EE16B, Spring 2018
UC Berkeley EECS

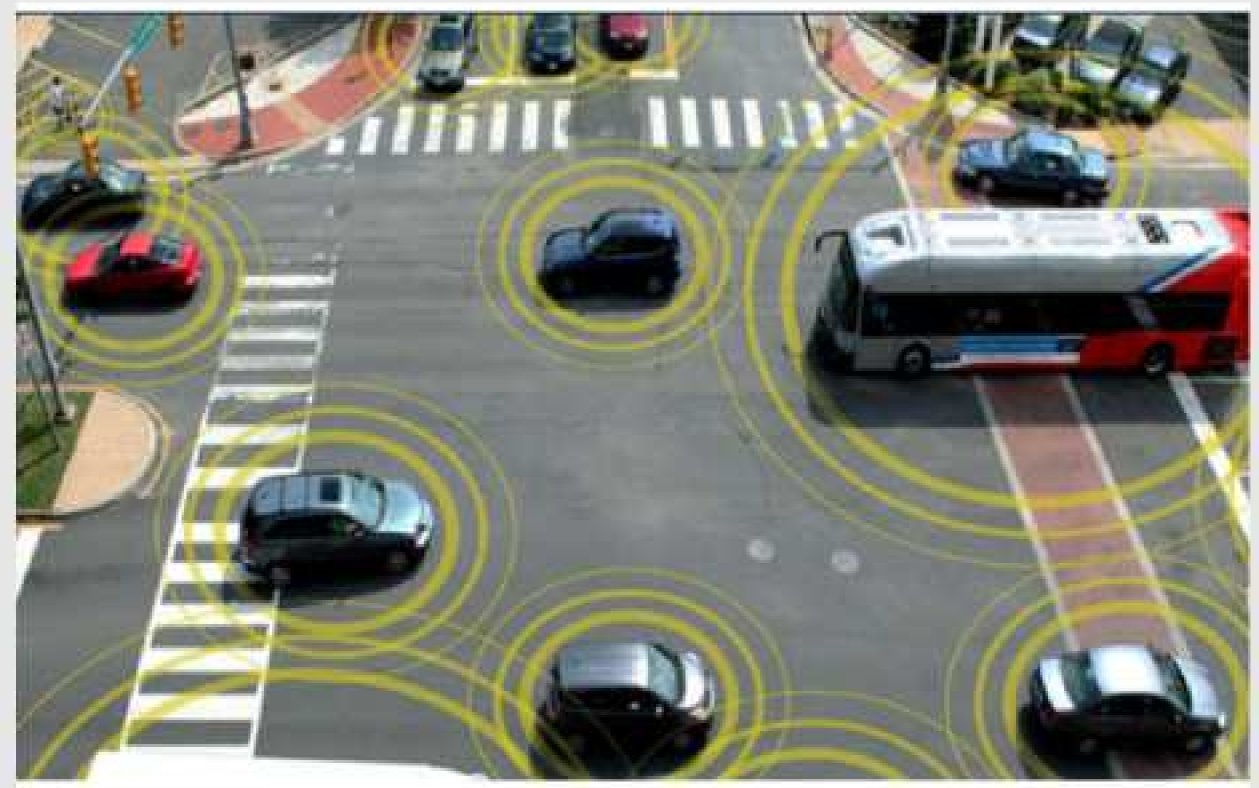
Maharbiz and Roychowdhury

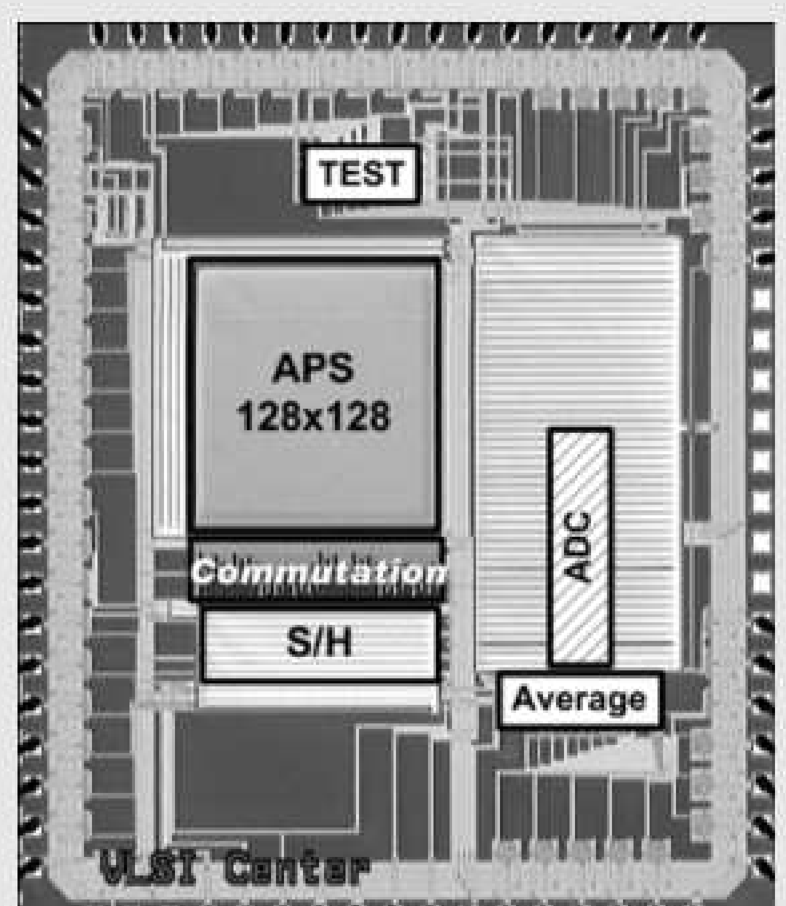
Lecture 4A: Overview Slides

State Space Representations

Systems

- Previously: circuits
- Now: systems
  - circuits + more: a broader concept





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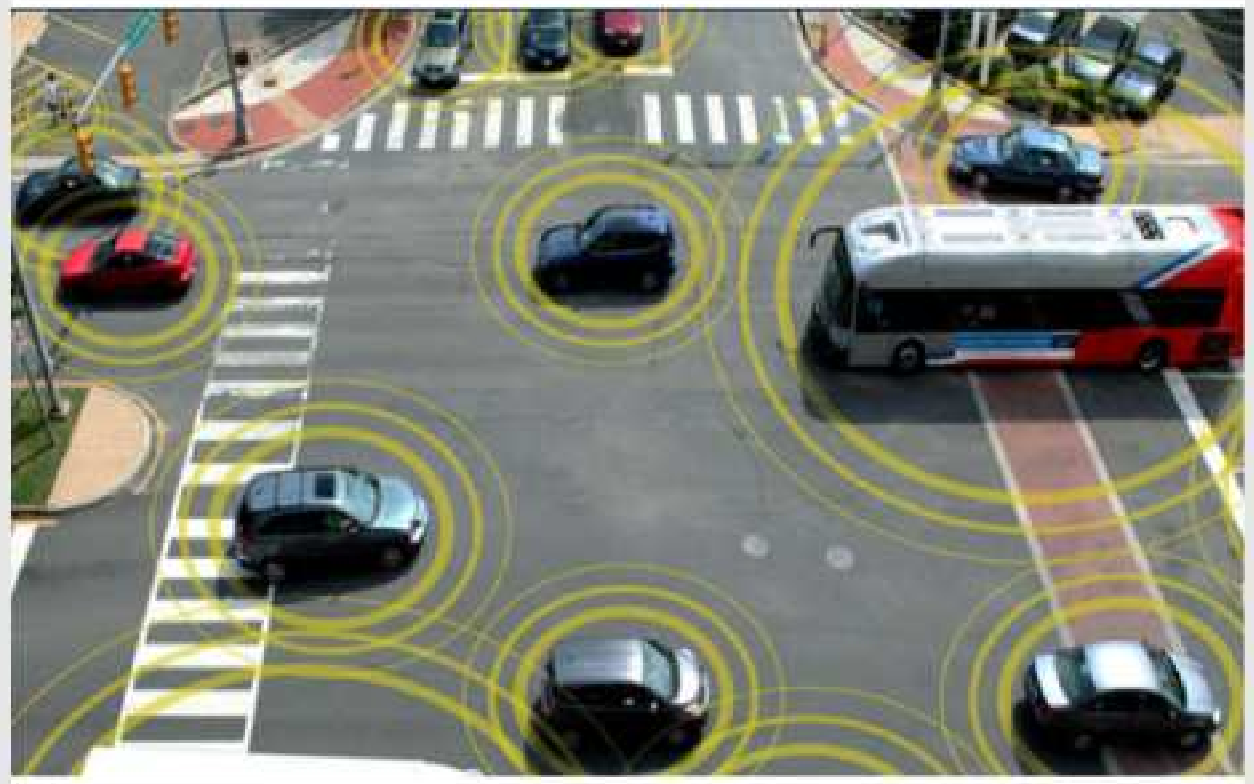

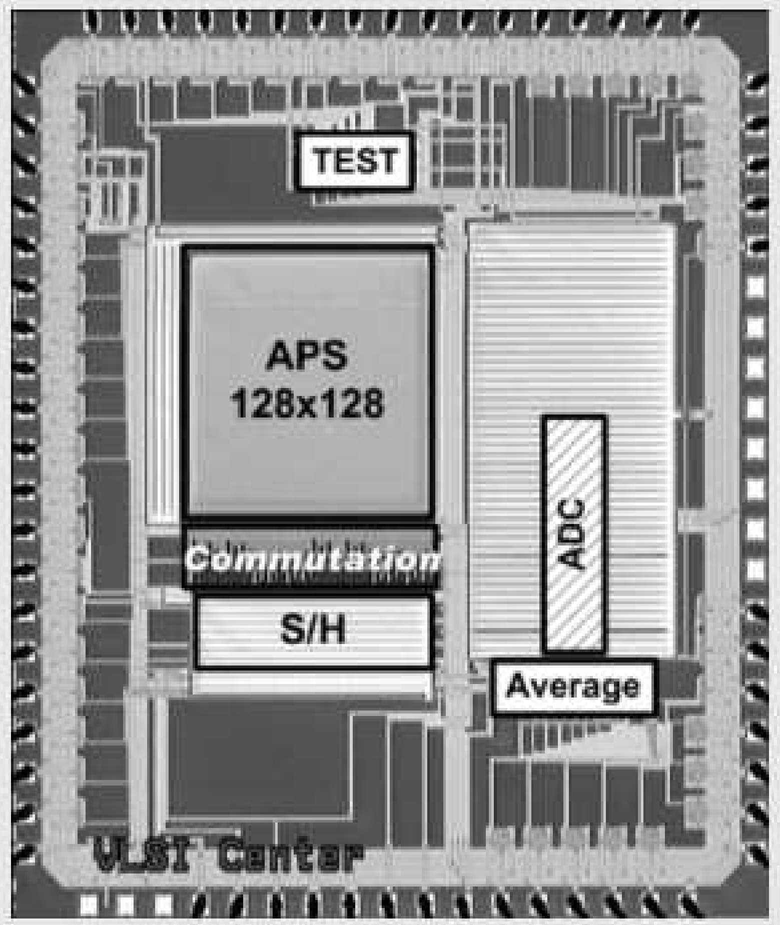

- Can be enormously complex
  - multi-domain
    - ➔ EE (control, comm., computing, ...)
    - ➔ mech., chem., optical, …

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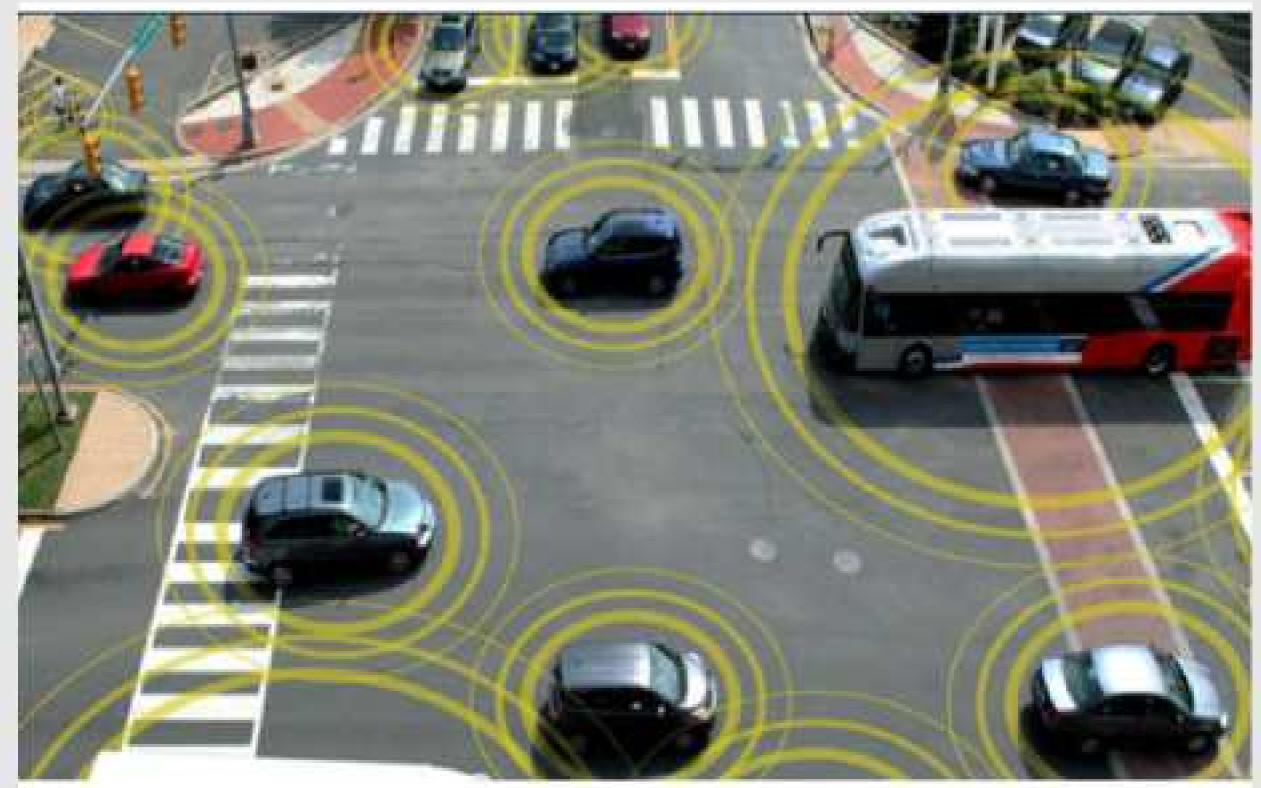
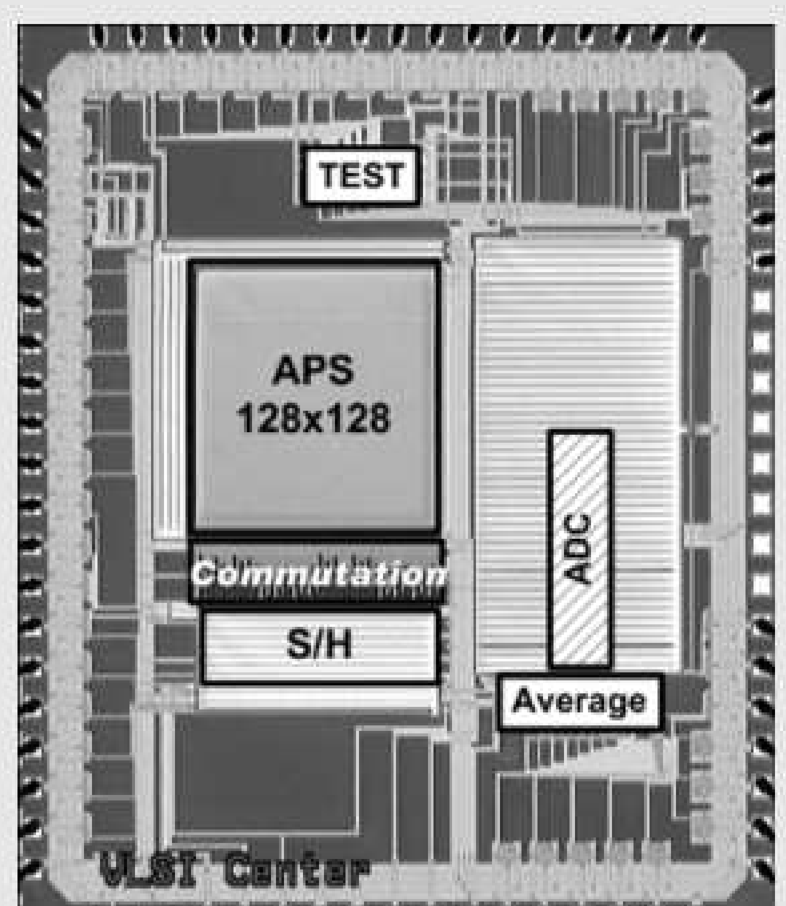


