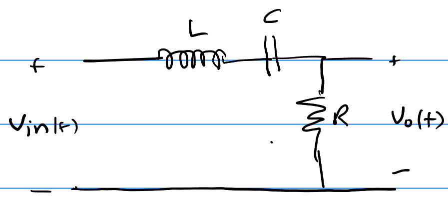
BW of SERIES RLC CKT w V_R AS OUTPUT



$$V_o(\omega) = \frac{R}{j\omega L + \frac{1}{j\omega C} + R} V_i(\omega) = \frac{j\omega RC}{1 - \omega^2 LC + j\omega RC} V_i(\omega)$$

$$\Rightarrow |H(\omega)| \triangleq \left|\frac{V_o(\omega)}{V_i(\omega)}\right| = \frac{\omega RC}{\sqrt{(1-\omega^2 LC)^2 + \omega^2 R^2 C^2}} = k \qquad (1)$$

$\leftarrow$ will set $= \frac{1}{\sqrt{2}}$ later

$$\omega^2 R^2 C^2 = k^2 \left[(1-\omega^2 LC)^2 + \omega^2 R^2 C^2 \right]$$

$$\Rightarrow \omega^2 R^2 C^2 (1-k^2) = k^2 (1-\omega^2 LC)^2$$

$$\Rightarrow \pm \omega RC \sqrt{\frac{1-k^2}{k^2}} = (1-\omega^2 LC)$$

$$\Rightarrow \underbrace{\omega^2 LC}_{a} \underbrace{\pm \omega RC \sqrt{\frac{1-k^2}{k^2}}}_{b} \underbrace{-1}_{c} = 0 \qquad (2)$$

$$\Rightarrow \omega_{1,2} = \mp \frac{R\cancel{C}}{2L\cancel{C}} \sqrt{\frac{1-k^2}{k^2}} \pm \frac{1}{2LC} \sqrt{R^2C^2\left(\frac{1-k^2}{k^2}\right) + 4LC} \qquad (3)$$

$-$ leads to −ve frequencies → ignore

$$\boxed{\text{Approximation: if } 4LC \gg R^2C^2\left(\frac{1-k^2}{k^2}\right), \text{ then } + \frac{1}{2LC}\sqrt{R^2C^2\left(\frac{1-k^2}{k^2}\right) + 4LC} \simeq \frac{1}{\sqrt{LC}} = \omega_0}$$

"R is small"

→ we don't need this approximation

$$\Rightarrow \omega_{1,2} = + \frac{1}{2LC} \sqrt{R^2C^2\left(\frac{1-k^2}{k^2}\right) + 4LC} \mp \frac{R}{2L} \sqrt{\frac{1-k^2}{k^2}} \qquad (4)$$

→ NOTE: ω_1 and ω_2 are NOT SYMMETRIC about $\omega_0 = \frac{1}{\sqrt{LC}}$

$$\Rightarrow \Delta\omega = \omega_2 - \omega_1 = \frac{R}{L}\sqrt{\frac{1-k^2}{k^2}} \qquad (5)$$

$$\boxed{\Rightarrow \frac{\omega_0}{\Delta\omega} = \frac{L}{\sqrt{LC}\,R}\sqrt{\frac{k^2}{1-k^2}} = \frac{1}{R}\sqrt{\frac{L}{C}}\sqrt{\frac{k^2}{1-k^2}} = \frac{\omega_0 L}{R}\sqrt{\frac{k^2}{1-k^2}}} \qquad (6)$$

$$\text{Choose } k^2 = \tfrac{1}{2} \Rightarrow \left|\frac{V_o}{V_i}\right| = \sqrt{\tfrac{1}{2}} \quad = \quad \frac{\omega_0}{\Delta\omega} = \frac{\omega_0 L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}} \qquad (7)$$

Q factor

char. impedance

PEAK FREQ. OF SERIES RLC CKT w V_R AS OUTPUT

From (1): $$|H(\omega)| \triangleq \left|\frac{V_o(\omega)}{V_i(\omega)}\right| = \frac{\omega RC}{\sqrt{(1-\omega^2LC)^2+\omega^2R^2C^2}}$$

$$\Rightarrow |H(\omega)|^2 = \frac{\omega^2R^2C^2}{(1-\omega^2LC)^2+\omega^2R^2C^2} \qquad (8)$$

— $(x)^2$ is a monotonic function for $x \geq 0$

$$\Rightarrow \text{peak of } |H(\omega)| = \text{peak of } |H(\omega)|^2 \qquad (9)$$

