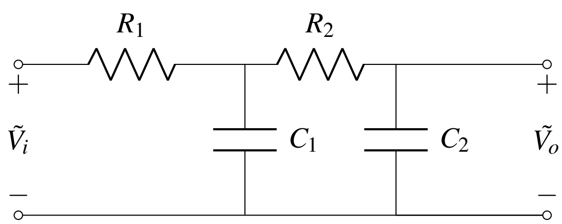
EECS 16B Designing Information Devices and Systems II
Spring 2017 Murat Arcak and Michel Maharbiz **Homework 3**

This homework is due February 15, 2017, at 17:00. This homework is **OPTIONAL**. However, you are highly encouraged to do this homework since everything on it, except for bode plots, is within the scope of midterm 1.

1. Transfer functions

Consider the circuit below.



The circuit has an input phasor voltage $\tilde{V}_i$ at frequency ω rad/sec applied at the input terminals shown in the illustration above, causing an output phasor voltage $\tilde{V}_o$ at output terminals.

(a) Obtain an expression for $H = \tilde{V}_o/\tilde{V}_i$ in terms of $Z_{R1}, Z_{R2}, Z_{C1}, Z_{C2}$ (they are the impedance of R_1, R_2, C_1, C_2, respectively).

(b) Obtain an expression for $H(\omega) = \tilde{V}_o/\tilde{V}_i$ in the form of

$$H(\omega) = \frac{\tilde{V}_o}{\tilde{V}_i} = \frac{1}{1 + j2\xi(\omega/\omega_c) + (j\omega/\omega_c)^2},$$

given that $R_1 = 1\Omega$, $R_2 = 2\Omega$, $C_1 = 1F$, and $C_2 = 2F$. What are the values of ξ and ω_c?

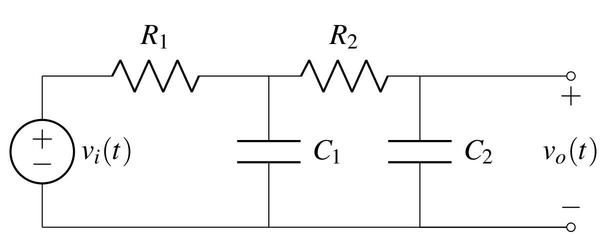
(c) We can express the transfer function $H(\omega)$ in the polar form. That is,

$$H(\omega) = M(\omega)e^{j\phi(\omega)}$$

The functions $M(\omega)$ and $\phi(\omega)$ are the magnitude and the phase angle of $H(\omega)$, respectively. Write down $M(\omega)$ and $\phi(\omega)$ using the transfer function you derived in part (b).

(d) Compute the phasors of $H(0)$, $H(\omega_c)$, and $H(\infty)$ using the results in part (b) and (c).

(e) Consider the circuit below.



The voltage source is given by

$$v_i(t) = 12\sin\left(\frac{1}{2}t - \frac{\pi}{4}\right)$$

The values of R_1, R_2, C_1, and C_2 are the ones given in part (b). Obtain an expression for $v_o(t)$ in the form of $\alpha\cos(\frac{1}{2}t+\theta)$.

2. **Active filter**

   (a) problem 9.44 from the textbook

Problem 9.44 Show that the transfer function of the circuit shown in Fig. P9.44 is given by

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_s} = -G\left(1 + j\frac{\omega}{\omega_c}\right),$$

and relate G and ω_c to R_1, R_2, and C.

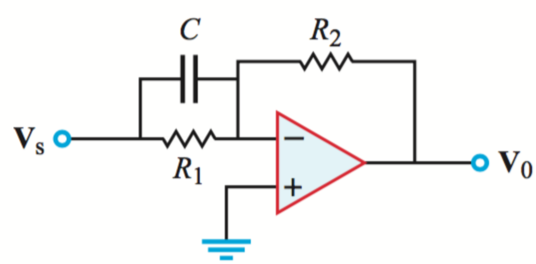


Figure P9.44 Circuit for Problem 9.44.

Figure 1: Reproduced from Ulaby, Maharbiz, Furse. Circuits. Third Edition. with permission

3. **Bass-booster**

   RC circuits and filters are very useful for altering the frequency content of signals. For example, audio equalization equipment can use these filters to adjust the pitch content of audio signals. Suppose we want to boost the bass of our favorite music, we can use the active filter circuit below to tune the frequency content of our favorite jams.