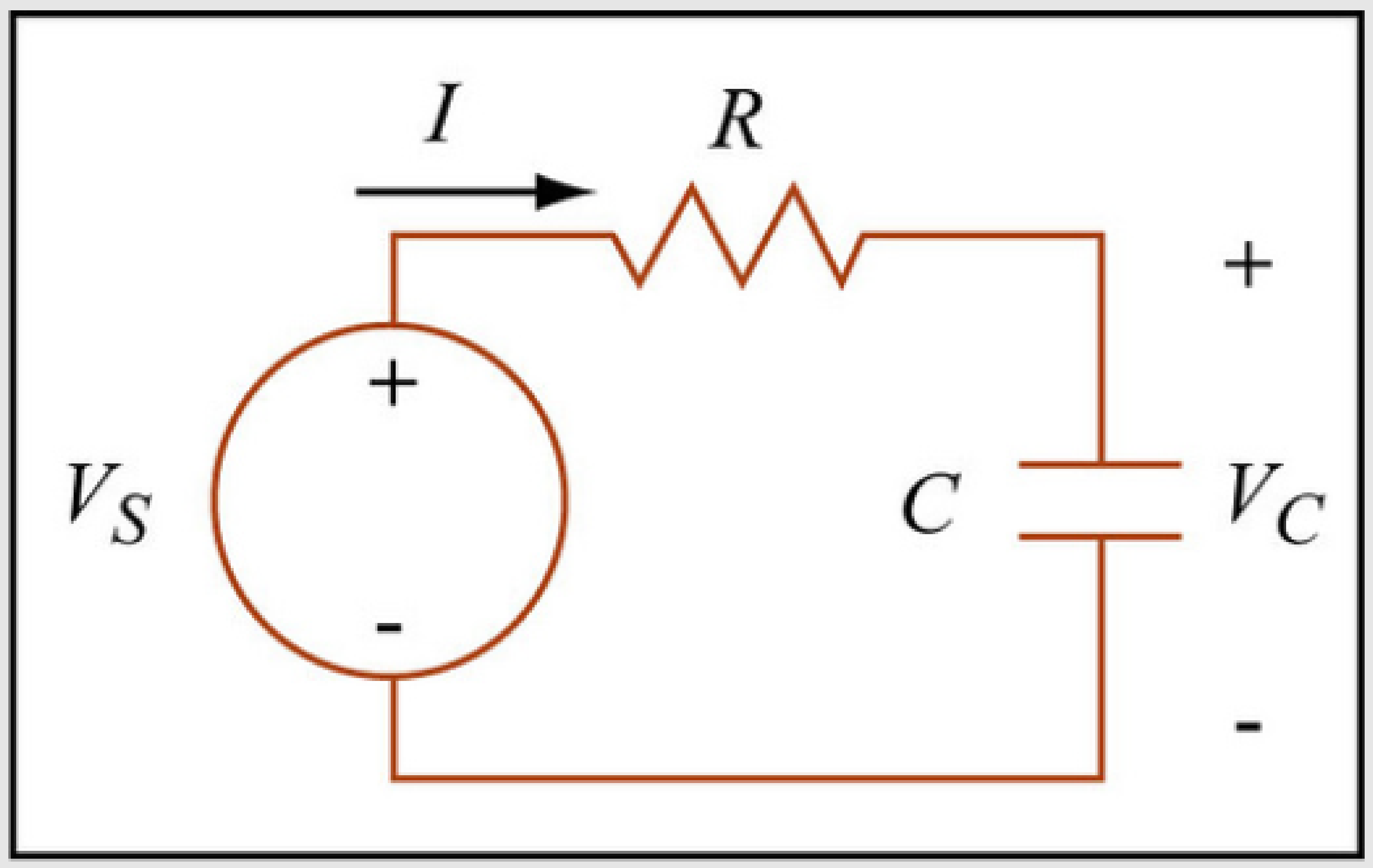


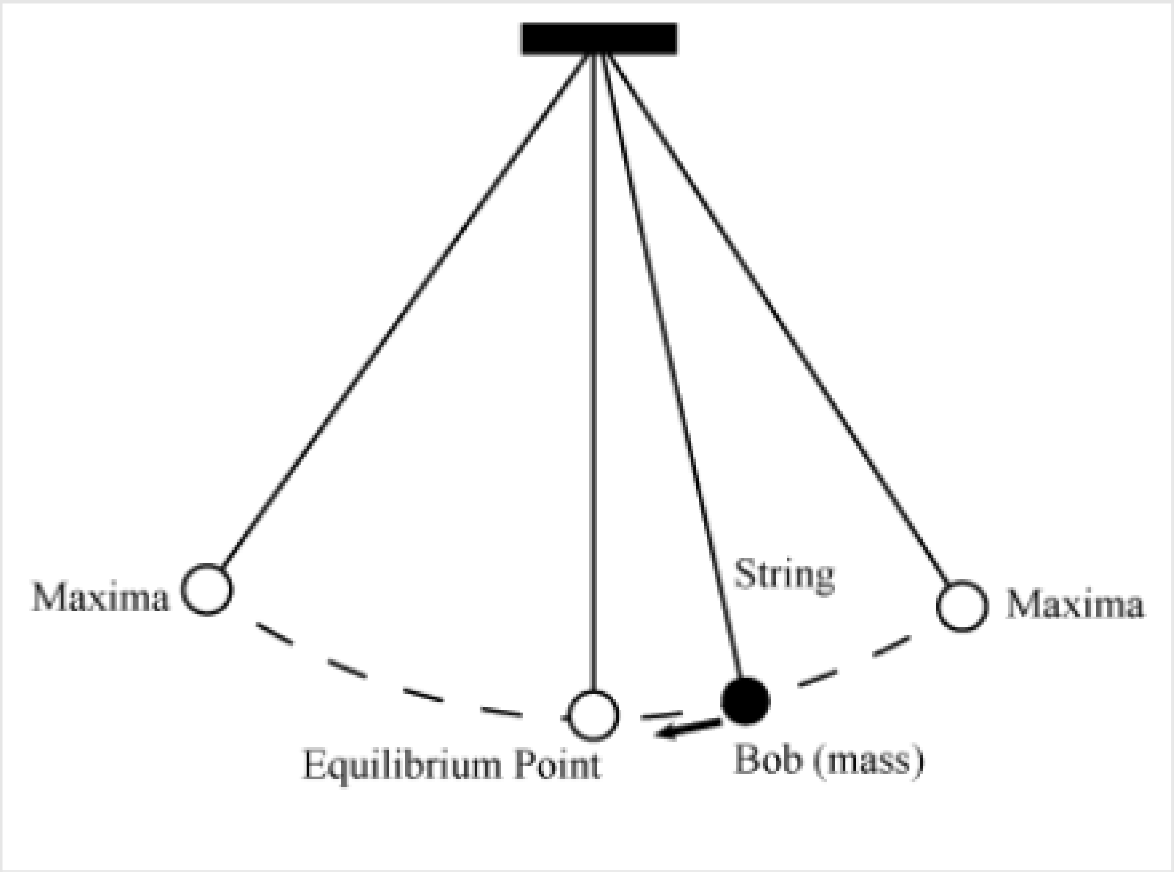


- Can be enormously complex
  - multi-domain
    - EE (control, comm., computing, ...)
    - mech., chem., optical, …
  - hierarchy of sub-systems

Simpler Systems

- … easier to understand and to work with





$$4H + O_2 \underset{k_2}{\overset{k_1}{\rightleftharpoons}} H_2O$$

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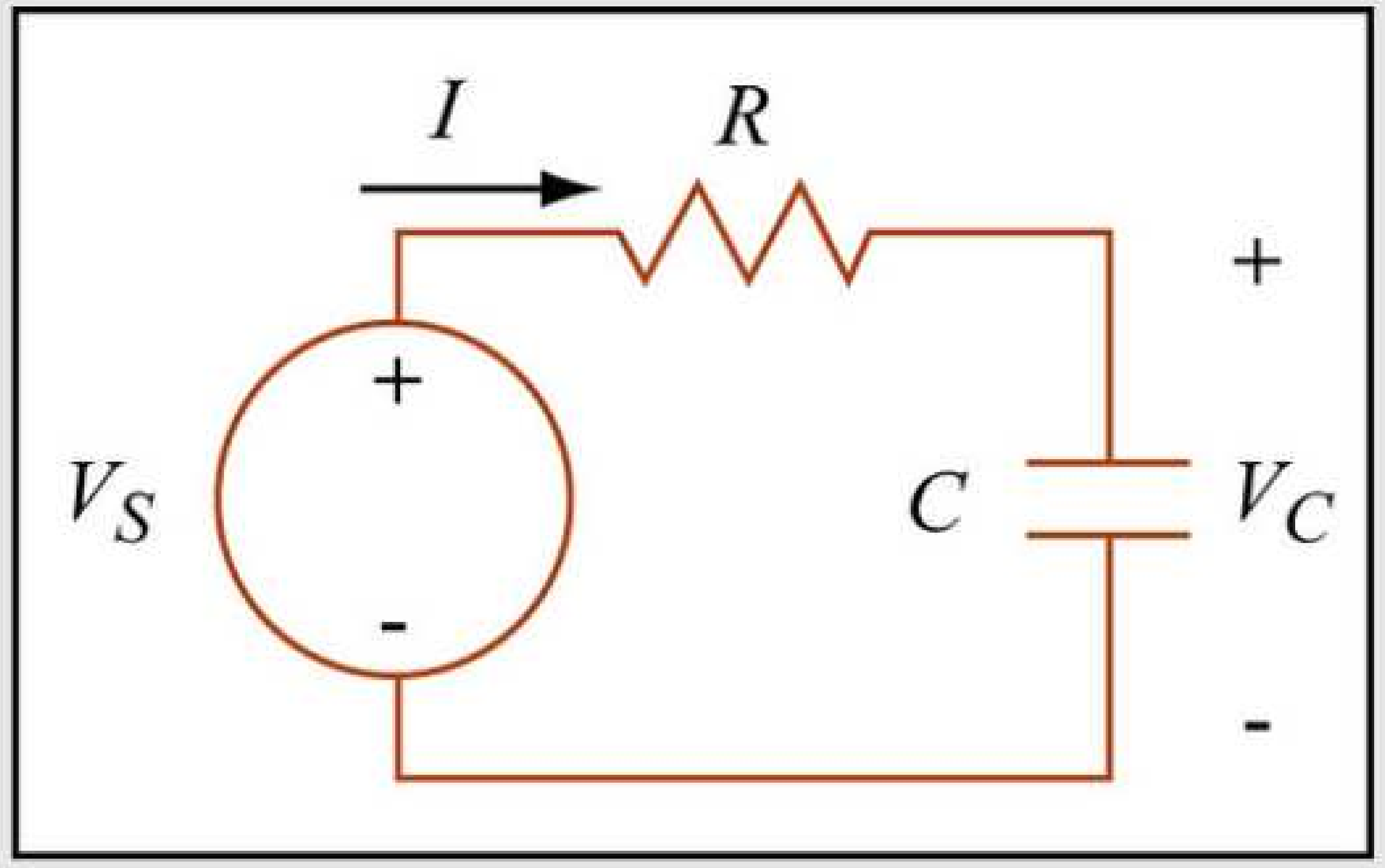

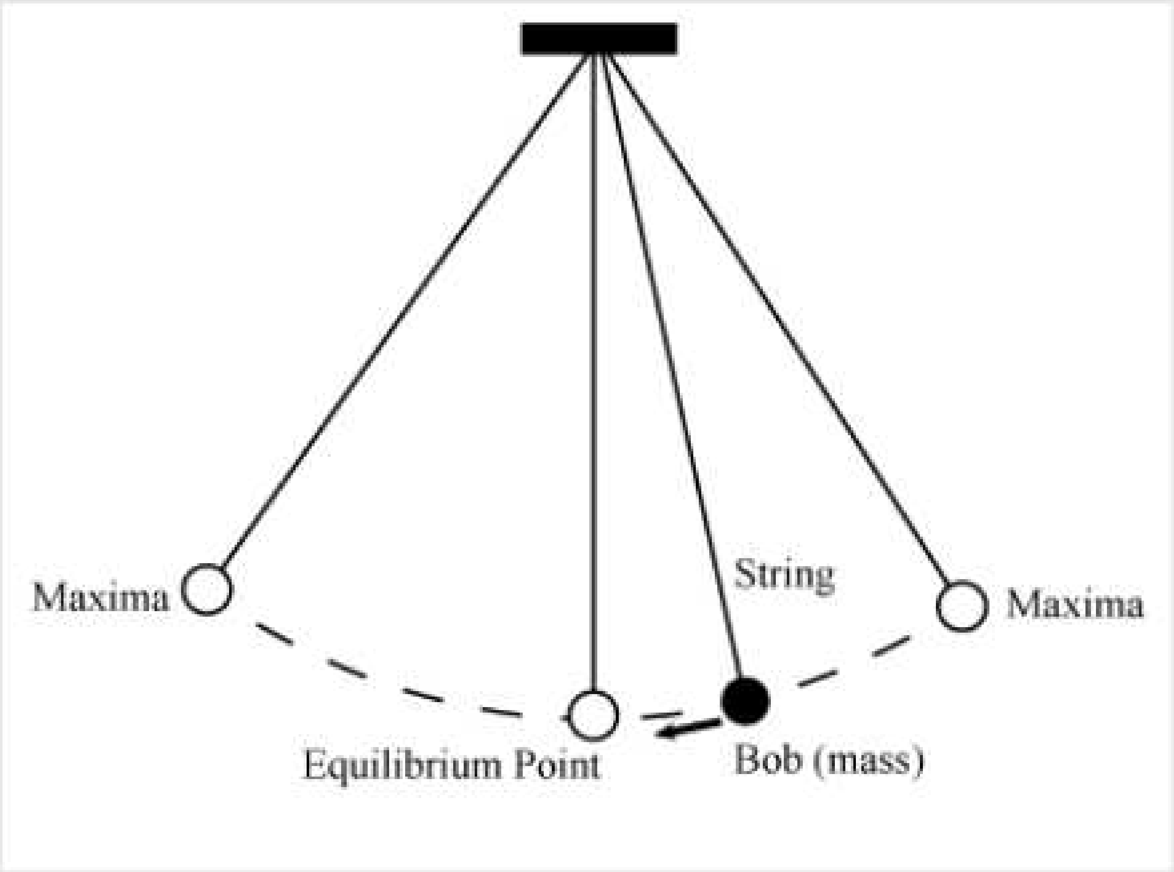

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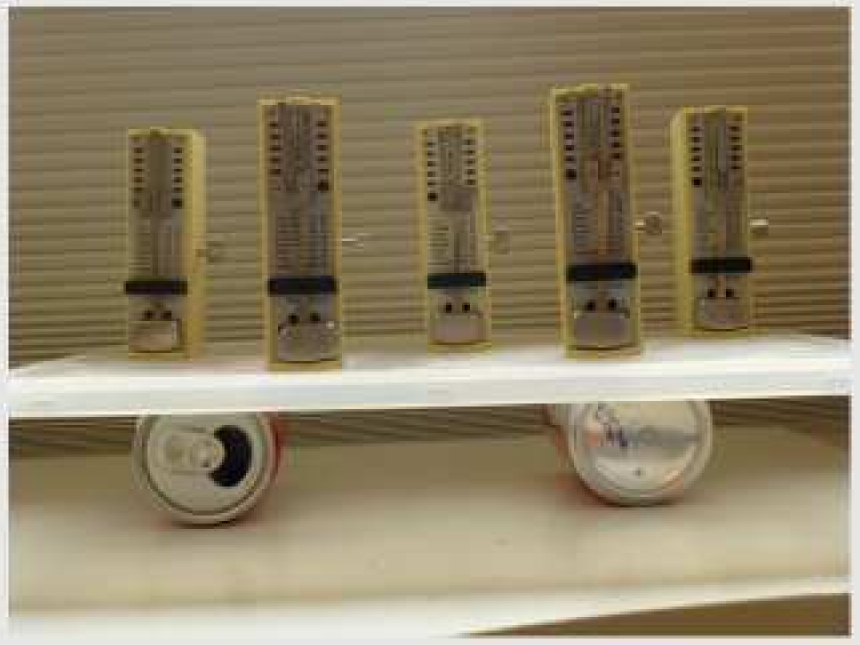
- Even small and simple systems can do interesting things



