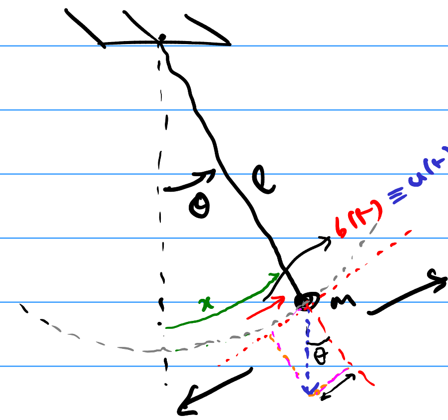
THE "SIMPLE" PENDULUM



$x(t) = \ell\,\theta(t)$ ← radians

$F = ma$

$\text{accel.} = \dfrac{d^2x}{dt^2} = \ell\,\dfrac{d^2\theta(t)}{dt^2}$

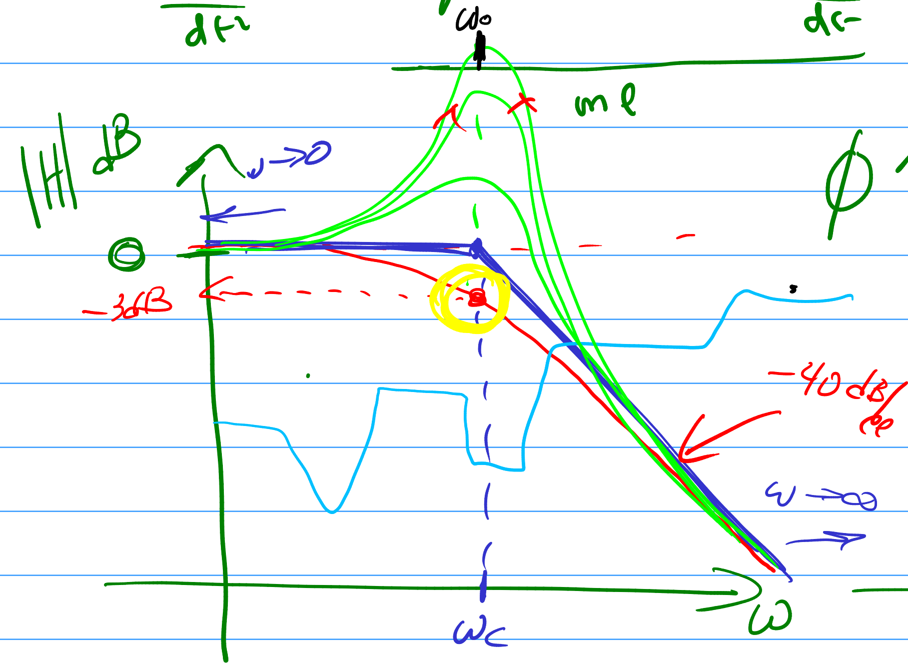
$a = \dfrac{F}{m}$

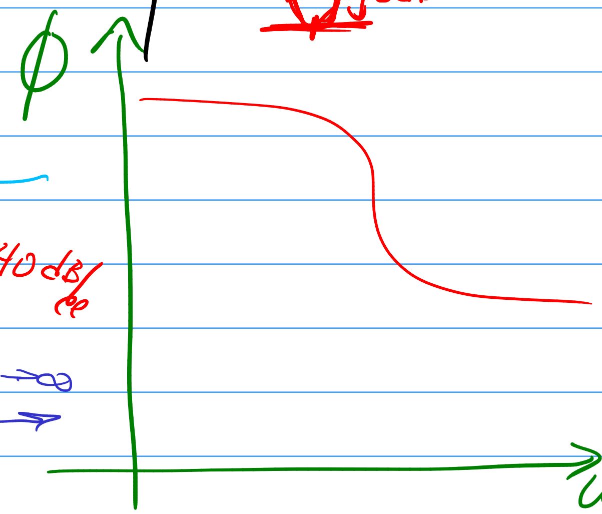
F

$-mg\sin(\theta) + b(t)$

$$m\ell\,\frac{d^2\theta(t)}{dt^2} = -mg\sin(\theta) + b(t) - k\ell\frac{d\theta}{dt}$$

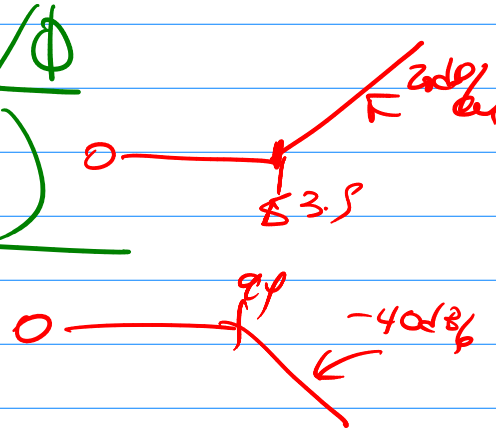
$$\frac{d^2\theta(t)}{dt^2} = \frac{-mg\sin(\theta) + b(t) - k\ell\dfrac{d\theta}{dt}}{m\ell}$$