— To find the max. value of $|H(\omega)|^2$, we use

$$\frac{d}{d\omega}|H(\omega)|^2 = 0 \qquad (10)$$

$$\Rightarrow \frac{d}{d\omega}\left[\frac{\omega^2R^2C^2}{(1-\omega^2LC)^2+\omega^2R^2C^2}\right] = 0$$

$$\Rightarrow \frac{2\omega R^2C^2}{(1-\omega^2LC)^2+\omega^2R^2C^2} - \frac{\omega^2R^2C^2[2\omega R^2C^2+2(1-\omega^2LC)(-2\omega LC)]}{[(1-\omega^2LC)^2+\omega^2R^2C^2]^2} = 0$$

$$\Rightarrow \frac{2\cancel{\omega}\cancel{R^2C^2}}{\cancel{(1-\omega^2LC)^2+\omega^2R^2C^2}} = \frac{\cancel{\omega^2R^2C^2}[2\omega R^2C^2+2(1-\omega^2LC)(-2\omega LC)]}{[(1-\omega^2LC)^2+\omega^2R^2C^2]^{\cancel{2}}}$$

$$\Rightarrow \cancel{2}[(1-\omega^2LC)^2+\omega^2R^2C^2] = \omega[2\omega R^2C^2+2(1-\omega^2LC)(-2\omega LC)]$$
$$= 2\omega[\omega R^2C^2 - 2\omega LC(1-\omega^2LC)]$$
$$= \cancel{2}[\omega^2R^2C^2 - 2\omega^2LC(1-\omega^2LC)]$$

$$\Rightarrow (1-\omega^2LC)^2 + \cancel{\omega^2R^2C^2} = \cancel{\omega^2R^2C^2} - 2\omega^2LC(1-\omega^2LC)$$

$$\Rightarrow (1-\omega^2LC)^2 = -2\omega^2LC(1-\omega^2LC)$$

$$\Rightarrow \quad 1 + \omega^4 L^2 C^2 - \cancel{2\omega^2 LC} = \cancel{-2\omega^2 LC} + 2\omega^4 L^2 C^2$$

$$\Rightarrow \quad 1 = \omega^4 L^2 C^2$$

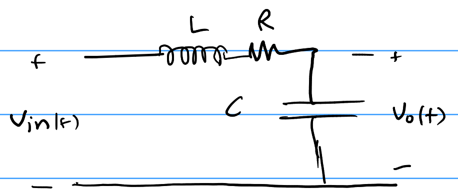
$$\Rightarrow \quad \omega^4 = \frac{1}{L^2 C^2} \Rightarrow \omega^2 = \frac{1}{LC} \Rightarrow \omega = \frac{1}{\sqrt{LC}} \qquad (10)$$

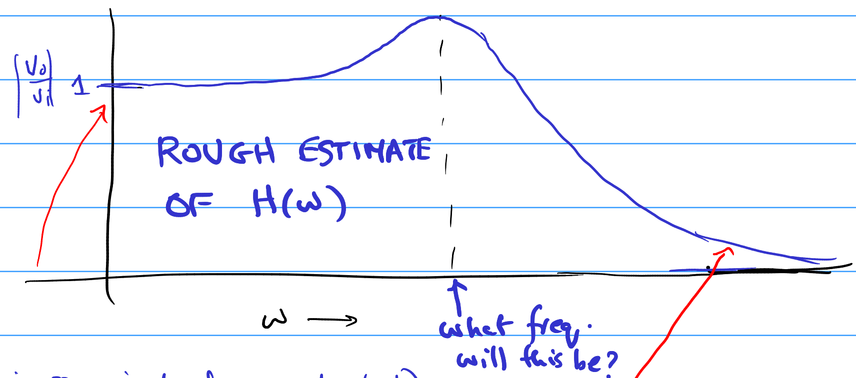
— i.e., the peak of this transfer function is at $\omega_0 = \frac{1}{\sqrt{LC}}$

— double-check that magnitude at $\omega_0 = 1$:

— from (1): $|H(\omega)| \triangleq \left|\frac{V_0(\omega)}{V_i(\omega)}\right| = \frac{\omega RC}{\sqrt{(1-\omega^2 LC)^2 + \omega^2 R^2 C^2}}$

$$\Rightarrow |H(\omega_0)| = \frac{\omega_0 RC}{\sqrt{(\underbrace{1-\omega_0^2 LC}_{0})^2 + \omega_0^2 R^2 C^2}} = \frac{\omega_0 RC}{\omega_0 RC} = 1 \leftarrow \text{OK!}$$





→ at DC : $\frac{V_o}{V_{in}} = 1$ (cap is open inductor is shorted)

→ as $\omega \to \infty$: $\frac{V_o}{V_{in}} \to 0$ (cap is shorted, inductor is open)

→ at $\omega = \omega_o = \sqrt{\frac{1}{LC}}$, what is the capacitor voltage?