A Note about Intuition vs Math

- Proper intuition: extremely important
  - quickly make mental leaps to solve problems and design new things
    - → … avoiding confusion that can arise from masses of detail

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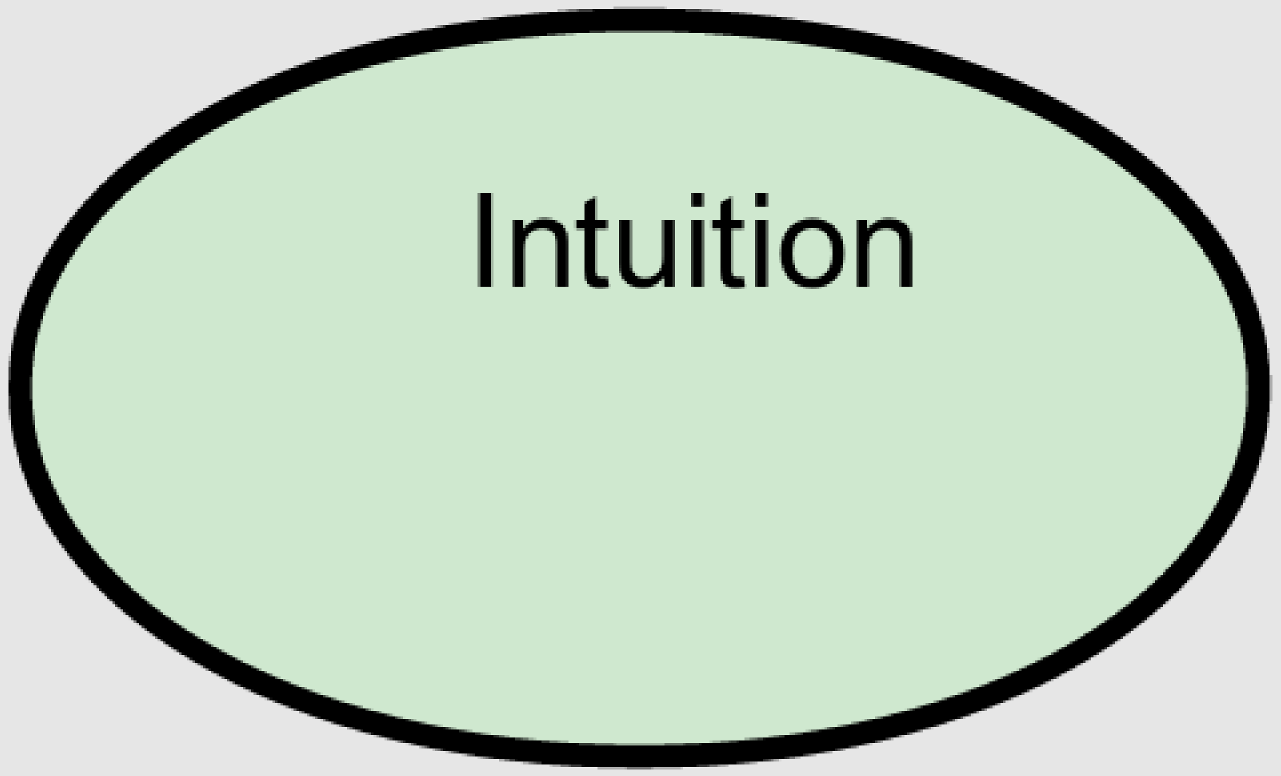

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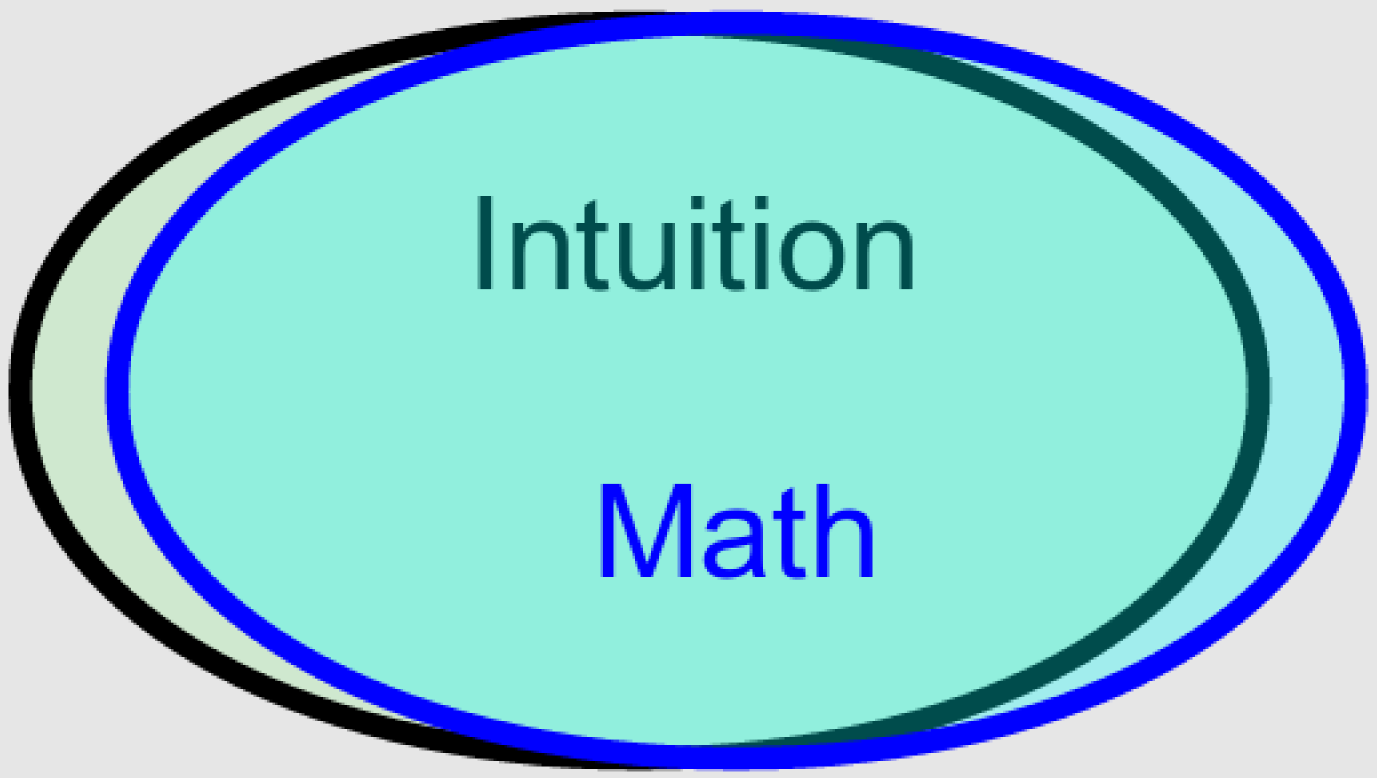

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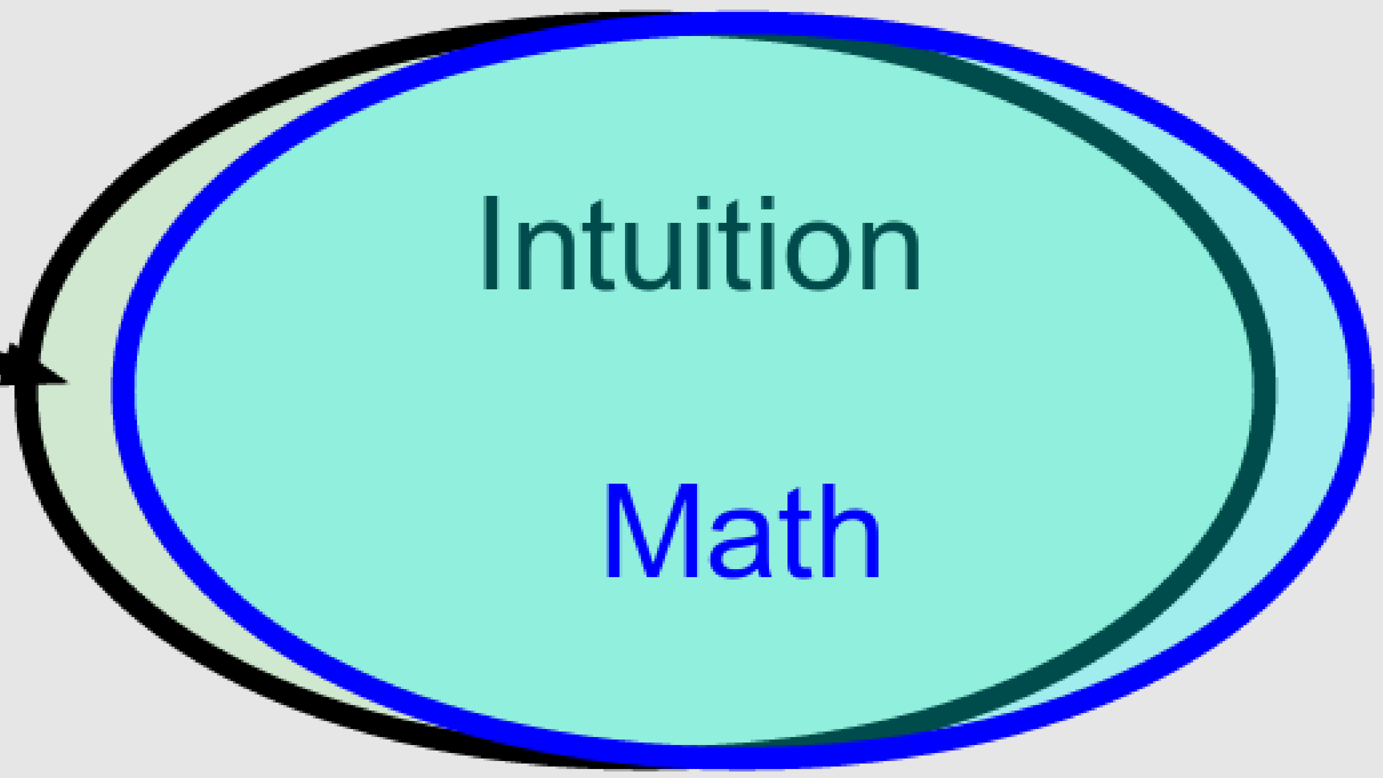

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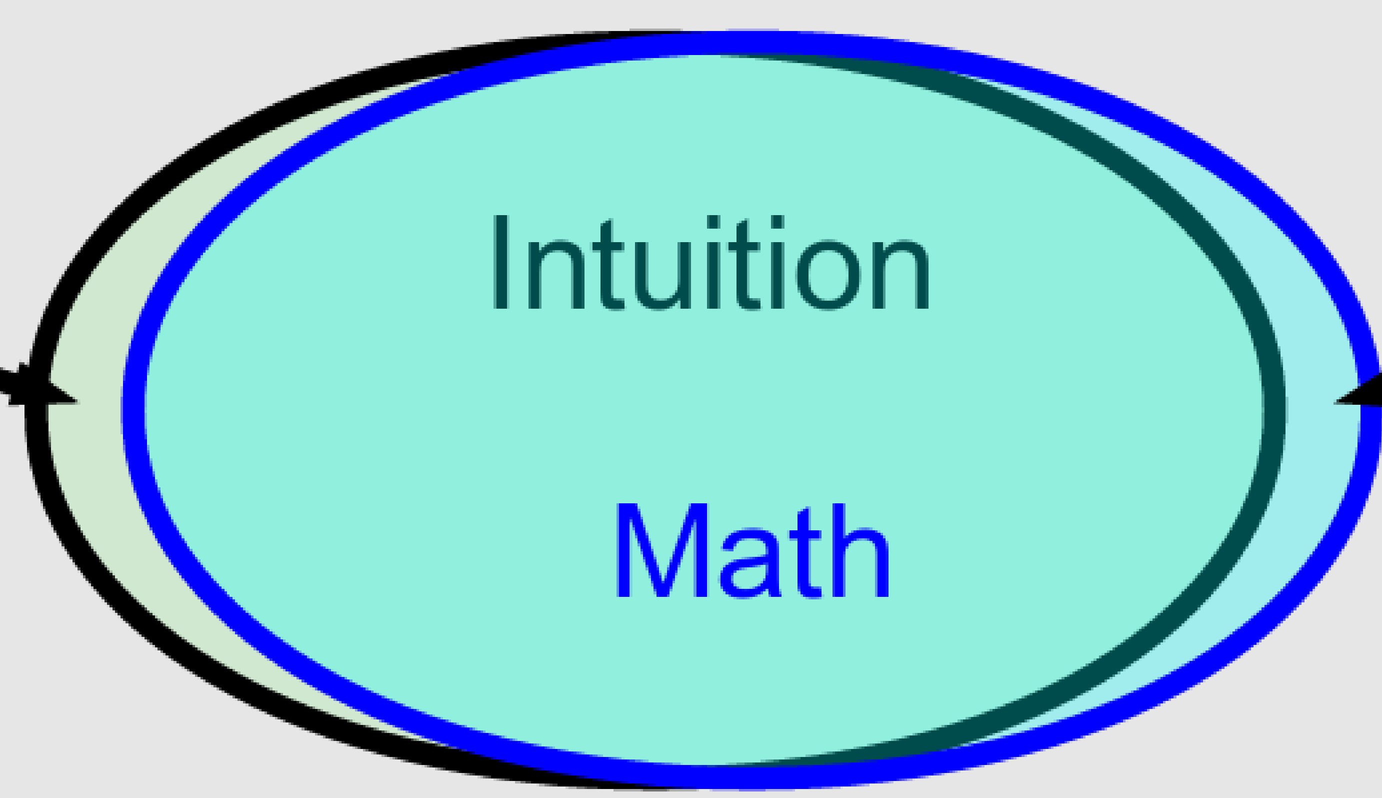

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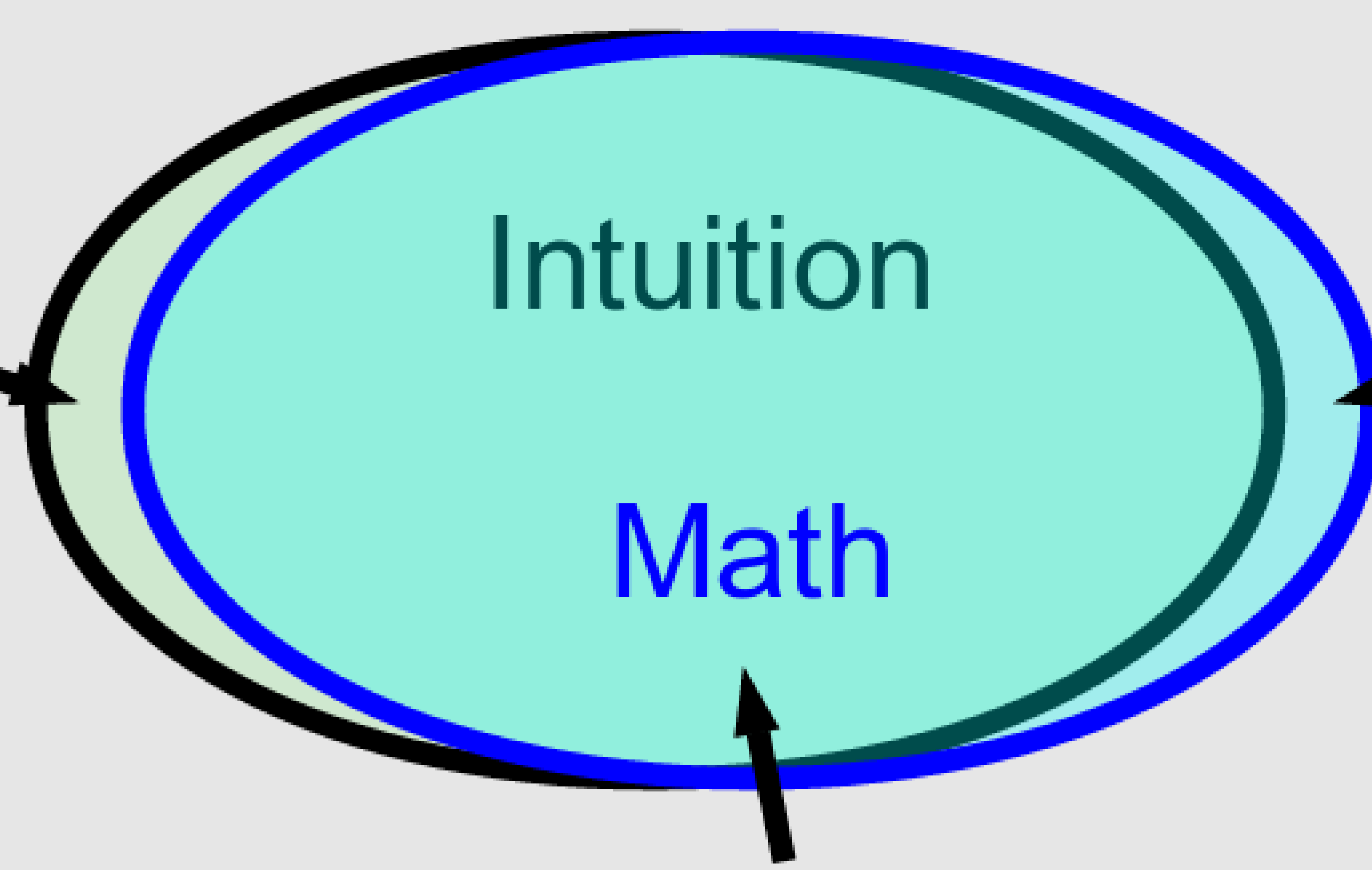