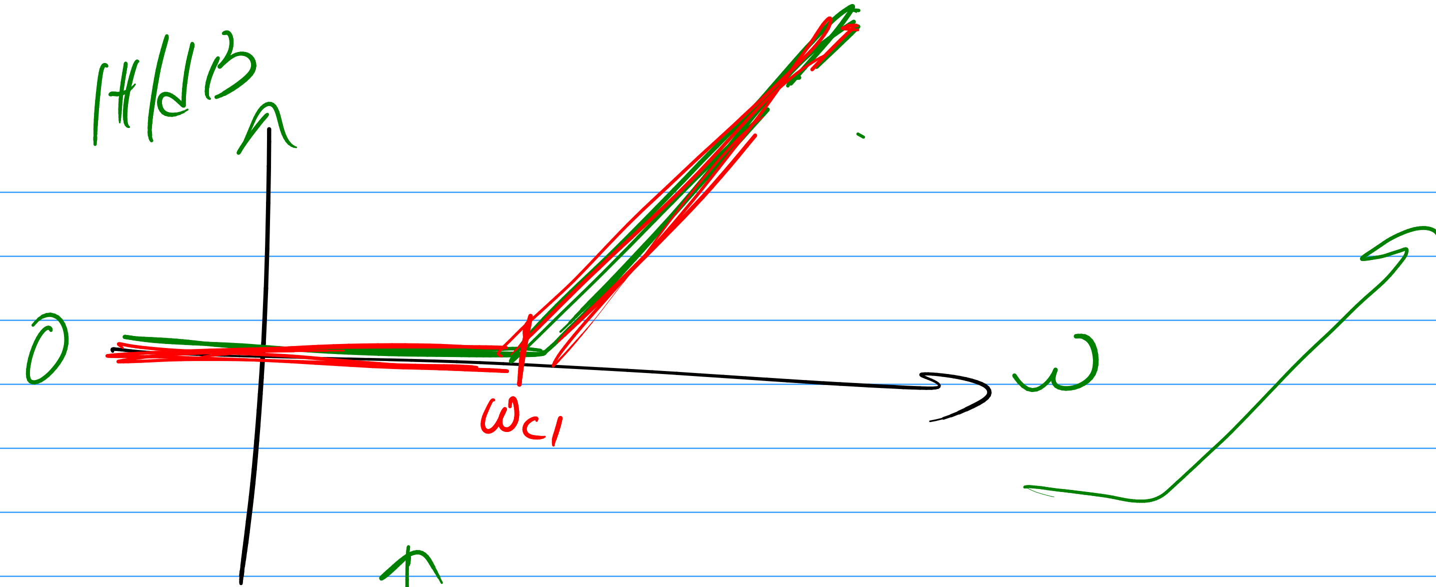




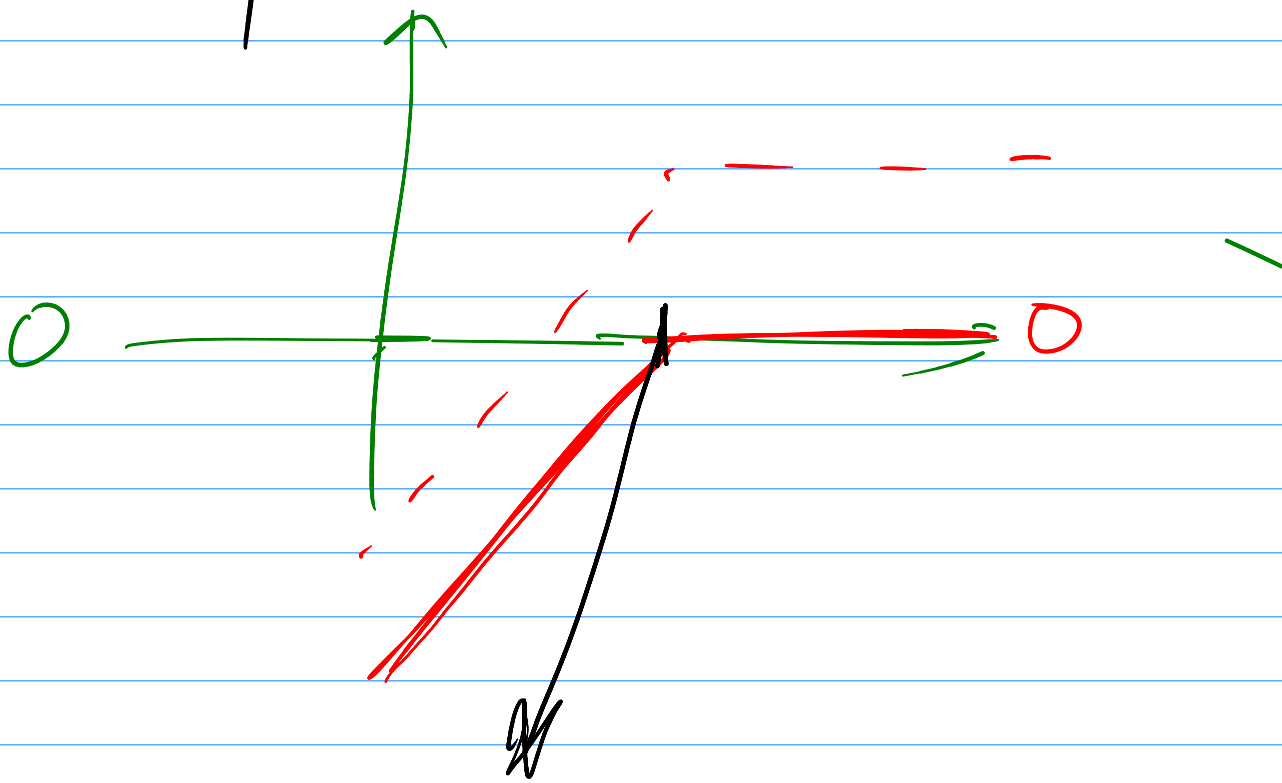
$H(\omega) \Rightarrow \text{complex number} = |H|e^{j\phi} = |H|\angle\phi$