→ the current is $\frac{V_{in}}{R}$ ⟹ cap voltage $= V_o = \frac{V_{in}}{j\omega_o CR}$ ⟹ $\frac{V_o}{V_{in}} = \frac{1}{j\omega_o CR}$

$$\Rightarrow \left|\frac{V_o}{V_{in}}\right| = \frac{1}{\omega_o CR} = \frac{\sqrt{LC}}{CR} = \frac{\sqrt{\frac{L}{C}}}{R}$$

characteristic impedance

this can be > 1 or < 1

Q: is $\omega_0 = \frac{1}{\sqrt{LC}}$ the freq. of the peak? (as it was for output = V_R?)

$$H(\omega) = \frac{\frac{1}{j\omega C}}{j\omega L + \frac{1}{j\omega C} + R} = \frac{\frac{1}{j\omega C}}{\frac{(1-\omega^2 LC) + j\omega RC}{j\omega C}} = \frac{1}{(1-\omega^2 LC) + j\omega RC} \qquad (11)$$

$$|H(\omega)| = \frac{1}{\sqrt{(1-\omega^2 LC)^2 + \omega^2 R^2 C^2}} \Rightarrow |H(\omega)|^2 = \frac{1}{(1-\omega^2 LC)^2 + \omega^2 R^2 C^2} \qquad (12)$$

→ want $\frac{d}{d\omega}|H(\omega)|^2 = 0$ to find the peak

$$\Rightarrow \quad \frac{-\left[2\omega R^2C^2 + 2(1-\omega^2LC)(-2\omega LC)\right]}{(1-\omega^2LC)^2 + \omega^2R^2C^2} = 0$$

$$\Rightarrow \quad 2\omega R^2C^2 = 4\omega LC(1-\omega^2LC)$$

$$\Rightarrow \quad R^2C^2 = 2LC(1-\omega^2LC) = 2LC - 2\omega^2L^2C^2$$

$$\Rightarrow \quad 2\omega^2L^2C^2 = 2LC - R^2C^2$$

$$\Rightarrow \quad \omega^2 = \frac{1}{LC} - \frac{R^2}{2L^2}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}} = \sqrt{\frac{2L^2 - R^2LC}{LC(2L^2)}} = \frac{1}{\sqrt{LC}} \times \sqrt{\frac{2L - R^2C}{2L}}$$

ω for peak magnitude

$$\Rightarrow \quad \omega_p = \omega_0\sqrt{\frac{2L-R^2C}{2L}} \qquad (13)$$

— i.e., the peak is NOT at $\omega = \omega_0$

Q: what is the magnitude at $\omega = \omega_p$?

— from (12): $|H(\omega)| = \dfrac{1}{\sqrt{(1-\omega^2LC)^2 + \omega^2R^2C^2}}$

$$\Rightarrow |H(\omega_p)| = \frac{1}{\sqrt{(1-\omega_p^2LC)^2 + \omega_p^2R^2C^2}} \qquad \text{call this } M \qquad (14)$$

$$\rightarrow \quad (1-\omega_p^2LC)^2 + \omega_p^2R^2C^2 = \left(1 - \underbrace{\omega_0^2LC}_{1}\frac{(2L-R^2C)}{2L}\right)^2 + \omega_0^2\left(\frac{2L-R^2C}{2L}\right)R^2C^2$$

$$= 1 + \frac{(2L-R^2C)^2}{4L^2} + \frac{\omega_0^2R^2C^2(2L-R^2C^2)}{2L}$$

$$= \frac{4L^2 + (2L-R^2C)^2 + 2\omega_0^2R^2C^2L(2L-R^2C^2)}{4L^2}$$

$$= \frac{4L^2 + (2L-R^2C)^2 + 2R^2C(2L-R^2C^2)}{4L^2}$$

$$= \frac{4L^2 + 4L^2 + R^4C^2 - \cancel{4}LR^2C + \cancel{4}LR^2C - 2R^4C^3}{4L^2}$$

$$= \frac{8L^2 + R^4C^2(1-2C)}{4L^2}$$

doesn't seem to reduce to any cleaner expression.

$$\Rightarrow M = \frac{2L}{\sqrt{8L^2 + R^4C^2(1-2C)}} \qquad (15)$$

→ if $R=0$, simplifies to $\frac{1}{\sqrt{2}}$

<u>NEXT Q:</u> IS THE BW the same (as when output = V_R) when the output = V_C?

– from (12): $|H(\omega)| = \frac{1}{\sqrt{(1-\omega^2LC)^2 + \omega^2R^2C^2}}$

– want $|H(\omega)| = k.\underbrace{M}_{\text{from (13)}}$, will set $k = \frac{1}{\sqrt{2}}$ later

$\Rightarrow |H(\omega)|^2 = k^2M^2$

$\Rightarrow \frac{1}{(1-\omega^2LC)^2 + \omega^2R^2C^2} = k^2M^2 \Rightarrow k^2M^2[(1-\omega^2LC)^2 + \omega^2R^2C^2] = 1$

$\Rightarrow k^2M^2[\omega^4L^2C^2 + 1 - 2\omega^2LC + \omega^2R^2C^2] = 1$

$\Rightarrow k^2M^2L^2C^2\omega^4 + k^2M^2(R^2C^2 - 2LC)\omega^2 - 1 = 0$

$$\Rightarrow \omega^2 = \pm\left[\frac{k^2M^2(2LC - R^2C^2) \pm \sqrt{k^4M^4(R^2C^2-2LC)^2 + 4k^2M^2L^2C^2}}{2k^2M^2L^2C^2}\right] \qquad (16)$$

– NOT AN ATTRACTIVE EXPRESSION

– ANYONE UP FOR SIMPLIFYING IT?