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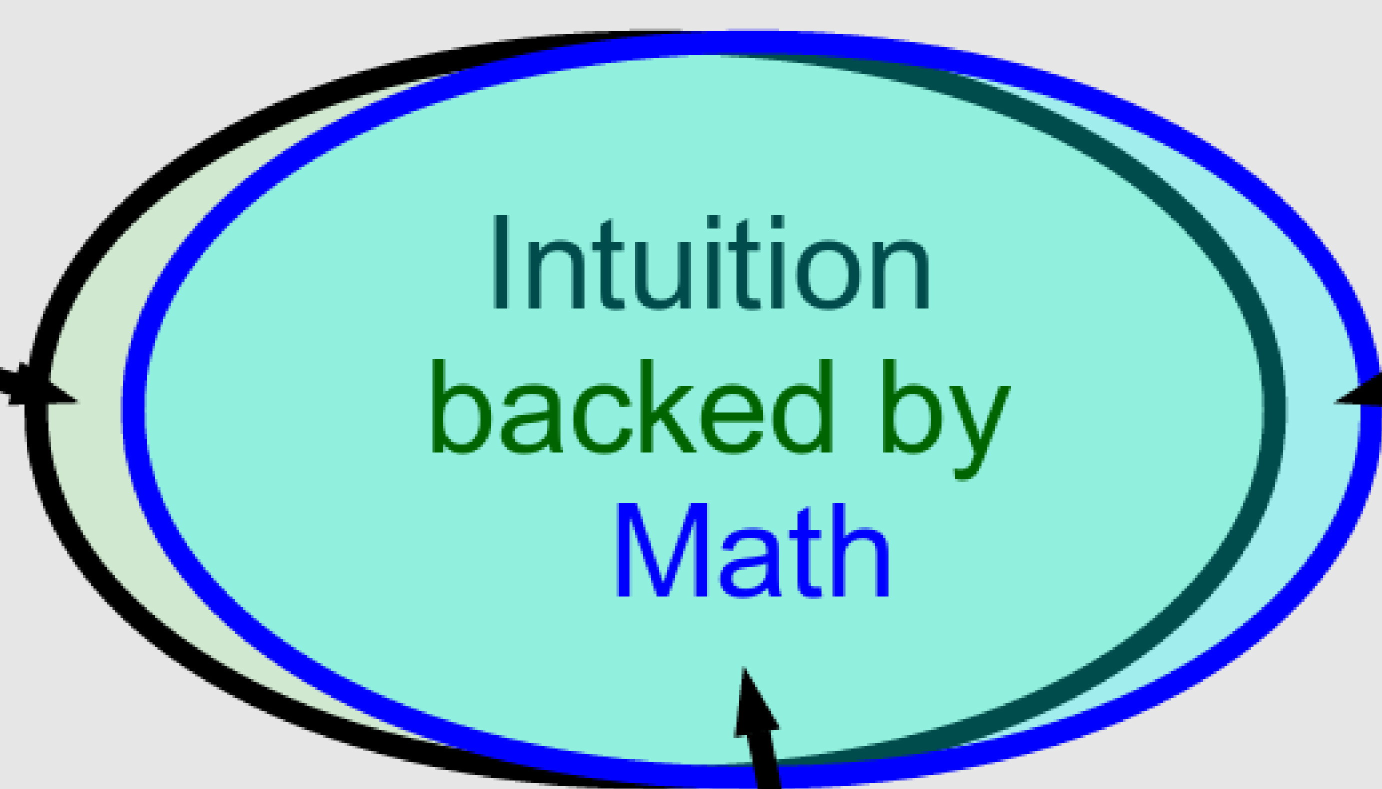

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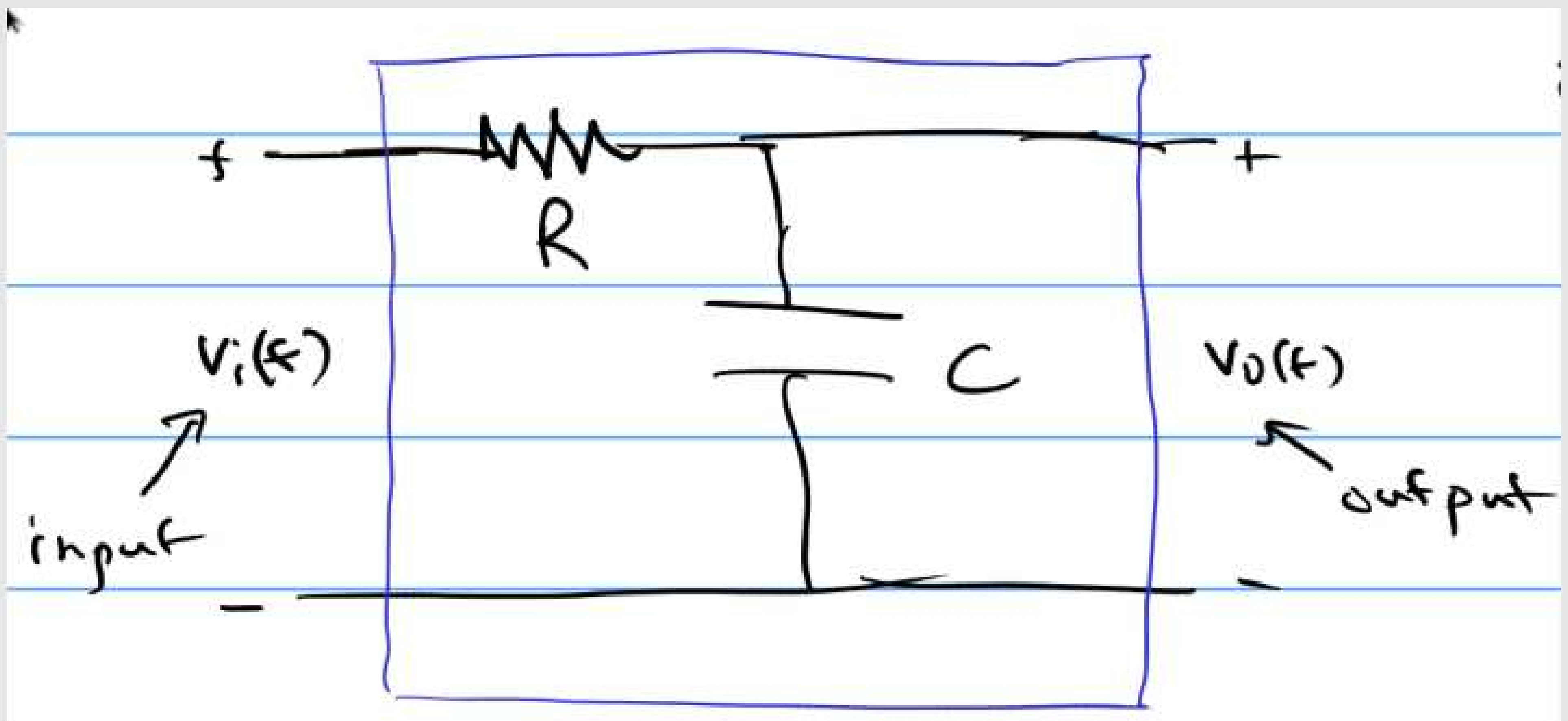
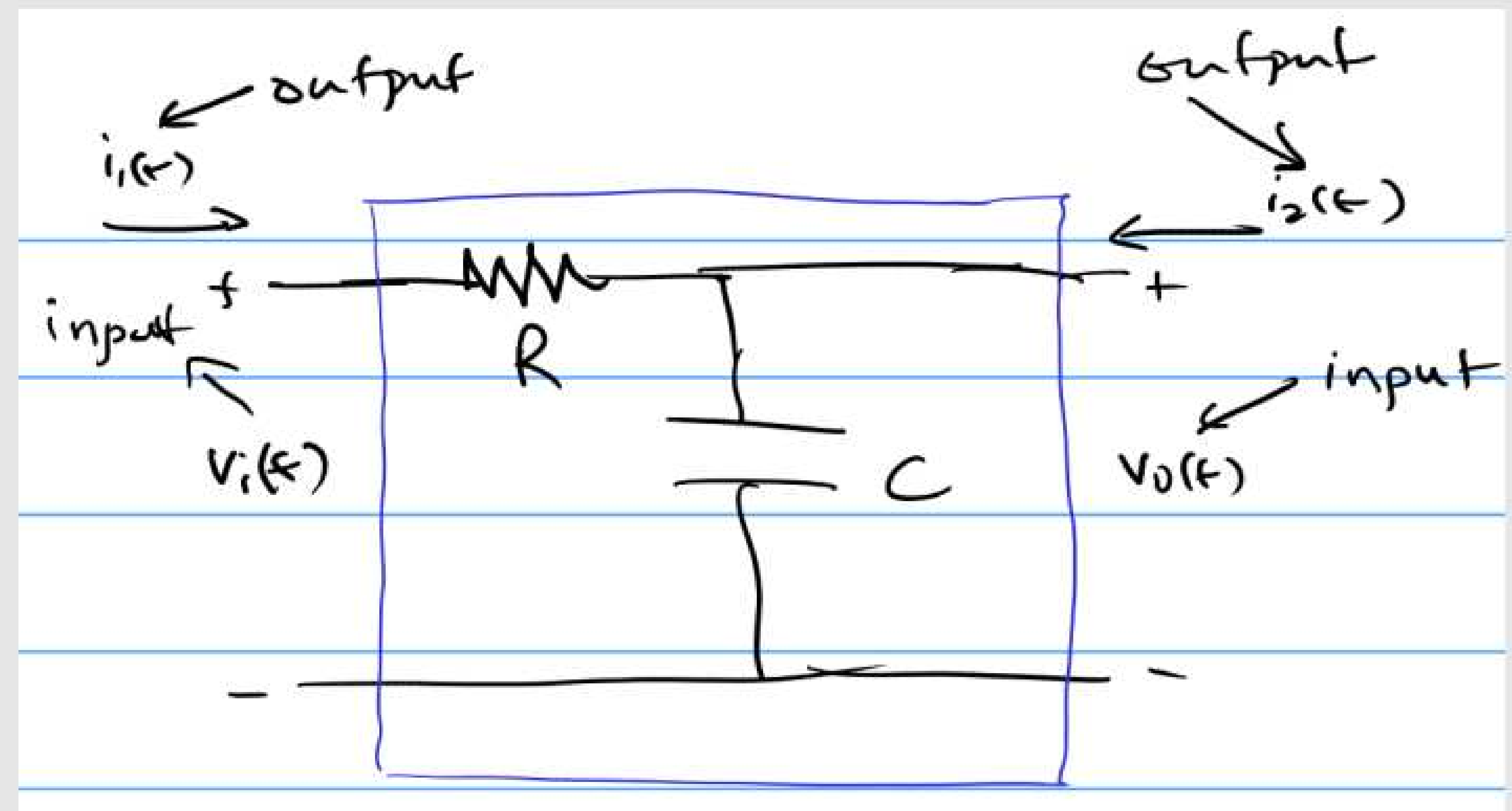
Inputs and Outputs as VECTORS

- (move to xournal)





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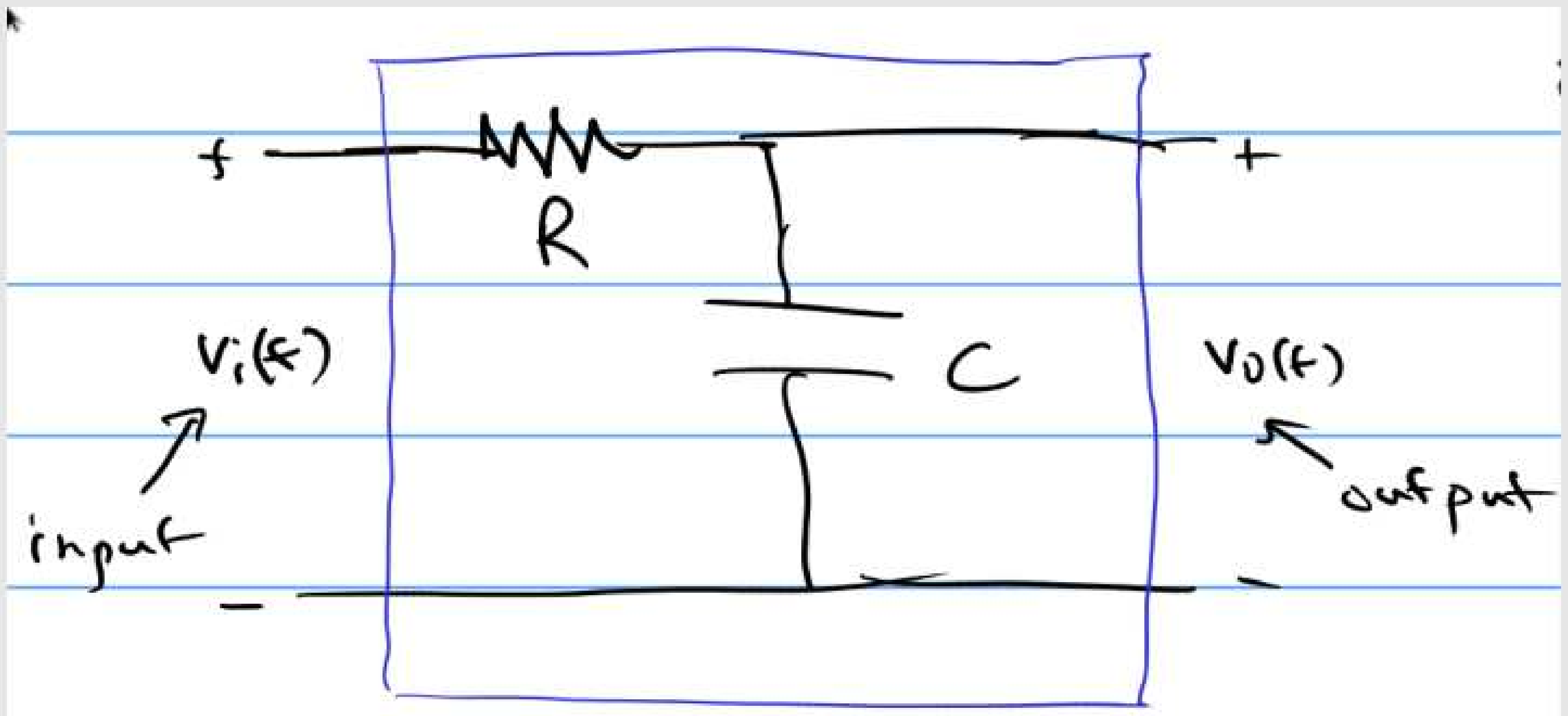