$$H(\omega) = \frac{35.4\left(1 + j\,\omega/53.5\right)}{\left(1 + j\dfrac{\omega}{74}\right)^{2}}$$





|H|dB
0
ω
ω_{c1}



0
0

$$\frac{d^2 v_c}{dt^2} + \frac{R}{L}\frac{dv_c}{dt} + \frac{1}{LC} v_c = 0$$

$$x_1 \triangleq v_c$$

$$x_2 \triangleq \frac{dv_c}{dt}$$

$$\begin{cases} \dfrac{dx_2}{dt} + \dfrac{R}{L} x_2 + \dfrac{1}{LC} x_1 = 0 \\[2ex] \dfrac{dv_c}{dt} = x_2 \end{cases}$$

$$\begin{cases} \dfrac{dx_1}{dt} = x_2 \\[2ex] \dfrac{dx_2}{dt} = -\dfrac{1}{LC} x_1 - \dfrac{R}{L} x_2 \end{cases}$$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\frac{d\vec{x}}{dt} = \overbrace{\begin{array}{cc} \left[\begin{array}{c:c} 0 & 1 \\ \hdashline -1/LC & -R/L \end{array}\right] \\ \begin{array}{cc} c \quad & \quad d \end{array} \end{array}}^{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

def

$$A\vec{\xi} = \lambda \vec{\xi} \Rightarrow (A - \lambda I)\vec{\xi} = 0$$

$$A - \lambda I = \begin{bmatrix} -\lambda & 1 \\ c & d-\lambda \end{bmatrix}; \quad \det(A - \lambda I) = -\lambda(d-\lambda) - c = 0$$

$$\Rightarrow \lambda^2 - \lambda d - c = 0$$

$$\begin{bmatrix} \lambda_1 & \\ & \lambda_2 \end{bmatrix}$$

$$\lambda_{1,2} = \frac{+d \pm \sqrt{d^2 + 4c}}{2}$$

$$A = P \Lambda P^{-1}$$

$$P = \begin{bmatrix} 1 & 1 \\ \lambda_1 & \lambda_2 \end{bmatrix}$$

$$A\vec{p} = \lambda \vec{p} \rightarrow \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}$$

$$\Rightarrow (A - \lambda I)\vec{p} = 0$$

$$-\lambda p_1 + p_2 = 0 \qquad \text{choose } p_1 = 1,\ p_2 = \lambda$$

if it is an eigenvector

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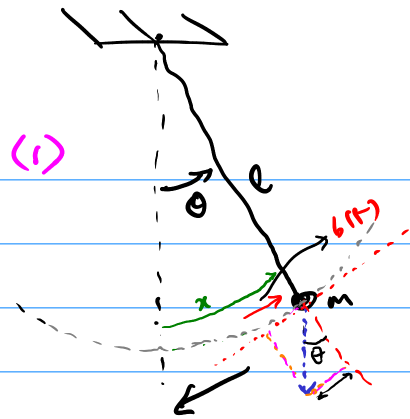


$x(t) = \ell\,\theta(t)$ ← radians $\qquad F = ma \qquad$ (1)

$$\text{accel.} = \frac{d^2x}{dt^2} = \ell\,\frac{d^2\theta(t)}{dt^2} \qquad (2)$$

$$a = \frac{F}{m} \qquad (3)$$

$$-mg\sin(\theta) + b(t) \qquad (4)$$

$$m\ell\,\frac{d^2\theta(t)}{dt^2} = \underbrace{-mg\sin(\theta) + b(t) - k\ell\frac{d\theta}{dt}}_{F} \qquad (5)$$

$$\frac{d^2\theta(t)}{dt^2} = -\frac{g}{\ell}\sin(\theta) - \frac{k}{m}\frac{d\theta}{dt} + \frac{b(t)}{m\ell} \qquad (6)$$

RECAP: Have got the "physics" eqns. for the "simple" pendulum.

NEXT: PUT (6) IN STATE-EQN FORM

$$\frac{d}{dt}\left[\frac{d\theta}{dt}\right] \qquad \frac{d^2\theta(t)}{dt^2} = -\frac{g}{\ell}\sin(\theta) - \frac{k}{m}\underbrace{\frac{d\theta}{dt}}_{x_2} + \underbrace{\frac{b(t)}{m\ell}}_{\text{input} = b(t)} \qquad (6)$$

$$\vec{x}(t) = \begin{array}{c} \begin{bmatrix} \theta(t) \\ \frac{d\theta}{dt} \end{bmatrix} \end{array} \begin{array}{l} \to x_1(t) \\ \to x_2(t) \end{array}$$

