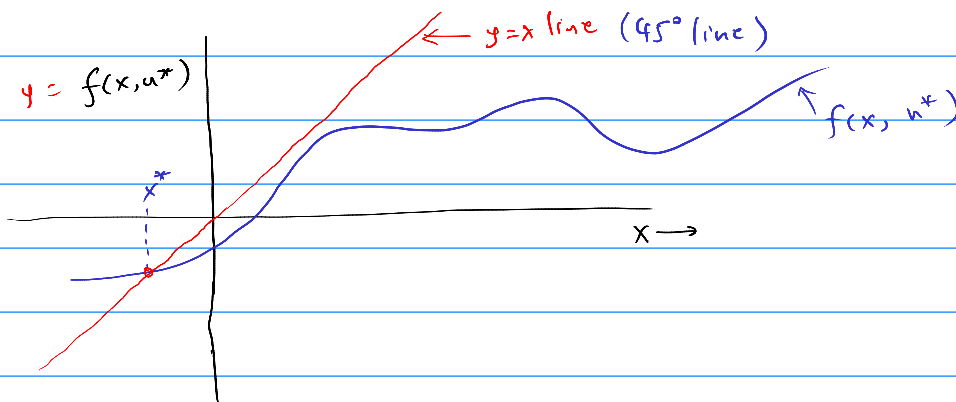
LINEARIZATION OF A DISCRETE-TIME SYSTEM:

$$\vec{x}[t+1] = \vec{f}(\vec{x}[t], u[t]) \qquad (1)$$

1. Choose a "DC" input: $u[t] = u^* \quad \forall t = 1,2,3,$ (2)
2. Find (assuming you can) a DC solution x^*
   - DC sol. means no change w t: $\Rightarrow x[t] = x^* \quad \forall t$ (3)
     - in particular, $x[t+1] = x[t] = x^*$ (4)
   - $\Rightarrow$ (1) becomes: $x^* = \vec{f}(\vec{x}^*, u^*)$ (5)
     - vector case: solution can be difficult
     - scalar case: easier
     - example: fix u^* (say=5)



(6)

3. Now one can linearize around $(\overset{\text{input}}{u^*}, \overset{\text{state}}{\vec{x}^*})$
   - Say $u[t] = u^* + \Delta u[t]$ "small" (7)
   - resultant $\vec{x}[t] = \vec{x}^* + \vec{\Delta x}[t]$ small (assumption) (8)

   - expand $\vec{f}(\vec{x}[t], u[t])$ in Taylor Series and ignore all higher order terms

$$\rightarrow \vec{f}[\vec{x}^* + \Delta\vec{x}[t], u^* + \Delta u[t]) \simeq \vec{f}(\vec{x}^*, u^*) + \underbrace{\frac{\partial \vec{f}}{\partial \vec{x}}\Big|_{\vec{x}^*, u^*}}_{J_x} \Delta\vec{x}[t] + \underbrace{\frac{\partial \vec{f}}{\partial u}\Big|_{\vec{x}^*, u^*}}_{J_u} \Delta u[t] \qquad (9)$$

$$= \vec{f}(\vec{x}^*, u^*) + \underset{n\times n}{J_x} \underset{n\times 1}{\Delta\vec{x}[t]} + J_u \Delta u[t] \qquad (10)$$

4. Insert (10) and (8) in (1): $\vec{x}[t+1] = \vec{f}(\vec{x}[t], u[t])$ (1)

$$\Rightarrow \underbrace{\vec{x}^* + \Delta\vec{x}[t+1] = \vec{f}(\vec{x}^*, u^*)}_{\text{these are the terms of the DC op pt eqn. (5)}} + J_x \Delta\vec{x}[t] + J_u \Delta u[t] \quad (11)$$

$$\Rightarrow \Delta\vec{x}[t+1] = \underbrace{J_x}_{A}^{n\times n} \Delta\vec{x}[t] + \underbrace{J_u}_{\vec{b}}^{n\times 1} \Delta u[t] \quad (12)$$

THE LINEARIZED EQN