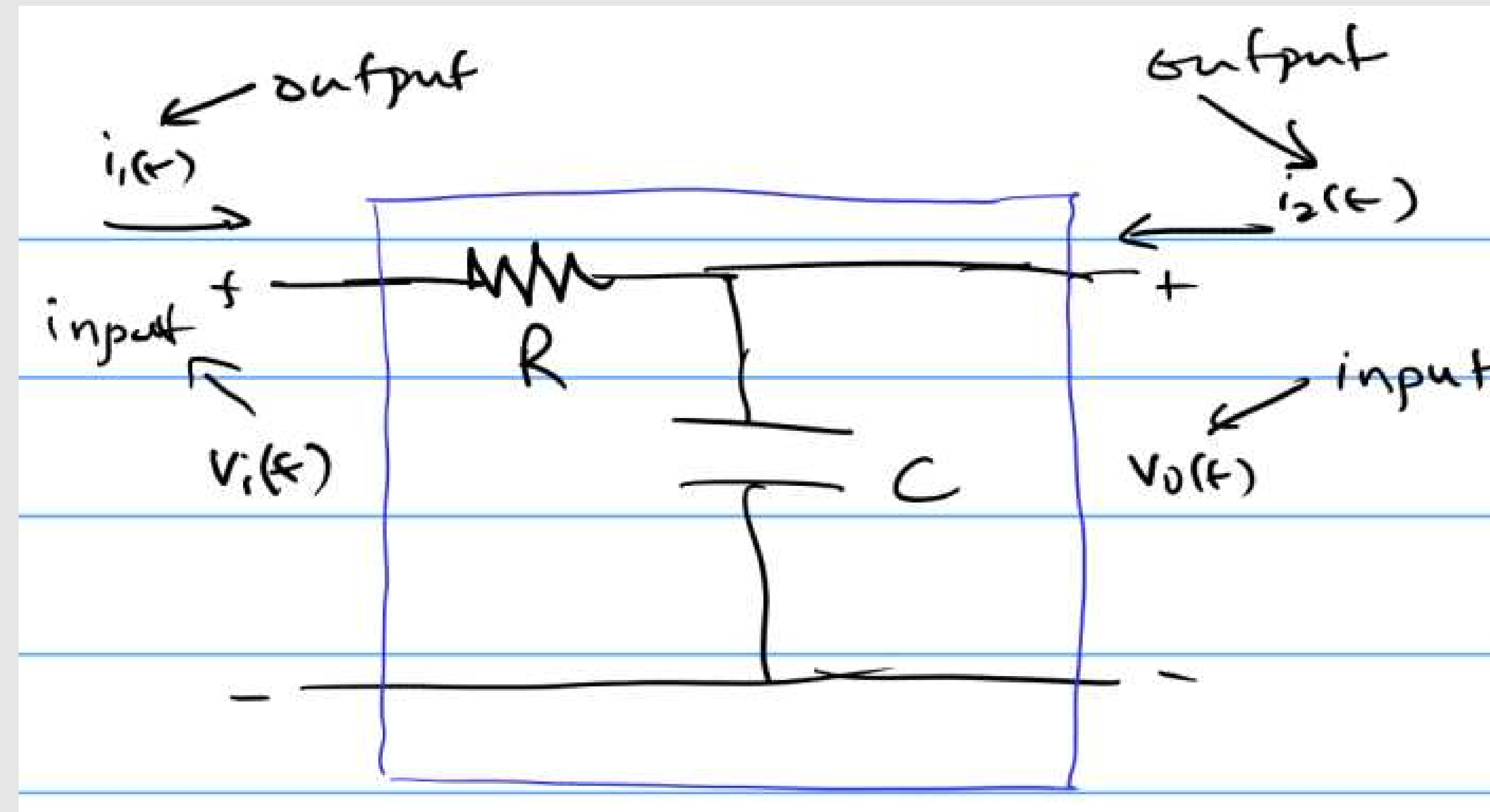

$$\vec{u}(t) = \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} ; \quad \vec{y}(t) = \begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix}$$

vector of inputs

vector of outputs

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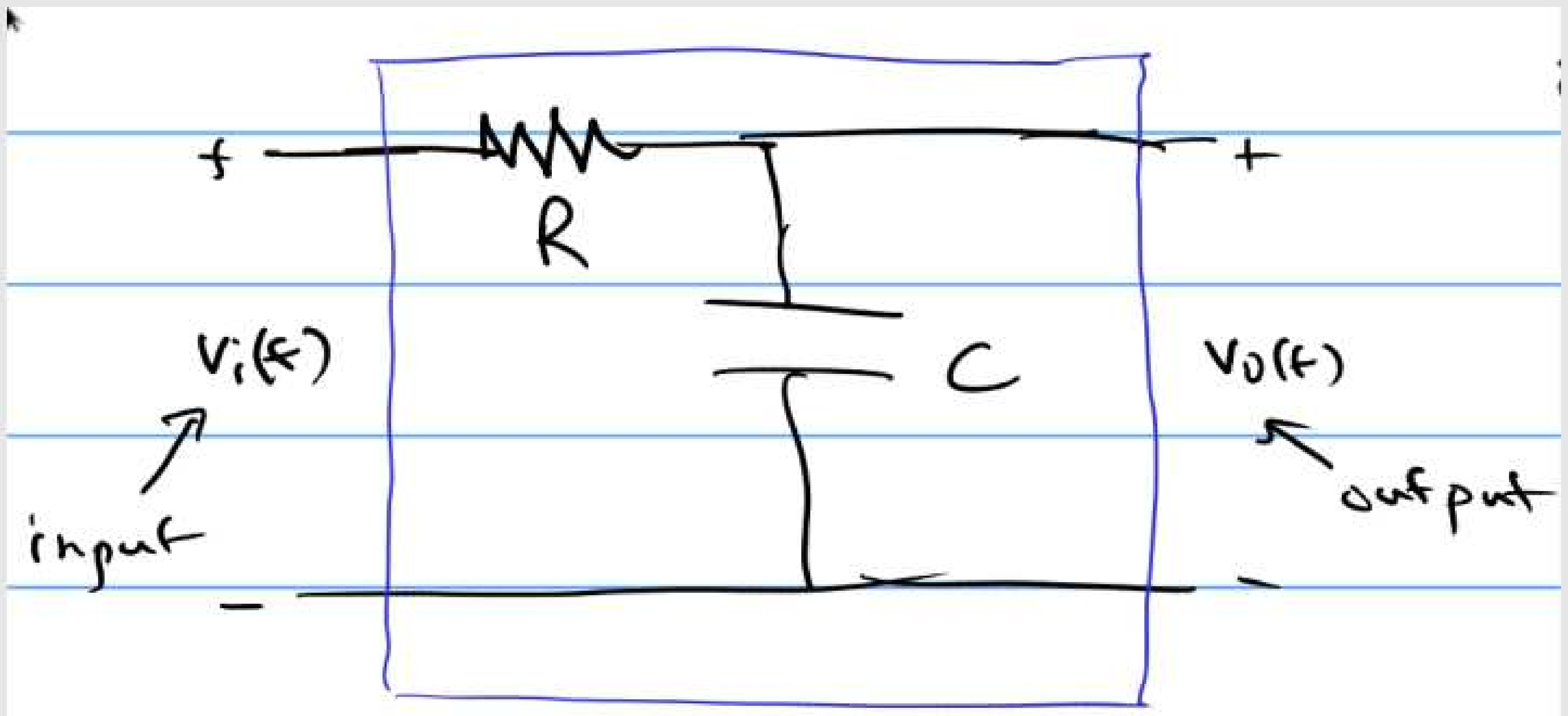

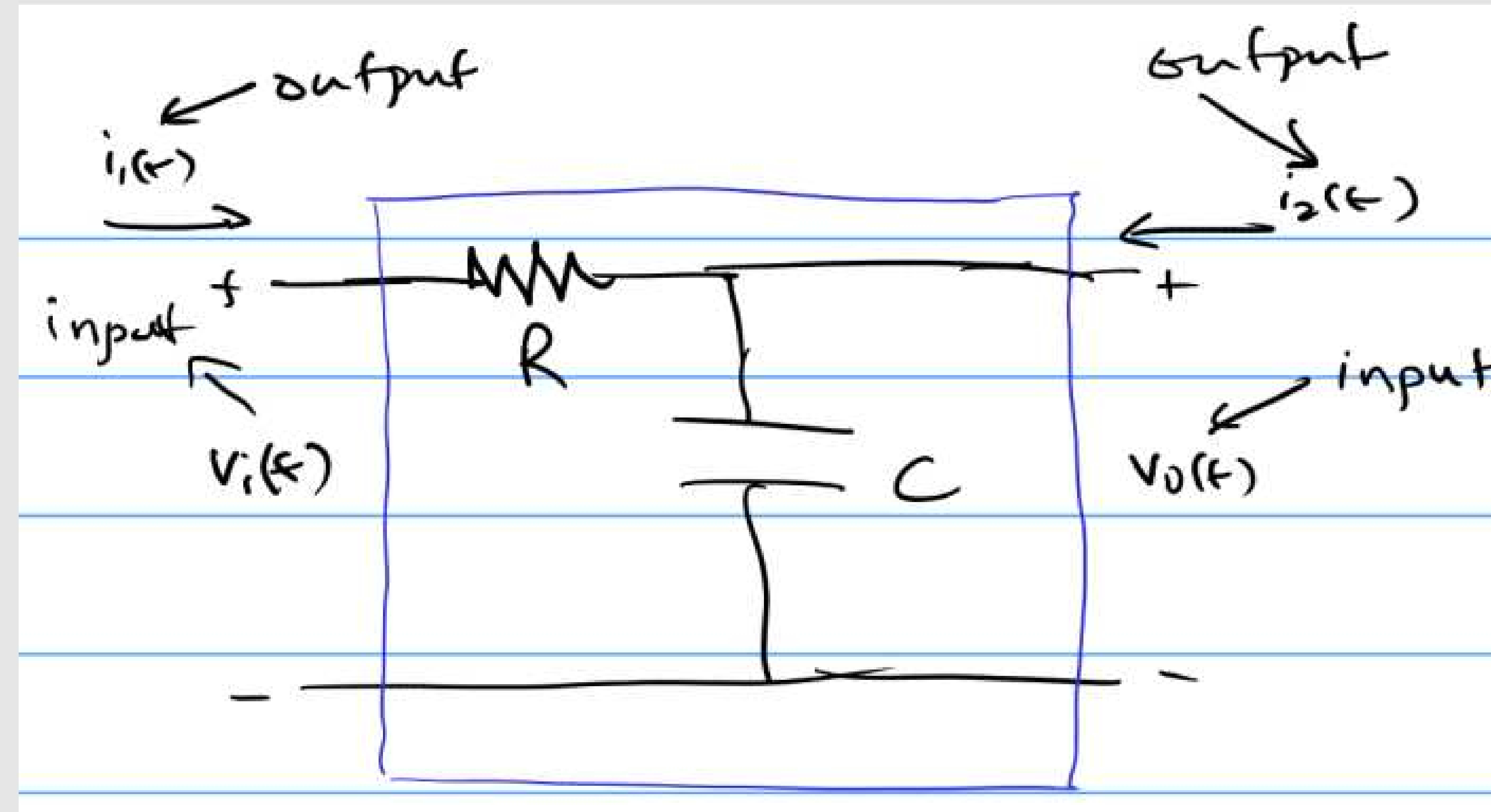

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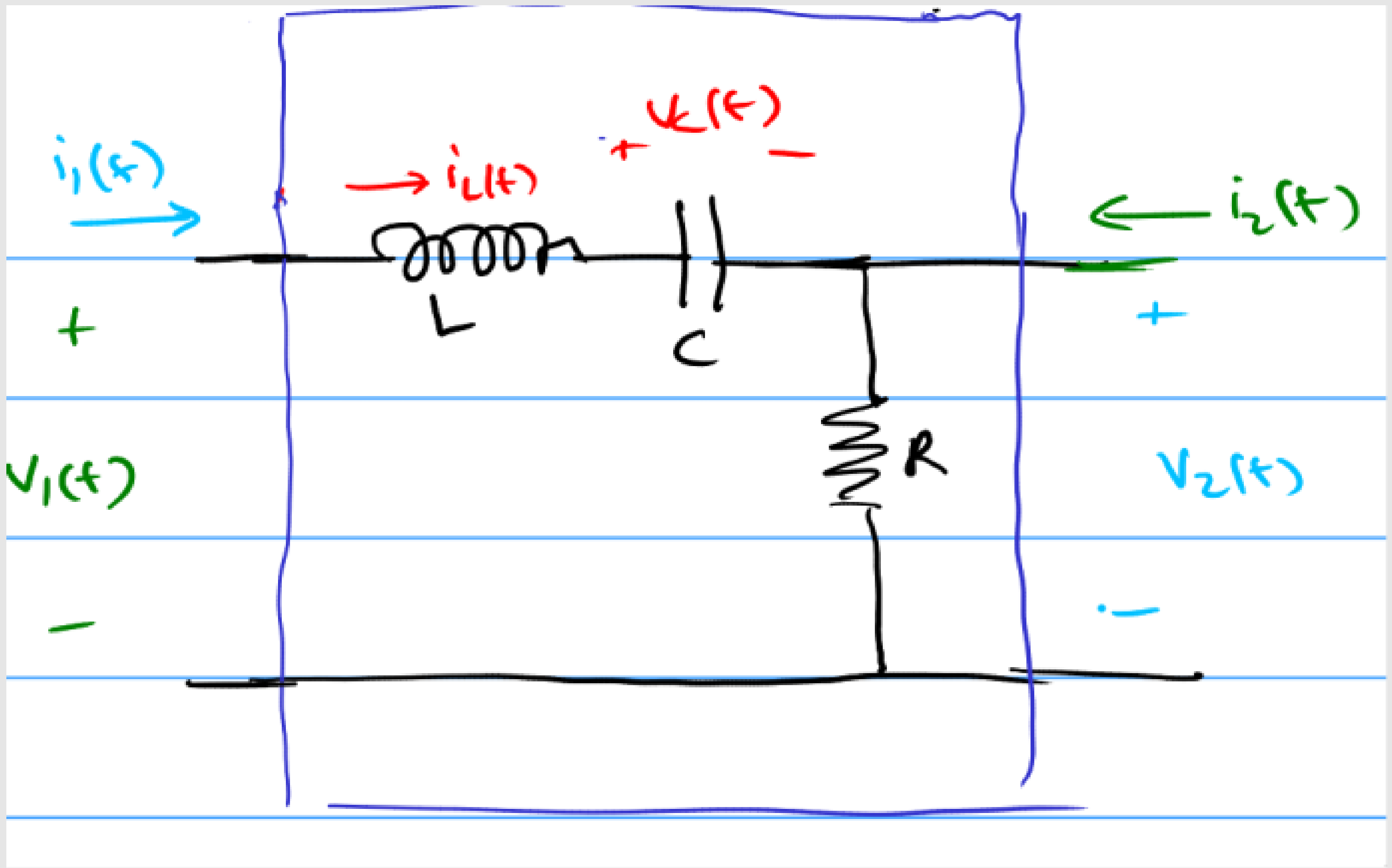
vector of inputs

vector of outputs

INPUTS AND OUTPUTS CAN BE ORGANIZED AS VECTORS

The Internal State

- (move to xournal)



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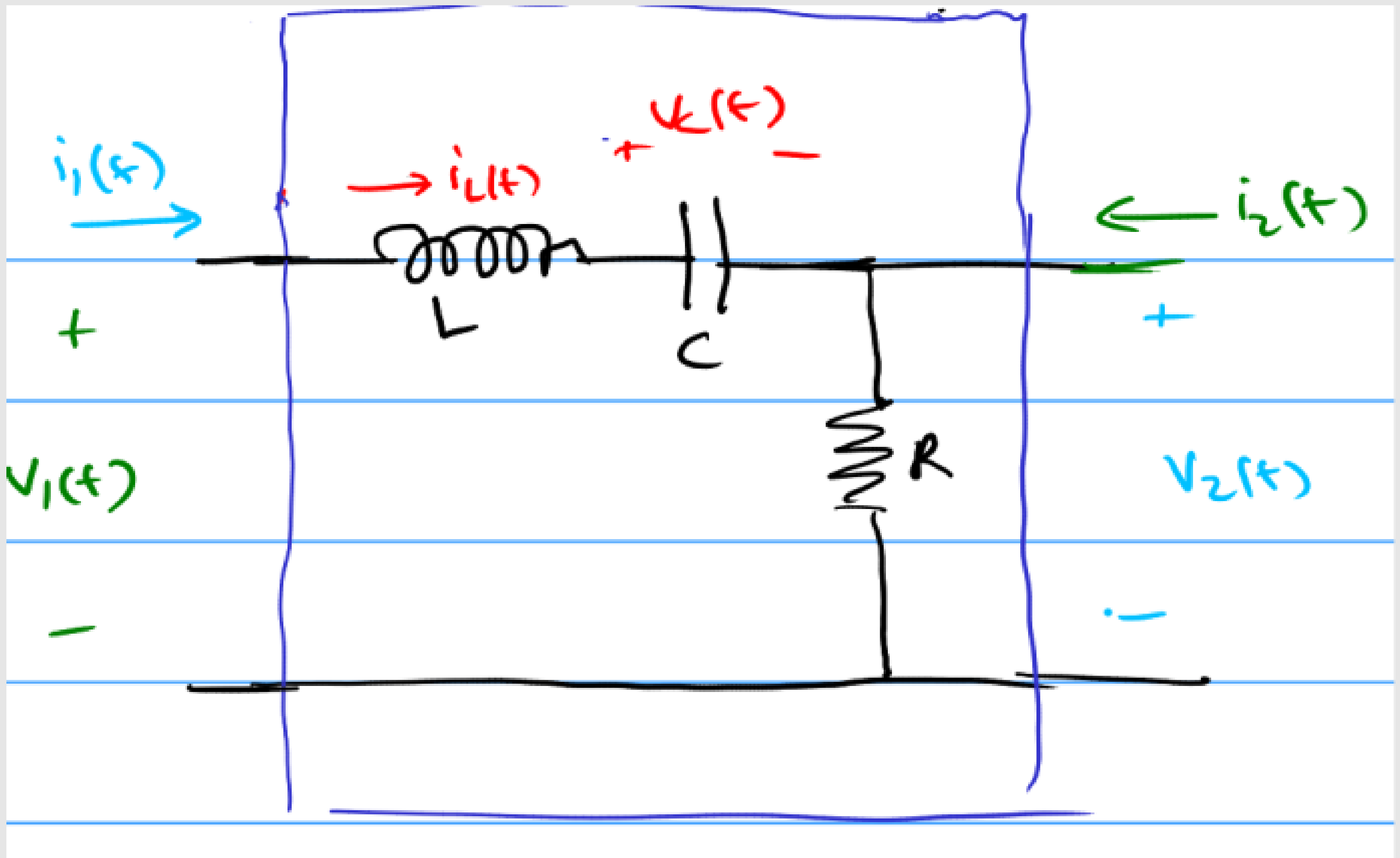

$$\vec{u}(t) = \begin{bmatrix} v_1(t) \\ i_2(t) \end{bmatrix} \qquad \vec{y}(t) = \begin{bmatrix} v_2(t) \\ i_1(t) \end{bmatrix}$$