→ define: $x_2(t) \triangleq \dfrac{d\theta(t)}{dt}$ ← x_1 $\qquad$ (7)

$$\frac{d\vec{x}}{dt}$$

$$\boxed{\begin{aligned} \frac{dx_1(t)}{dt} &= x_2 & (8) \\ \frac{d}{dt}x_2(t) &= -\frac{g}{\ell}\sin(x_1(t)) - \frac{k}{m}x_2(t) + \frac{b(t)}{m\ell} & (9) \end{aligned}}$$

$$\vec{x} = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, \quad u(t) \equiv b(t)$$

$$\frac{d\vec{x}}{dt} = \underbrace{\begin{bmatrix} f_1(x_1, x_2; u) \\ f_2(x_1, x_2; u) \end{bmatrix}}_{\triangleq\, \vec{f}(\vec{x}, u)}, \text{ with } \quad f_1(x_1, x_2; u) \triangleq x_2 \qquad (10)$$

$$f_2(x_1, x_2; u) \triangleq -\frac{g}{\ell}\sin(x_1(t)) - \frac{k}{m}x_2(t) + \frac{b(t)}{m\ell} \qquad (11)$$

ATTEMPT: can we write:

$$\vec{f}(\vec{x},u) \text{ as } \underbrace{A}_{\text{const matrix}}\vec{x} + \underbrace{\vec{b}}_{\text{const (vector)}} u \; ?$$

VECTOR 2×1

$$\vec{f}(\vec{x},u) = \begin{bmatrix} x_2 \\ -\frac{g}{l}\sin(x_1(t)) - \frac{k}{m}x_2(t) + \frac{u(t)}{ml} \end{bmatrix} \qquad (12)$$

$$\cancel{\overset{A}{\begin{bmatrix} 0 & 1 \\ & -\frac{k}{m} \end{bmatrix}} \overset{\vec{x}}{\begin{bmatrix} \\ x_2 \end{bmatrix}} + \overset{\vec{b}}{\begin{bmatrix} \\ \\ \end{bmatrix}} u(t)}$$

STATE-SPACE FORM: (GENERAL - includes nonlinear)

$$\frac{d\vec{x}}{dt} = \vec{f}(\vec{x},\vec{u}) \longleftarrow \boxed{\text{MAY NOT BE EXPRESSIBLE AS } A\vec{x} + B\vec{u}}$$

— So our system is nonlinear, not in $A\vec{x} + B\vec{u}$ form?

— WE'D LIKE VERY MUCH TO PUT IT IN $\nearrow$, EVEN IF WE HAVE TO APPROXIMATE

$\downarrow$

LINEARIZATION

LINEARIZATION

— STEP 1: — select constant (wrt time) input $\vec{u}^*$: "DC"

— ASSUME $\vec{x}(t)$ ALSO DC. $\equiv \vec{x}^*$

$$-\; \frac{d\overset{x^*}{x(t)}}{dt} = \vec{f}(\overset{x^*}{\vec{x}(t)}, \overset{u^*}{\vec{u}(t)}) \qquad (13)$$

$$\boxed{0 = \vec{f}(\vec{x}^*, \vec{u}^*)} \qquad (14)$$

← DC op. pt.
← equilibrium pt
← quiescent point

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} x_2^* \\ -\frac{g}{l}\sin(x_1^*) - \frac{k}{m}x_2^* + \frac{b^*}{ml} \end{bmatrix}$$

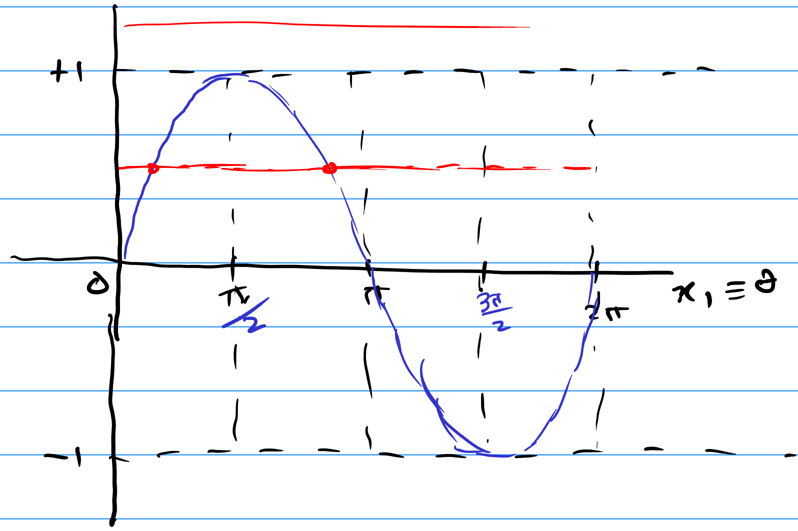
$\frac{dx_1}{dt} = x_2^* = 0$

$x_2^* = 0$

$$-\frac{g}{l}\sin(x_1^*) + \frac{b^*}{ml} = 0$$

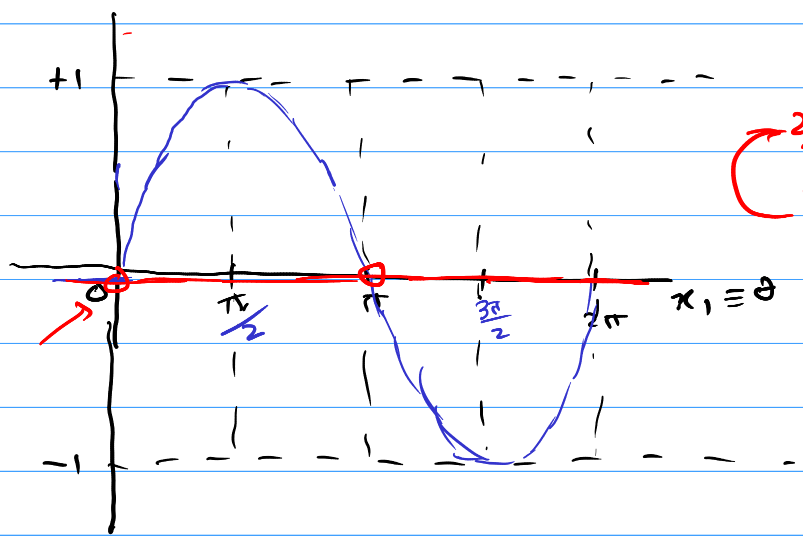
$$\Rightarrow \boxed{\sin(x_1^*) = \frac{b^*}{mg}} \qquad (15)$$

