in f between k and $k+1$.

(d) Consider the function,

$$g(x) = f(k)\phi(x - k) + f(k+1)\phi(x - (k+1))$$

What is the equation of the line formed between $[k, k+1]$, where $[k, k+1]$ is within $[k_1, k_2]$? Write it once again in the form,

$$y = mx + c.$$

This should match your previous answer.

(e) Given the answers to the previous parts, we have shown that we can break down f into a linear sum of shift ϕ functions. Find the coefficients α_k such that,

$$f(x) = \sum_{k \in \mathbb{Z}} \alpha_k \phi(x - k)$$

3. Sampling a continuous-time control system to get a discrete-time control system

Recall from Lecture 12A that a continuous-time system

$$\begin{aligned} \frac{d}{dt}\vec{x}(t) &= A\vec{x}(t) \\ y(t) &= C\vec{x}(t) \end{aligned} \tag{1}$$

can be represented with a discrete-time model

$$\begin{aligned} \vec{x}_d(k+1) &= A_d\vec{x}(k) \\ y_d(k) &= C\vec{x}_d(k) \end{aligned} \tag{2}$$

where $\vec{x}_d(k)$ and $y_d(k)$ are the values of the state $\vec{x}(t)$ and output $y(t)$ at time instants $t = kT$, $k = 1, 2, 3, \ldots$ In this problem we will see that the observability of (1) does not necessarily imply observability of (2): there may be sampling periods T that fail to preserve observability. Since observability depends on A and C alone we have omitted the inputs in the equations above.

(a) Suppose A is diagonalizable; that is, there exists a matrix P such that

$$P^{-1}AP = \begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix}.$$

Show that

$$A_d = P \begin{bmatrix} e^{\lambda_1 T} & & \\ & \ddots & \\ & & e^{\lambda_n T} \end{bmatrix} P^{-1}.$$

To do so, you can introduce the new state vector $\vec{z} = P^{-1}\vec{x}$ and then use the result from Lecture 12A for the discretization of a diagonal A matrix to obtain a discrete-time model for $\vec{z}_d(k)$. You would then return to the original state with $\vec{x}_d(k) = P\vec{z}_d(k)$ to obtain (2).

(b) Use the result of part (a) to calculate A_d as a function of the sampling period T when

$$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.$$

(c) Let $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$ so that (A,C) is an observable pair. Show that there exist values of T for which (A_d,C) is *not* observable. (Hint: compare the discrete-time model to the example in Lecture 7B.)

4. Aliasing

Watch the following video: `https://www.youtube.com/watch?v=jQDjJRYmeWg`.

Assume the video camera running at 30 frames per second. That is to say, the camera takes 30 photos within a second, with the time between photos being constant.

(a) Given that the main rotor has 5 blades, list *all* the possible rates at which the main rotor is spinning in revolutions per second assuming no physical limitations.
Hint: Your answer should depend on k where k can be any integer.

(b) Given that the back rotor has 3 blades and completes 2 revolutions in 1 second **in the video**, list *all* the possible rates at which the back rotor is spinning in revolutions per second assuming no physical limitations.
Hint: Your answer should depend on k where k can be any integer.

Contributors:

- John Maidens.
- Siddharth Iyer.
- Murat Arcak.