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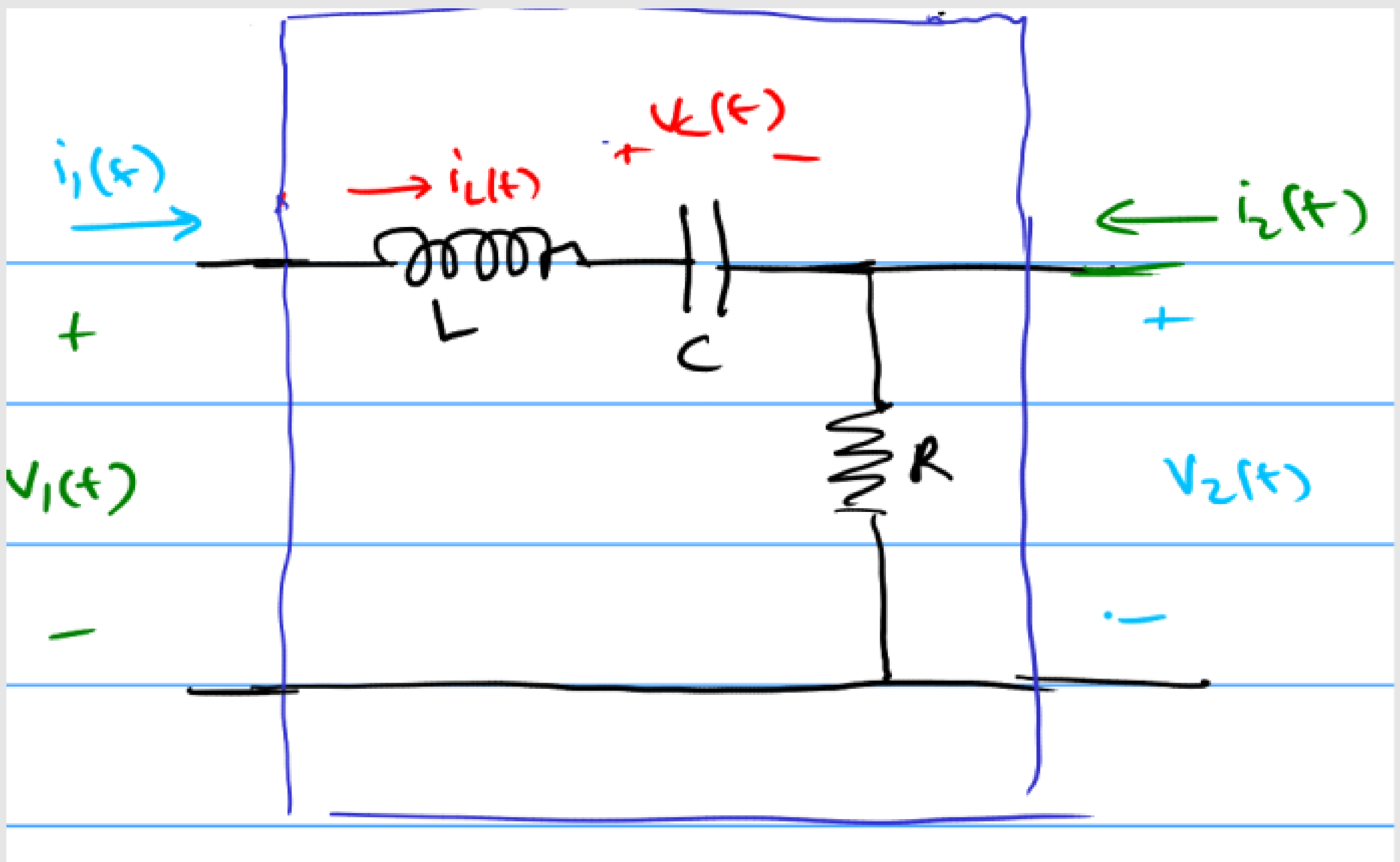
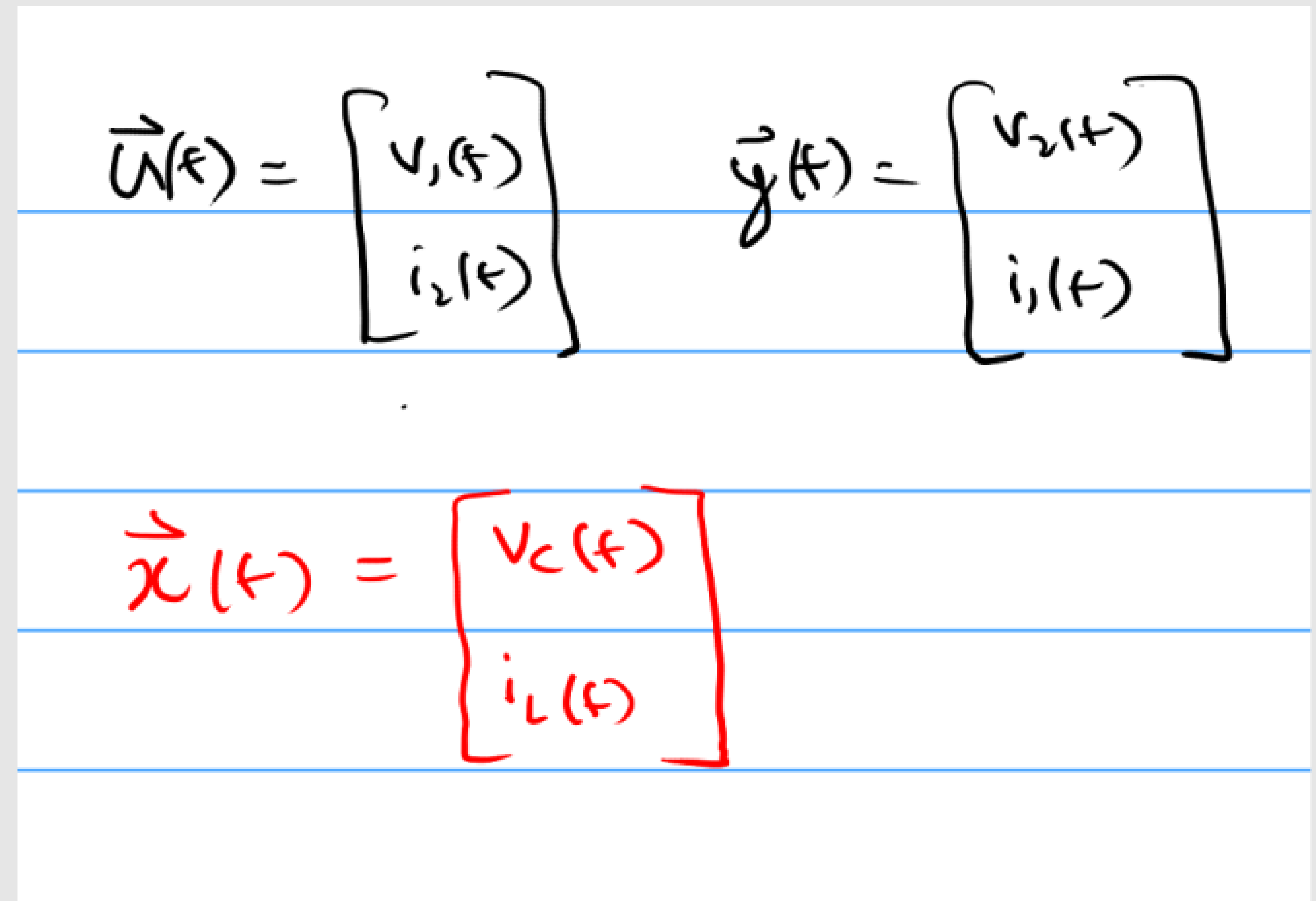


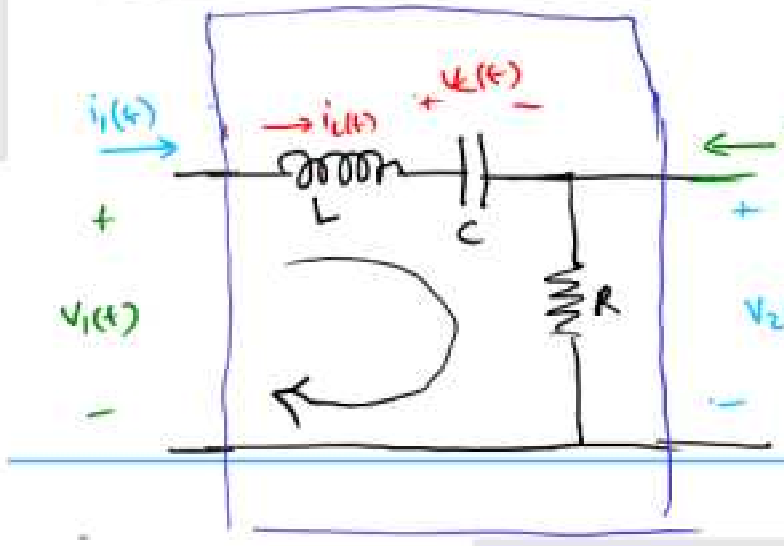


- Internal voltages/currents (unknowns): the **state**
  - also written as a vector: $\vec{x}(t)$

The System's Equations

- (move to xournal)

INTERNAL UNKNOWNS (STATE) AS VECTORS



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EQUATIONS

capacitor + KCL : $C\dfrac{dv_c(t)}{dt} - i_L(t) = 0$

KVL : $L\dfrac{di_L(t)}{dt} + v_c(t) + R\left(i_L(t) + i_2(t)\right) - v_1(t) = 0$

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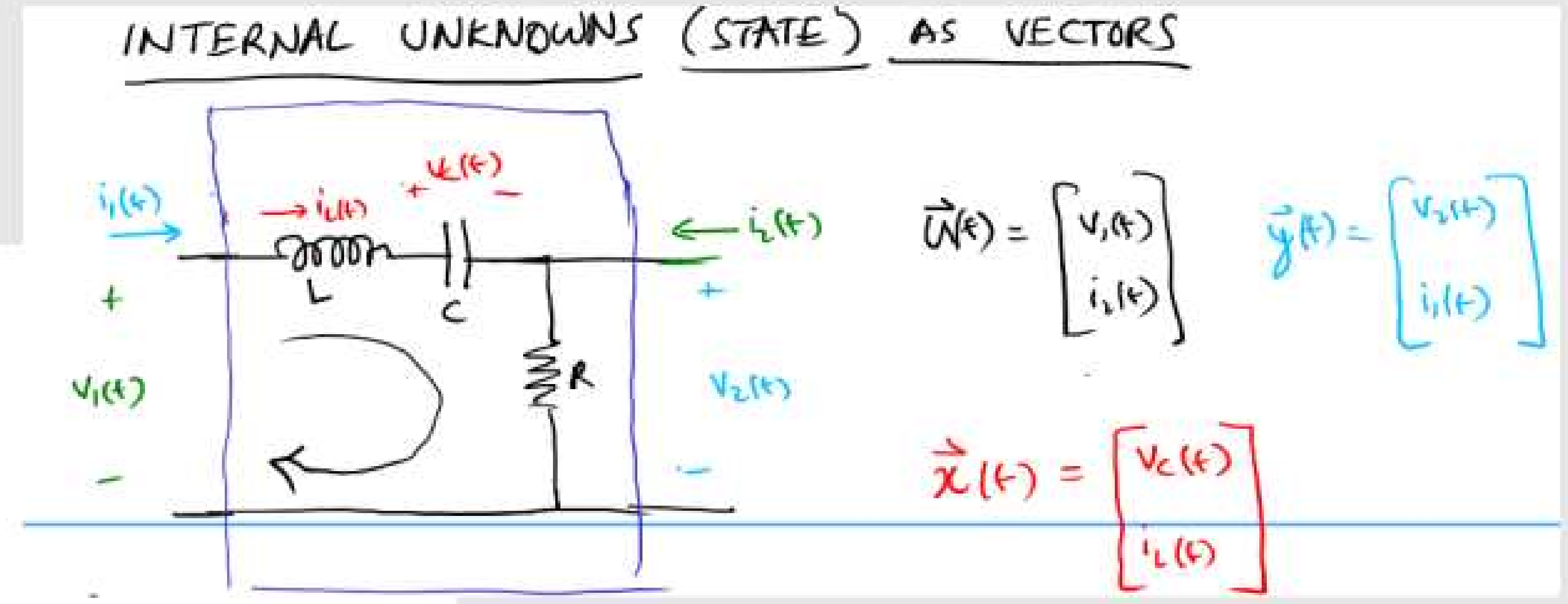

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$$\underbrace{\begin{bmatrix} C & 0 \\ 0 & L \end{bmatrix}}_{A} \overbrace{\frac{d}{dt}\begin{bmatrix} v_c(t) \\ i_L(t) \end{bmatrix}}^{\frac{d}{dt}\vec{x}(t)} + \underbrace{\begin{bmatrix} 0 & -1 \\ 1 & R \end{bmatrix}}_{B} \overbrace{\begin{bmatrix} v_c(t) \\ i_L(t) \end{bmatrix}}^{\vec{x}(t)} + \underbrace{\begin{bmatrix} 0 & 0 \\ -1 & R \end{bmatrix}}_{C} \overbrace{\begin{bmatrix} v_1(t) \\ i_2(t) \end{bmatrix}}^{\vec{u}(t)} = \overbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}^{\vec{0}}$$

$$\frac{d}{dt}\begin{bmatrix} v_c(t) \\ i_L(t) \end{bmatrix} = \begin{bmatrix} & 1/C \\ -1/L & -R/L \end{bmatrix}\begin{bmatrix} v_c(t) \\ i_L(t) \end{bmatrix} + \begin{bmatrix} & \\ 1/L & -R/L \end{bmatrix}\begin{bmatrix} v_1(t) \\ i_2(t) \end{bmatrix}$$

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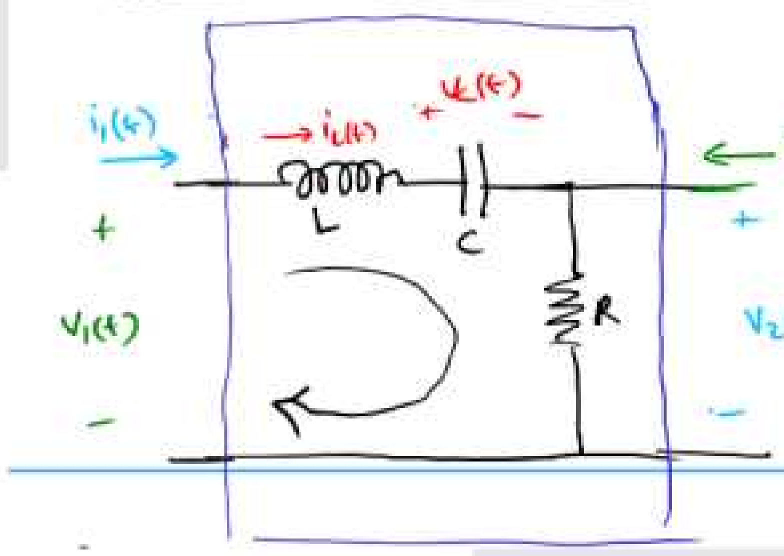

$$\vec{w}(t) = \begin{bmatrix} v_1(t) \\ i_1(t) \end{bmatrix} \qquad \vec{y}(t) = \begin{bmatrix} v_2(t) \\ i_1(t) \end{bmatrix}$$