$\left(\frac{b^*}{mg} = 0.5\right)$ say



(16)

$\rightarrow$ Now choose a "natural" $b^* = 0$ $\leftarrow$ no tangential force applied by me



different 2 solutions

$\begin{cases} x_1^* = 0 \leftarrow \text{one solution} \\ x_1^* = \pi \end{cases}$

(17)

$$\frac{d}{dt}\left[\frac{d\theta}{dt}\right] \to x_2$$

$$\frac{d^3\theta}{dt^3} + a_1 \frac{d^2\theta(t)}{dt^2} + \frac{g}{l}\sin(\theta) + \frac{k}{m}\frac{d\theta}{dt} - \frac{b(t)}{ml} = 0 \qquad (18)$$

$$\frac{d^3\theta}{dt^3} \to \frac{d}{dt}\left[\frac{d^2\theta}{dt^2}\right], \quad \frac{d^2\theta}{dt^2} \to x_3$$

x_1

1) $\dfrac{dx_3}{dt} + a_1 x_3 + \dfrac{g}{l}\sin(x_1) + \dfrac{k}{m}x_2 - \dfrac{b(t)}{ml} = 0$

2) $\dfrac{dx_1}{dt} = x_2$

3) $\dfrac{dx_2}{dt} = x_3$

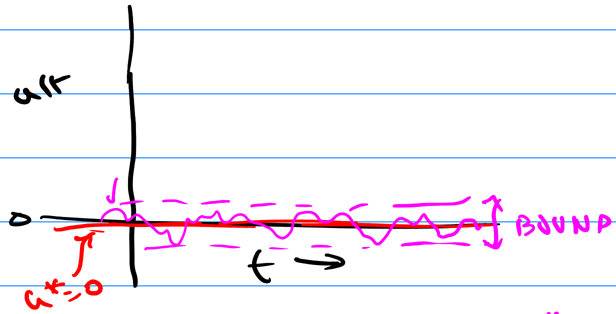
$x_1 \equiv \theta$

$x_2 = \dfrac{d\theta}{dt}$

$x_3 = \dfrac{d^2\theta}{dt^2} = \dfrac{dx_2}{dt}$ (19)

the other one is $\begin{bmatrix} \pi \\ 0 \end{bmatrix}$ ← will deal with this one later

— Choose $\vec{x}^* = \begin{bmatrix} x_1^* \\ x_2^* \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ (20)



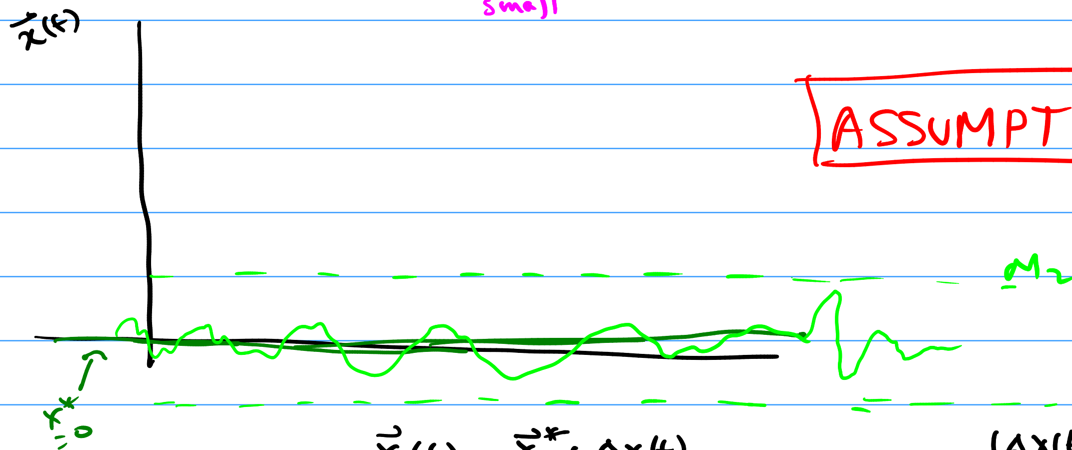
$|\Delta u(t)| < M$ forall t

small const

our current definition of "small"

$u(t) = u^* + \Delta u(t)$

"small"