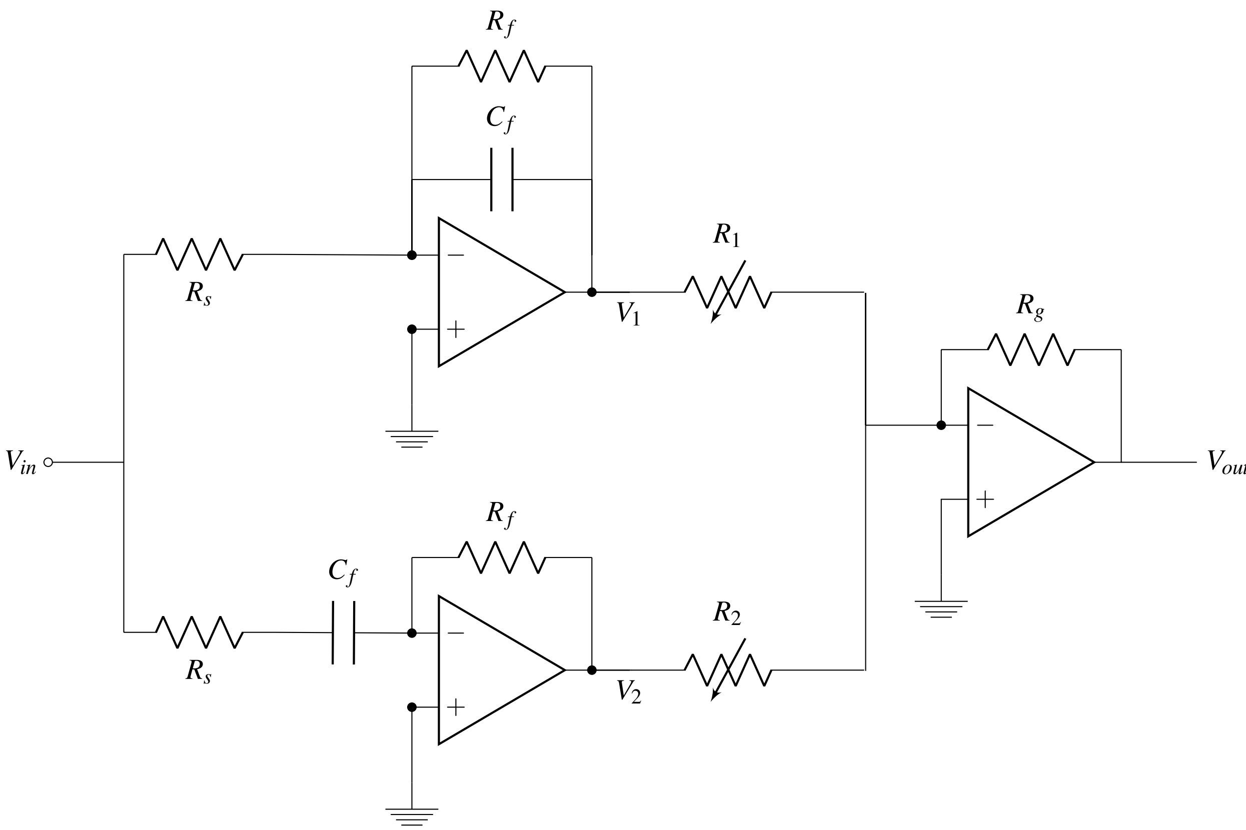


Figure 2: Audio Equalizer Circuit

$C_f = 400$ nF, $R_f = 1\text{k}\Omega$, $R_g = 100\Omega$, and $R_s = 1\text{k}\Omega$. R_1 and R_2 are both variable capacitors that can be used to tune the frequency balance of our output signal. Assume the input audio signal is $V_{in} = \cos(\omega t)$.

(a) First treat the first two active filters as disconnected and independent from the third (see figure below). Find expressions for $H_1(\omega)$= $\frac{\tilde{V_1}}{\tilde{V_{in}}}$ and $H_2(\omega) = \frac{\tilde{V_2}}{\tilde{V_{in}}}$, where $\tilde{V_{in}}$ and $\tilde{V_1}$, and $\tilde{V_2}$ are the phasor transformations of the voltages labeled on the circuit. Sketch the bode plots for the magnitude of these transfer functions. What kind of filter is each transfer function?

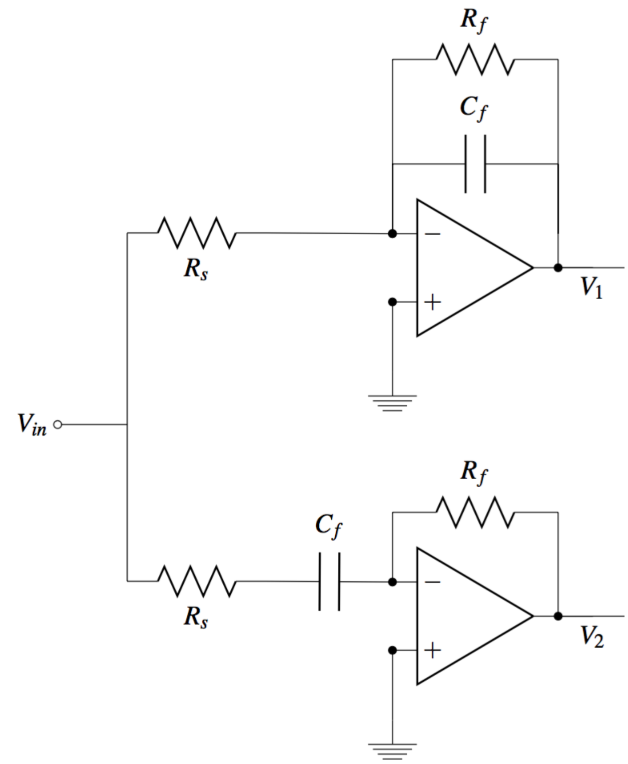


Figure 3: First Stage of Audio Equalizer

(b) If we connect the outputs, $\tilde{V_1}$ and $\tilde{V_2}$, to R_1 and R_2, respectively, will $\tilde{V_1}$ and $\tilde{V_2}$ change? Can we use our expressions for $\tilde{V_1}$ and $\tilde{V_2}$ that we found in the previous part to represent $\tilde{V_1}$ and $\tilde{V_2}$ in the full three op amp circuit?

(c) Find a function that describes $\tilde{V_{out}}$ in terms of $\tilde{V_1}$ and $\tilde{V_2}$

(d) Combine the results of the last two parts to find an overall transfer function for $H_{ov}(\omega) = \frac{\tilde{V_{out}}}{\tilde{V_{in}}}$.

(e) Using this circuit, how could we set R_1 and R_2 to boost our bass frequency signals ($f < 400$ Hz) without affecting mid and treble range signals? Sketch a bode plot of the magnitude of your overall bass-boosting function.

Contributors:

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