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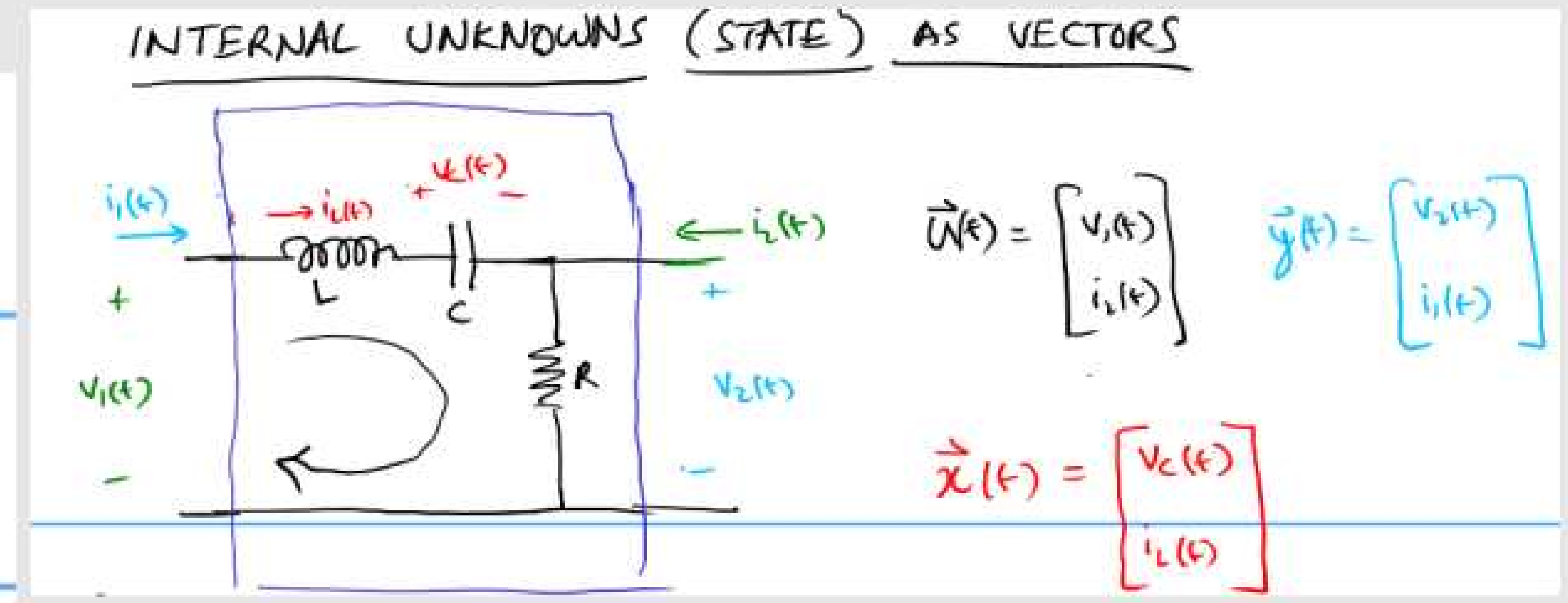
- can be written using the **input** and **state vectors**

The Outputs

- (move to xournal)

$$v_2(t) = R\,(i_L(t) + i_2(t))$$

$$i_1(t) = i_2(t)$$



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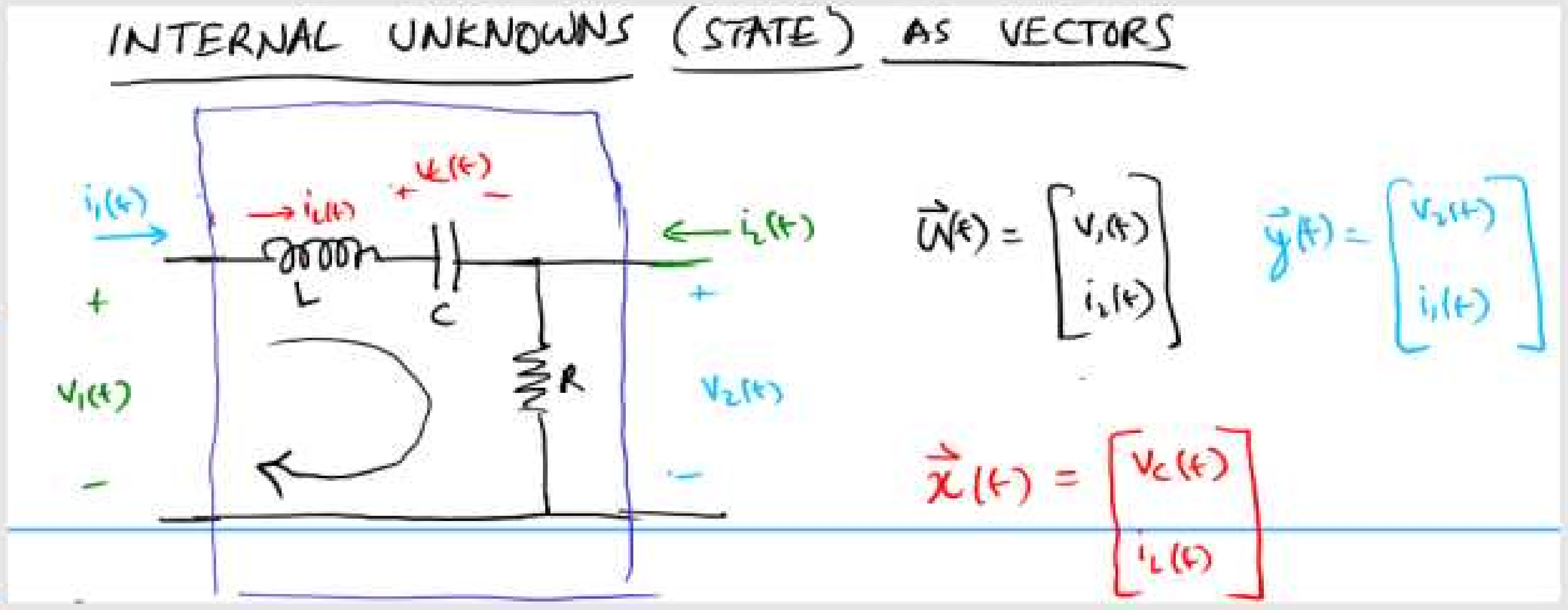

$$\underbrace{\begin{bmatrix} v_2(t) \\ i_1(t) \end{bmatrix}}_{\vec{y}(t)} = \underbrace{\begin{bmatrix} 0 & R \\ 0 & 1 \end{bmatrix}}_{D} \underbrace{\begin{bmatrix} v_c(t) \\ i_L(t) \end{bmatrix}}_{\vec{x}(t)} + \underbrace{\begin{bmatrix} 0 & R \\ 0 & 0 \end{bmatrix}}_{E} \underbrace{\begin{bmatrix} v_1(t) \\ i_2(t) \end{bmatrix}}_{\vec{u}(t)}$$

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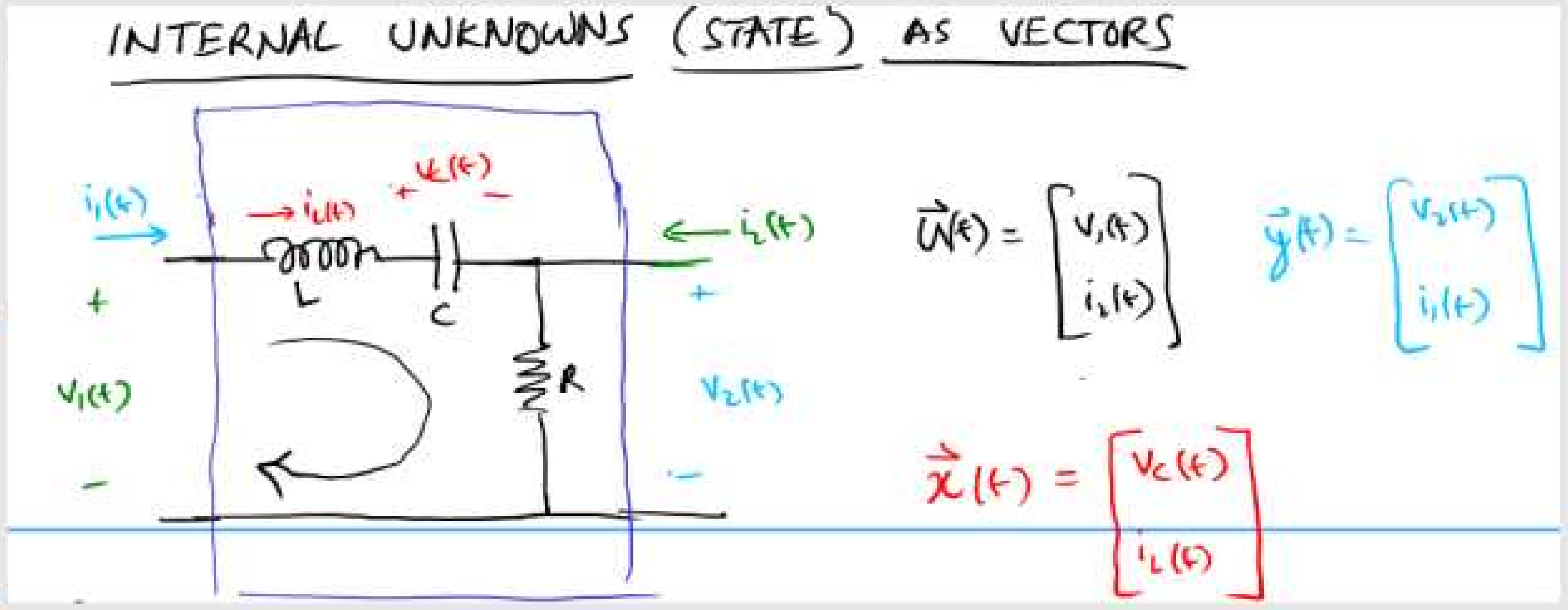


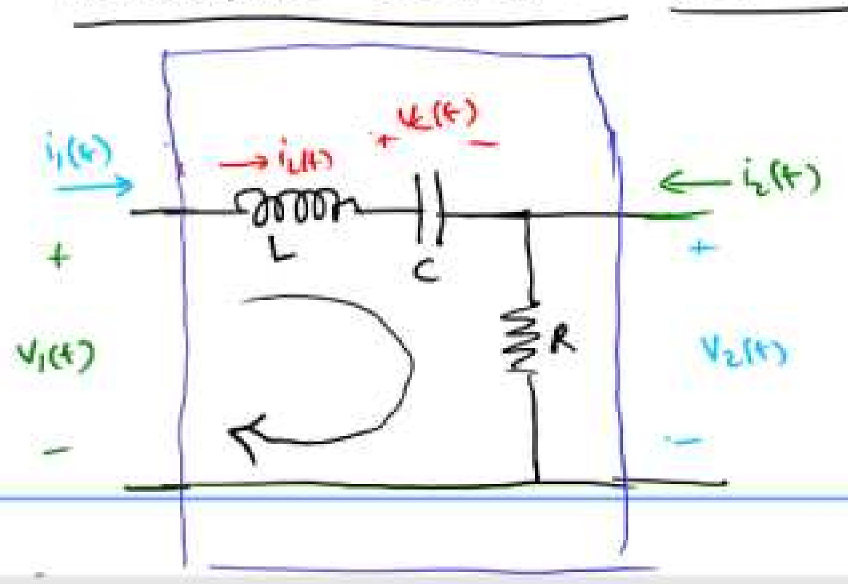
$$\underbrace{\begin{bmatrix} v_2(t) \\ i_1(t) \end{bmatrix}}_{\vec{y}(t)} = \underbrace{\begin{bmatrix} 0 & R \\ 0 & 1 \end{bmatrix}}_{D} \underbrace{\begin{bmatrix} v_C(t) \\ i_L(t) \end{bmatrix}}_{\vec{x}(t)} + \underbrace{\begin{bmatrix} 0 & R \\ 0 & 0 \end{bmatrix}}_{E} \underbrace{\begin{bmatrix} v_1(t) \\ i_2(t) \end{bmatrix}}_{\vec{u}(t)}$$

- also expressed using the **input** and **state vectors**

State+Output Eqns Together

$$\frac{d}{dt}\begin{bmatrix} v_C(t) \\ i_L(t) \end{bmatrix} = \begin{bmatrix} & \frac{1}{C} \\ -\frac{1}{L} & -R/L \end{bmatrix}\begin{bmatrix} v_C(t) \\ i_L(t) \end{bmatrix} + \begin{bmatrix} & \\ \frac{1}{L} & -\frac{R}{L} \end{bmatrix}\begin{bmatrix} v_1(t) \\ i_2(t) \end{bmatrix}$$

INTERNAL UNKNOWNS (STATE) AS VECTORS



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- **general form:** $\frac{d}{dt}\vec{x}(t) = \vec{f}(\vec{x}(t), \vec{u}(t))$, $\vec{y}(t) = \vec{g}(\vec{x}(t), \vec{u}(t))$

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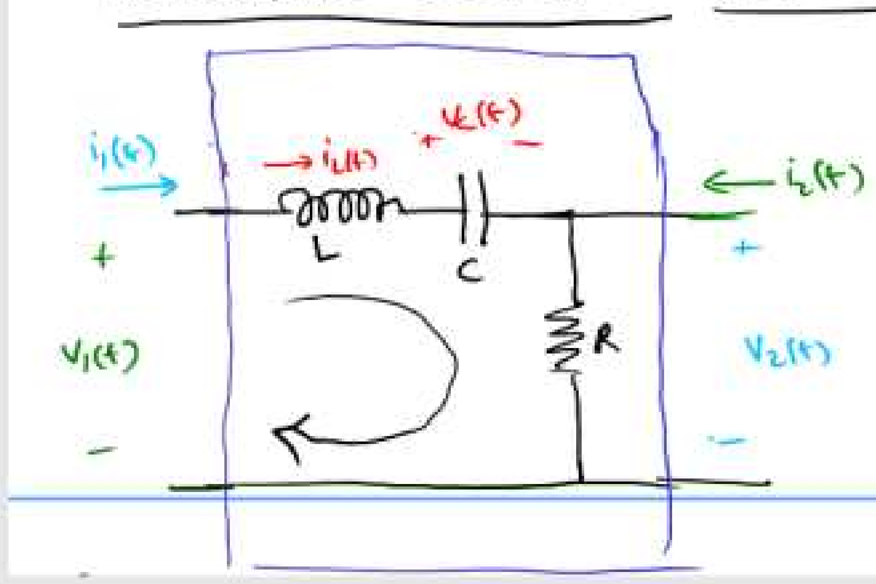

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$\vec{f}(\vec{x}, \vec{u})$ $\vec{g}(\vec{x}, \vec{u})$

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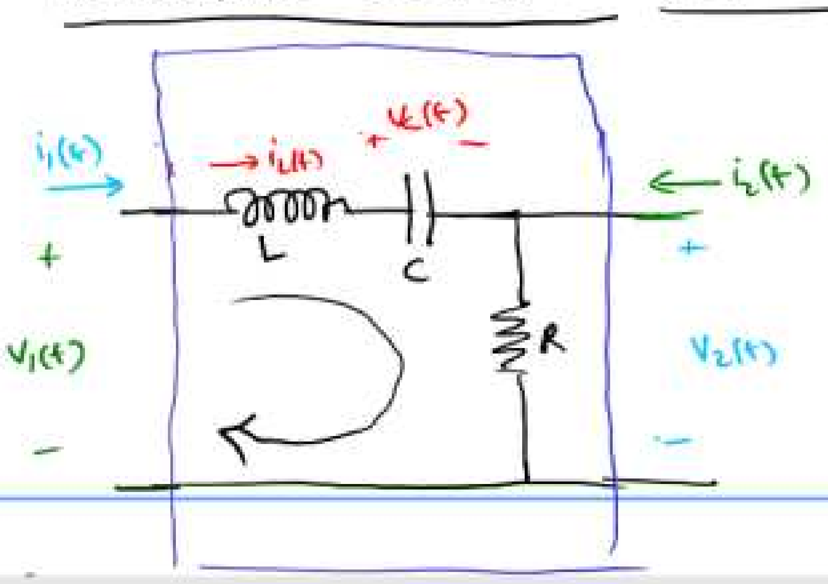

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$$\overset{\vec{y}(t)}{\begin{bmatrix} v_2(t) \\ i_1(t) \end{bmatrix}} = \underbrace{\begin{bmatrix} 0 & R \\ 0 & 1 \end{bmatrix}}_{D}\overset{\vec{x}(t)}{\begin{bmatrix} v_C(t) \\ i_L(t) \end{bmatrix}} + \underbrace{\begin{bmatrix} 0 & R \\ 0 & 0 \end{bmatrix}}_{E}\overset{\vec{u}(t)}{\begin{bmatrix} v_1(t) \\ i_2(t) \end{bmatrix}}$$