ASSUMPTION

$\forall$ → forall

$\vec{x}(t) = \vec{x}^* + \Delta x(t)$

↳ small/bounded

$|\Delta x(t)| \le M_2 \;\forall t$

RETURN TO FULL STATE EQUATIONS

$$\frac{d\vec{x}}{dt} = \vec{f}(\vec{x}(t), \vec{u}(t)) \longrightarrow \boxed{\text{DC. o.p.}: \quad \vec{f}(\vec{x}^*, \vec{u}^*) = 0}$$

$$\rightarrow \left(\begin{array}{l} \vec{u}(t) = \vec{u}^* + \Delta\vec{u}(t) \\ \vec{x}(t) = \vec{x}^* + \Delta\vec{x}(t) \end{array} \right) \qquad (21)$$

$$\frac{d}{dt}\left[\vec{x}^* + \Delta\vec{x}(t)\right] = \vec{f}\left(\vec{x}^* + \Delta\vec{x}(t), \vec{u}^* + \Delta\vec{u}(t)\right) \qquad (22)$$

$$\underbrace{\cancel{\frac{d}{dt}\vec{x}^*}}_{\begin{bmatrix} 0 \\ 0 \end{bmatrix}} + \frac{d}{dt}\Delta\vec{x}(t) = \vec{f}\left(\vec{x}^* + \Delta\vec{x}(t), \vec{u}^* + \Delta\vec{u}(t)\right)$$

$$\rightarrow \frac{d}{dt}\Delta\vec{x}(t) = \vec{f}\left(\vec{x}^* + \Delta\vec{x}(t),\ \vec{u}^* + \Delta\vec{u}(t)\right) \qquad \Delta\vec{x} = \begin{bmatrix}\Delta x_1\\ \Delta x_2\end{bmatrix} \quad \Delta u \qquad \vec{x}^* = \begin{bmatrix}x_1^*\\ x_2^*\end{bmatrix}$$

2×2 case as an example:

$$\underset{\downarrow 2}{\vec{f}}(\underset{\downarrow 2}{\vec{x}}, \underset{\downarrow 1}{u}) = \begin{bmatrix} f_1(x_1, x_2; u) \\ f_2(x_1, x_2; u)\end{bmatrix}$$

$$\vec{f}(\vec{x}^*+\Delta\vec{x}, u^*+\Delta u) = \begin{bmatrix} f_1(x_1^*+\Delta x_1, x_2^*+\Delta x_2;\ u^*+\Delta u) \\ f_2(x_1^*+\Delta x_1, x_2^*+\Delta x_2;\ u^*+\Delta u)\end{bmatrix} \qquad (23)$$

Taylor Series on : $f_1(x_1^*+\Delta x_1, x_2^*+\Delta x_2;\ u^*+\Delta u)$

$f_1(x_1^*+\Delta x_1, x_2^*+\Delta x_2;\ u^*+\Delta u) =$

APPROXIMATION

$$f_1(x_1^*+\Delta x_1) = f_1(x_1^*) + \left.\frac{df_1}{dx}\right|_{x_1^*} \Delta x_1 + \cancel{\frac{1}{2}\left.\frac{d^2f}{dx^2}\right|_{x_1^*} \Delta x_1^2} + \cancel{\frac{1}{6?}\left.\frac{d^3f}{dx^3}\right|_{x_1^*} \Delta x^3}$$

$$\frac{0.1}{0.01} \qquad \frac{0.01}{10^{-4}} \qquad \frac{0.001}{10^{-6}}$$

$$f_1(x_1^*+\Delta x_1, x_2^*+\Delta x_2;\ u^*+\Delta u) = \boxed{f_1(x_1^*, x_2^*, u^*)}$$

$$\boxed{+ \left.\frac{\partial f_1}{\partial x_1}\right|_{x_1^*, x_2^*, u^*} \Delta x_1} + \cancel{\frac{1}{2}\left.\frac{\partial^2 f_1}{\partial x_1^2}\right|_{x_1^*, x_2^*, u^*} \Delta x_1^2} + \cdots$$

$$\cancel{+ ? \left.\frac{\partial^2 f_1}{\partial x_1 \partial x_2}\right|_{x_1^*, x_2^*, u^*} \Delta x_1 \Delta x_2}$$

$$\boxed{+ \left.\frac{\partial f_1}{\partial x_2}\right|_{x_1^*, x_2^*, u^*} \Delta x_2} + \cancel{\cdots}$$

$$\cancel{+ ? \left.\frac{\partial^2 f}{\partial x_1 \partial u}\right|_{\cdots} \Delta x_1 \Delta u}$$

$$\boxed{+ \left.\frac{\partial f_1}{\partial u}\right|_{x_1^*, x_2^*, u^*} \Delta u} + \cancel{\cdots}$$

$$f_1(x_1^*+\Delta x_1, x_2^*+\Delta x_2; u^*+\Delta u) = f_1(x_1^*, x_2^*, u^*) + \underbrace{\overbrace{\frac{\partial f_1}{\partial x_1}\Big|_* \Delta x_1 + \frac{\partial f_1}{\partial x_2}\Delta x_2 + \frac{\partial f_1}{\partial u}\Delta u}^{\text{FIRST ORDER APPROXIMATION}}}_{\text{TOTAL DERIVATIVE}} \qquad (24)$$

$$f_2(x_1^*+\Delta x_1, x_2^*+\Delta x_2; u^*+\Delta u) = f_2(x_1^*, x_2^*, u^*) + \left(\frac{\partial f_2}{\partial x_1}\Big|_* \Delta x_1 + \frac{\partial f_2}{\partial x_2}\Delta x_2 + \frac{\partial f_2}{\partial u}\Delta u\right. \qquad (25)$$

