EECS 16B Designing Information Devices and Systems II
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Discussion 10A

1. Controls

Consider the following system:

$$\frac{dx_1(t)}{dt} = -x_1(t)^2 + x_2(t)u(t)$$
$$\frac{dx_2(t)}{dt} = 2x_1(t) - 2x_2(t)u(t)$$

(a) Choose states and write a state space model for the system in the form $\frac{d\vec{x}(t)}{dt} = f(\vec{x}(t), u(t))$.

(b) Find the equilibrium $\vec{x}^*$ and input u^* when $x_2^* = 1$ and $u^* = 1$.

(c) Linearize the system around the equilibrium state and input from the previous part. Your answer should be in the form $\frac{d\tilde{\vec{x}}(t)}{dt} = A\tilde{\vec{x}}(t) + B\tilde{u}(t)$.

(d) Is this system controllable? Is it stable?

(e) Convert this system into controller canonical form. Your answer should be in the form $\frac{d\vec{z}(t)}{dt} = \tilde{A}\vec{z}(t) + \tilde{B}\tilde{u}(t)$, where $\vec{z}(t) = T\tilde{\vec{x}}(t)$.

(f) Find a state feedback controller $\tilde{K}$ to place both system eigenvalues at $\lambda = -1$, where $\tilde{u}(t) = -\tilde{K}\vec{z}(t)$.

(g) Convert this feedback controller back into the non-CCF domain, i.e., find K, such that $\tilde{u}(t) = -K\tilde{\vec{x}}(t)$.
(*Hint:* Remember that $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.)

(h) Is our system observable? Our observed output is $y(t) = -\tilde{x}_1(t)$.

(i) Construct an observer system for this system.

(j) How does the error evolve over time?

(k) Pick L, such that our error signals converge to 0. Place both eigenvalues at $\lambda = -1$ again.

(l) Write out the entire closed-loop system with feedback control and observer.

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