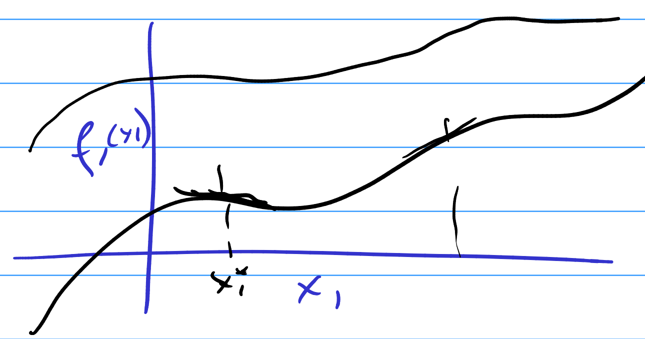
$$\frac{d}{dt}\Delta\vec{x}(t) = \vec{f}\left(\vec{x}^* + \Delta\vec{x}(t),\ \vec{u}^* + \Delta\vec{u}(t)\right)$$

$$\frac{d}{dt}\begin{bmatrix}\Delta x_1(t)\\ \Delta x_2(t)\end{bmatrix} = \begin{bmatrix}\cancel{f_1(x_1^*, x_2^*, u^*)} + \frac{\partial f_1}{\partial x_1}\Big|^* \Delta x_1(t) + \frac{\partial f_1}{\partial x_2}\Big|^* \Delta x_2(t) + \frac{\partial f_1}{\partial u}\Big|^* \Delta u(t)\\ \cancel{f_2(x_1^*, x_2^*, u^*)} + \frac{\partial f_2}{\partial x_1}\Big|^* \Delta x_1(t) + \frac{\partial f_2}{\partial x_2}\Big|^* \Delta x_2(t) + \frac{\partial f_2}{\partial u}\Big|^* \Delta u(t)\end{bmatrix} \qquad (26)$$

$$\vec{f}(\vec{x}^*, \vec{u}^*) = \vec{0} \leftarrow \text{DC o. p.}$$

$$\begin{bmatrix}f_1(x_1^*, x_2^*, u^*)\\ f_2(x_1^*, x_2^*, u^*)\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

$$\frac{d}{dt}\overset{\Delta\vec{x}}{\begin{bmatrix}\Delta x_1(t)\\ \Delta x_2(t)\end{bmatrix}} = \overset{2\times 3}{\begin{bmatrix}\frac{\partial f_1}{\partial x_1}^* & \frac{\partial f_1}{\partial x_2}^* & \frac{\partial f_1}{\partial u}^*\\ \frac{\partial f_2}{\partial x_1}^* & \frac{\partial f_2}{\partial x_2}^* & \frac{\partial f_2}{\partial u}^*\end{bmatrix}} \overset{3\times 1}{\begin{bmatrix}\Delta x_1(t)\\ \Delta x_2(t)\\ \Delta u(t)\end{bmatrix}} = \overset{A}{\begin{bmatrix}\frac{\partial f_1}{\partial x_1}^* & \frac{\partial f_1}{\partial x_2}^*\\ \frac{\partial f_2}{\partial x_1}^* & \frac{\partial f_2}{\partial x_2}^*\end{bmatrix}} \overset{\Delta\vec{x}(t)}{\begin{bmatrix}\Delta x_1(t)\\ \Delta x_2(t)\end{bmatrix}} + \overset{\vec{b}}{\begin{bmatrix}\frac{\partial f_1}{\partial u}^*\\ \frac{\partial f_2}{\partial u}^*\end{bmatrix}} \Delta u(t) \qquad (27)$$



— SO FAR, WE HAVE DERIVED THE LINEARIZATION FORMULA w/ JACOBIANS

— return to our DC op pt (20): $\vec{x}^* = \begin{bmatrix} x_1^* \\ x_2^* \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ ← hanging down (θ, v_θ)

— Just apply (27) to the $\vec{f}(\cdot,\cdot)$ for the pendulum (12):

$$\longrightarrow \vec{f}(\vec{x}, u) = \begin{bmatrix} x_2 \\ -\frac{g}{\ell}\sin(x_1(t)) - \frac{k}{m}x_2(t) + \frac{b(t)}{m\ell} \end{bmatrix} \quad (12)$$

$$\longrightarrow J_x = \left.\frac{\partial \vec{f}}{\partial \vec{x}}\right|_{\vec{x}^*, u^* = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, 0} = \begin{bmatrix} 0 & 1 \\ -\frac{g}{\ell}\underbrace{\cos(x_1^*)}_{1} & -\frac{k}{m} \end{bmatrix}; \quad J_u = \left.\frac{\partial \vec{f}}{\partial u}\right|_{\vec{x}^*, u^* = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, 0} = \begin{bmatrix} 0 \\ \frac{1}{m\ell} \end{bmatrix} \quad (28)$$

→ The linearized system is

$$\frac{d}{dt}\begin{bmatrix} \Delta x_1(t) \\ \Delta x_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -g/\ell & -\frac{k}{m} \end{bmatrix}\begin{bmatrix} \Delta x_1(t) \\ \Delta x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m\ell} \end{bmatrix}\Delta u(t) \quad (29)$$

— Now we choose a different DC operating point to linearize around

— see (17), we choose $\vec{x}^* = \begin{bmatrix} \pi \\ 0 \end{bmatrix}$, i.e., the inverted position

— we just redo (28), but now, $x_1^* = \pi$ (not 0)

$$\longrightarrow J_x = \left.\frac{\partial \vec{f}}{\partial \vec{x}}\right|_{\vec{x}^*, u^* = \begin{bmatrix} \pi \\ 0 \end{bmatrix}, 0} = \begin{bmatrix} 0 & 1 \\ -\frac{g}{\ell}\underbrace{\cos(x_1^*)}_{\text{no longer } = 1, \text{ but now } = -1} & -\frac{k}{m} \end{bmatrix}; \quad J_u = \left.\frac{\partial \vec{f}}{\partial u}\right|_{\vec{x}^*, u^* = \begin{bmatrix} \pi \\ 0 \end{bmatrix}, 0} = \begin{bmatrix} 0 \\ \frac{1}{m\ell} \end{bmatrix} \quad (30)$$

$$\frac{d}{dt}\begin{bmatrix} \Delta x_1(t) \\ \Delta x_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ +\frac{g}{\ell} & -\frac{k}{m} \end{bmatrix}\begin{bmatrix} \Delta x_1(t) \\ \Delta x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m\ell} \end{bmatrix}\Delta u(t) \quad (31)$$

← linearization of the inverted pendulum

STABILITY: just look at the eigenvalues of the "A" $\xrightarrow{} J_x$ matrix

→ Say $A_d = \left[\begin{array}{c|c} 0 & 1 \\ \hline -g/\ell & -\frac{k}{m} \end{array}\right]$ (linearization around the down position) (32)

$$\rightarrow \det(A_d - \lambda I) = \det\left(\left[\begin{array}{c|c} -\lambda & 1 \\ \hline -g/\ell & -\frac{k}{m}-\lambda \end{array}\right]\right) = \lambda\left(\frac{k}{m}+\lambda\right) + \frac{g}{\ell} = 0 \quad (33)$$

$$\Rightarrow \lambda^2 + \frac{k}{m}\lambda + \frac{g}{\ell} = 0 \quad (34)$$

friction

$$\Rightarrow \lambda_{1,2} = \frac{-k}{2m} \pm \frac{1}{2}\sqrt{\frac{k^2}{m^2} - \frac{4g}{\ell}} = \underbrace{\frac{-k}{2m} \pm \sqrt{\left(\frac{k}{2m}\right)^2 - \frac{g}{\ell}}}_{\text{ALWAYS STABLE IF FRICTION} > 0} \quad (35)$$

→ Now try A for the inverted pendulum: $A_i = \left[\begin{array}{c|c} 0 & 1 \\ \hline +\frac{g}{\ell} & -\frac{k}{m} \end{array}\right]$ (36)

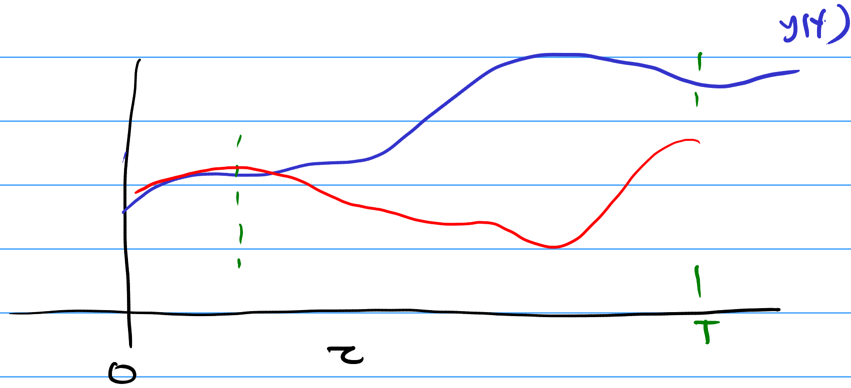
$$\lambda_{1,2} = \underbrace{-\frac{k}{2m} \pm \sqrt{\left(\frac{k}{2m}\right)^2 + \frac{g}{\ell}}}_{\text{ALWAYS UNSTABLE}} \quad (37)$$

$$\int_0^t e^{a(t-\tau)} b\, u(\tau)\, d\tau$$

$$x(t) = x(0)\, e^{at} + \underbrace{(e^{at}) * (b u(t))}_{\text{C.T conv}} \qquad \text{C. T.}$$

$$x[t] = x[0]\, a^t + \underbrace{\sum_{i=1}^{t} a^{t-i}\, b\, u[t-i]}_{\text{D.T. conv}}$$

$= x(t) \quad y(t)$

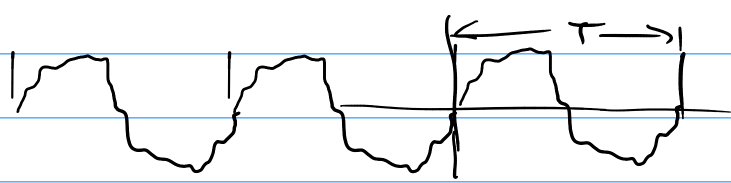


$$= \int_0^T x(T-\tau)\, y(\tau)\, d\tau$$

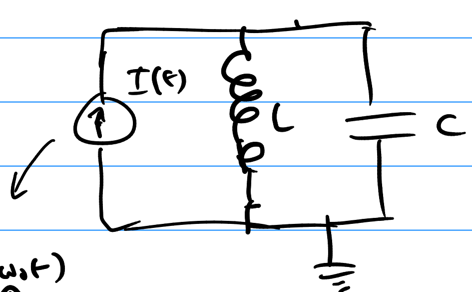
$x(T-\tau)$

$x(\tau) \mapsto$ flip around 0 $x(-\tau)$

— shift right by T



$$\omega_0 = \frac{1}{\sqrt{LC}} = 2\pi f_0 = \frac{2\pi}{T_0}$$



$\underline{\underline{\epsilon}} \sin(\omega_0 t)$