- **general form:** $\frac{d}{dt}\vec{x}(t) = \vec{f}(\vec{x}(t), \vec{u}(t))$, $\vec{y}(t) = \vec{g}(\vec{x}(t), \vec{u}(t))$

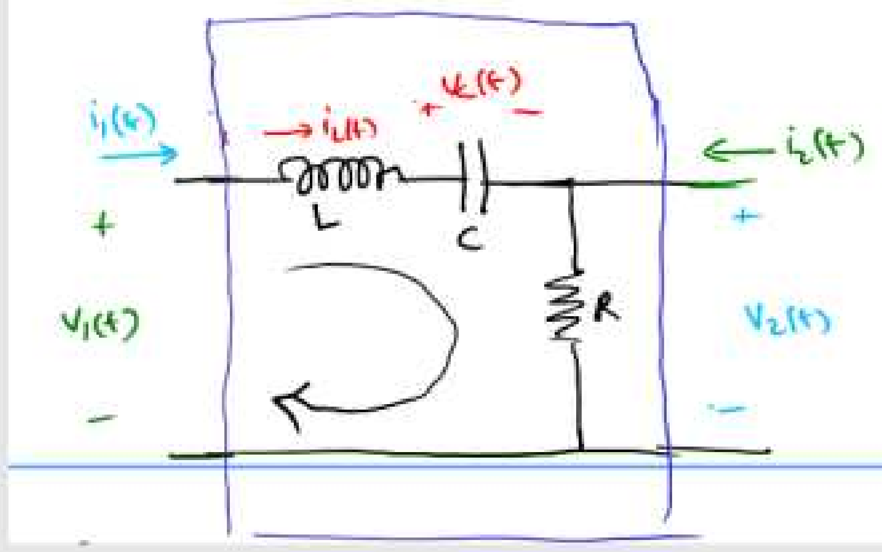
State+Output Eqns Together

$$\frac{d}{dt}\begin{bmatrix} v_C(t) \\ i_L(t) \end{bmatrix} = \begin{bmatrix} & 1/C \\ -1/L & -R/L \end{bmatrix}\begin{bmatrix} v_C(t) \\ i_L(t) \end{bmatrix} + \begin{bmatrix} & \\ 1/L & -R/L \end{bmatrix}\begin{bmatrix} v_1(t) \\ i_2(t) \end{bmatrix}$$

$\vec{f}(\vec{x}, \vec{u})$

$\vec{g}(\vec{x}, \vec{u})$

INTERNAL UNKNOWNS (STATE) AS VECTORS



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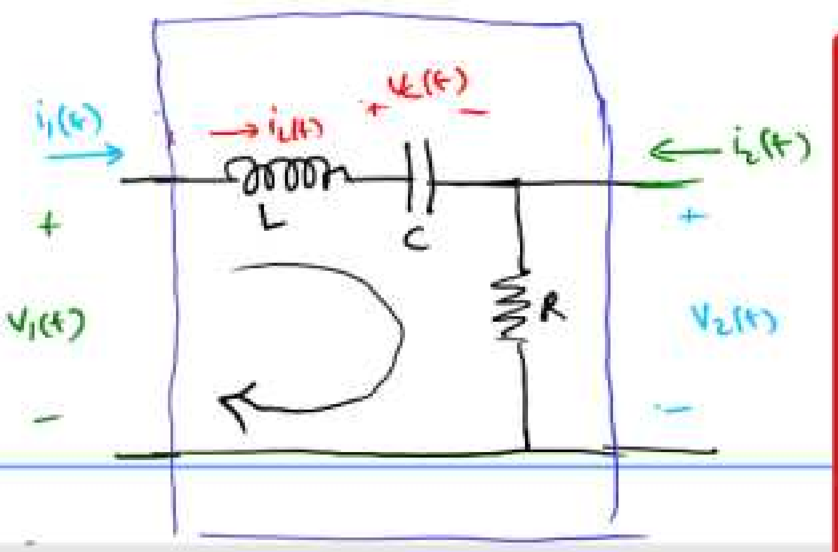
$$\frac{d}{dt}\begin{bmatrix} v_C(t) \\ i_L(t) \end{bmatrix} = \boxed{\begin{bmatrix} & \frac{1}{C} \\ -\frac{1}{L} & -R/L \end{bmatrix}\begin{bmatrix} v_C(t) \\ i_L(t) \end{bmatrix} + \begin{bmatrix} & \\ \frac{1}{L} & -\frac{R}{L} \end{bmatrix}\begin{bmatrix} v_1(t) \\ i_2(t) \end{bmatrix}} \leftarrow \vec{f}(\vec{x}, \vec{u})$$

INTERNAL UNKNOWNS (STATE) AS VECTORS



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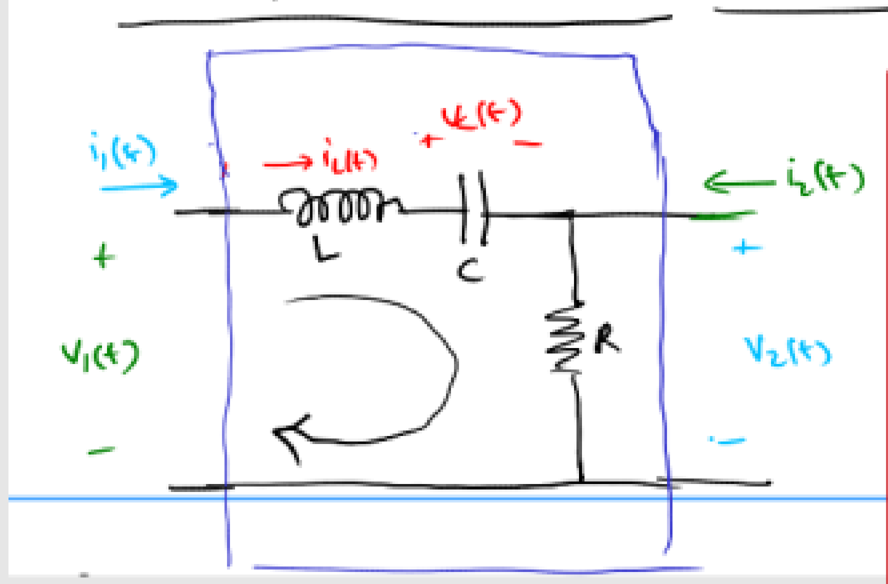

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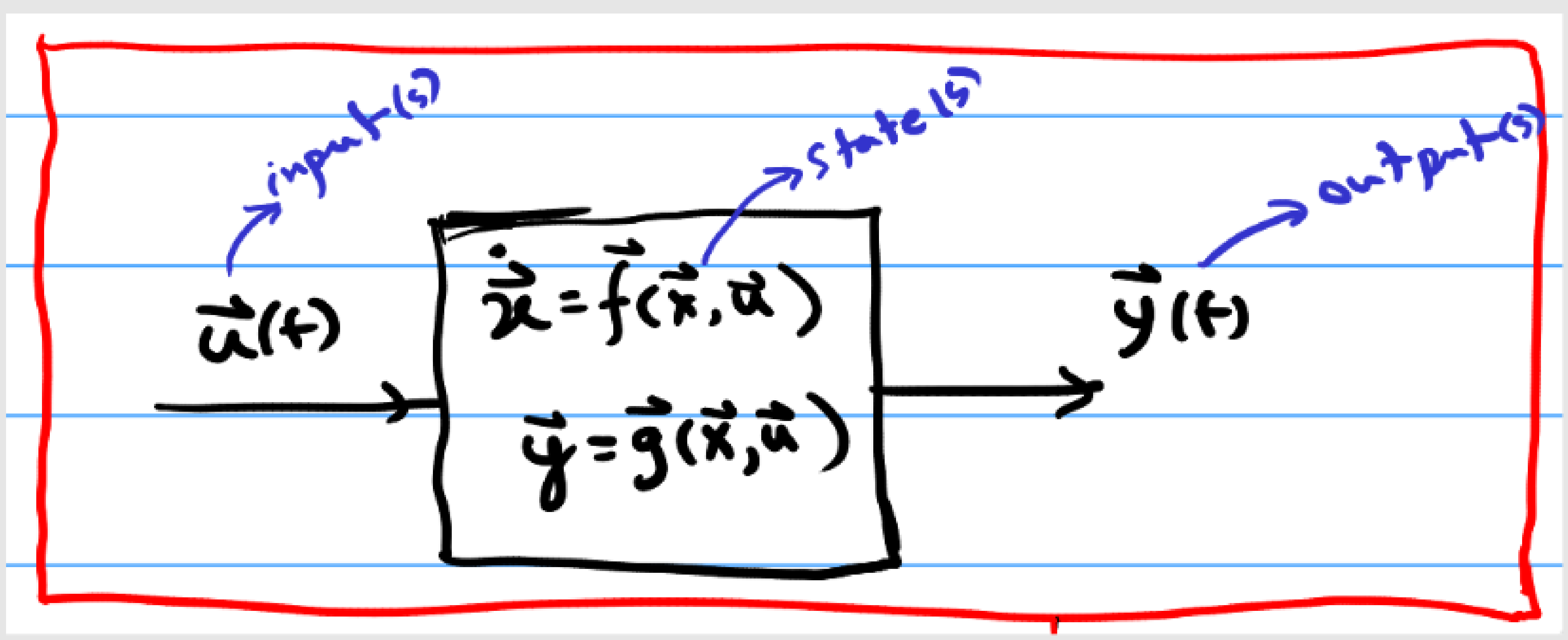

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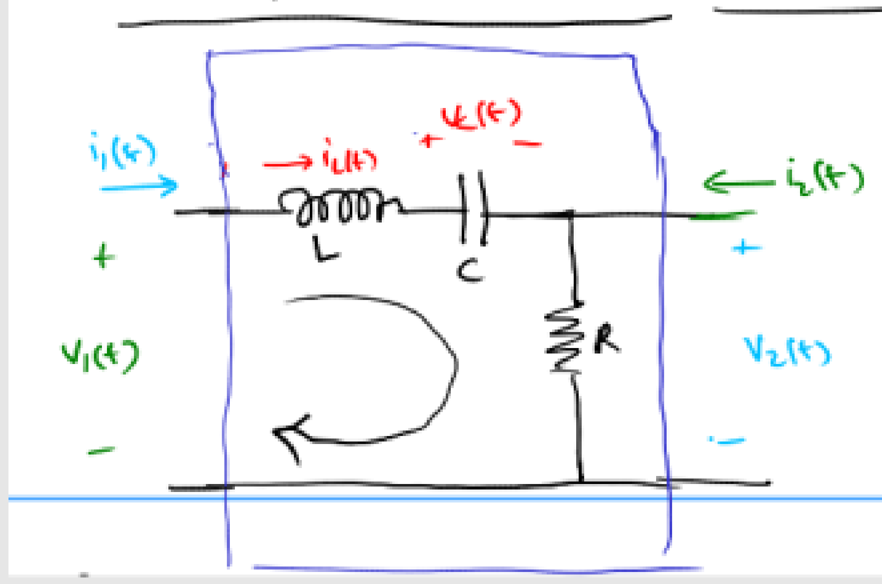

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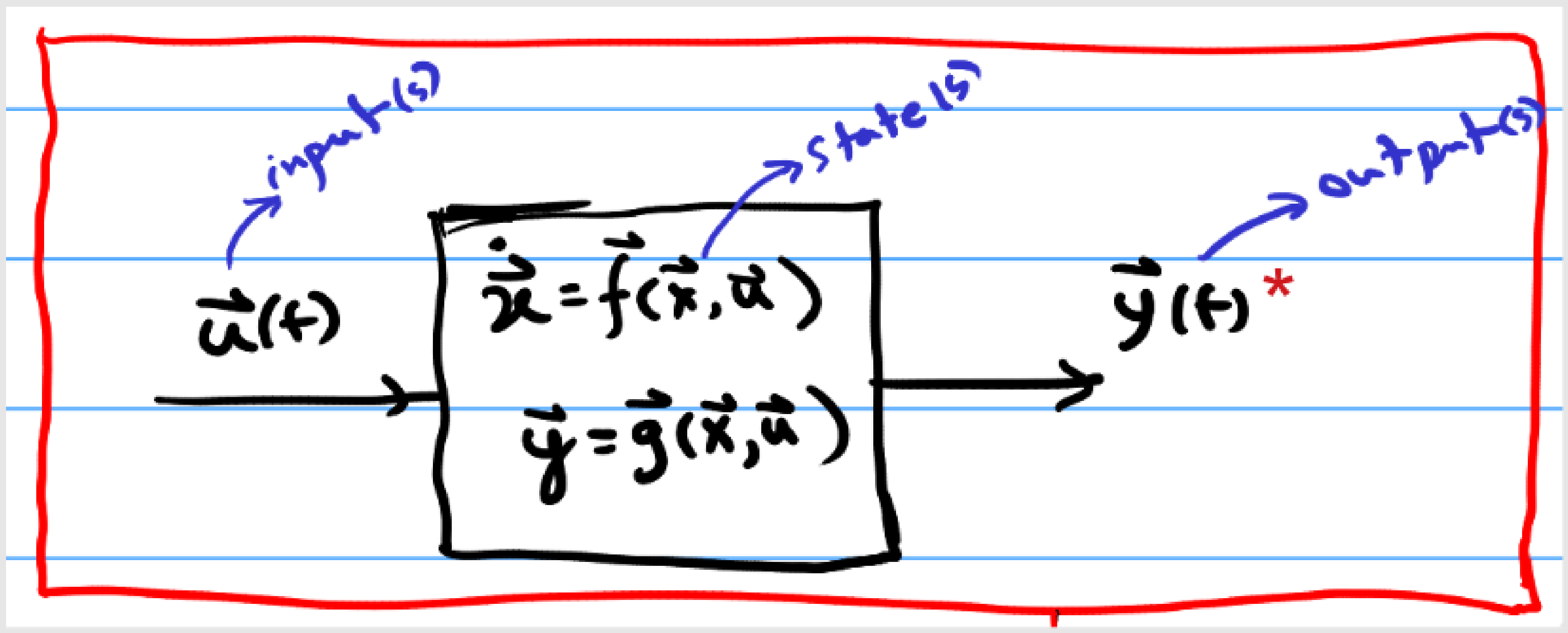

STATE SPACE FORMULATION

* if not explicitly given, the entire state will be the output.

State Space Formulation: Benefits

- why is it useful?

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  - any circuit can be written like this (not just this one)*
    - however big or complicated

* some additional generalization needed – won't cover in this class

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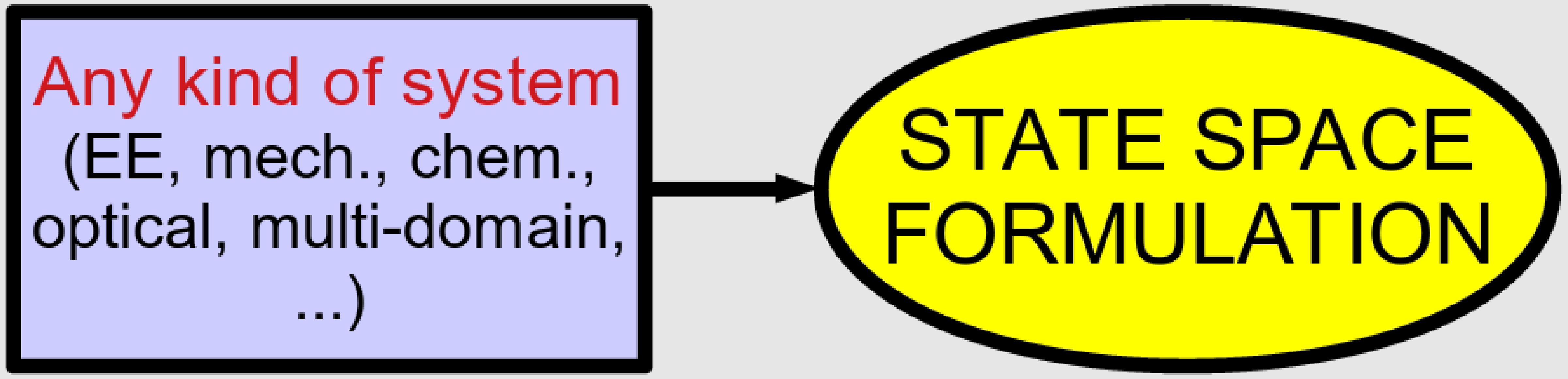

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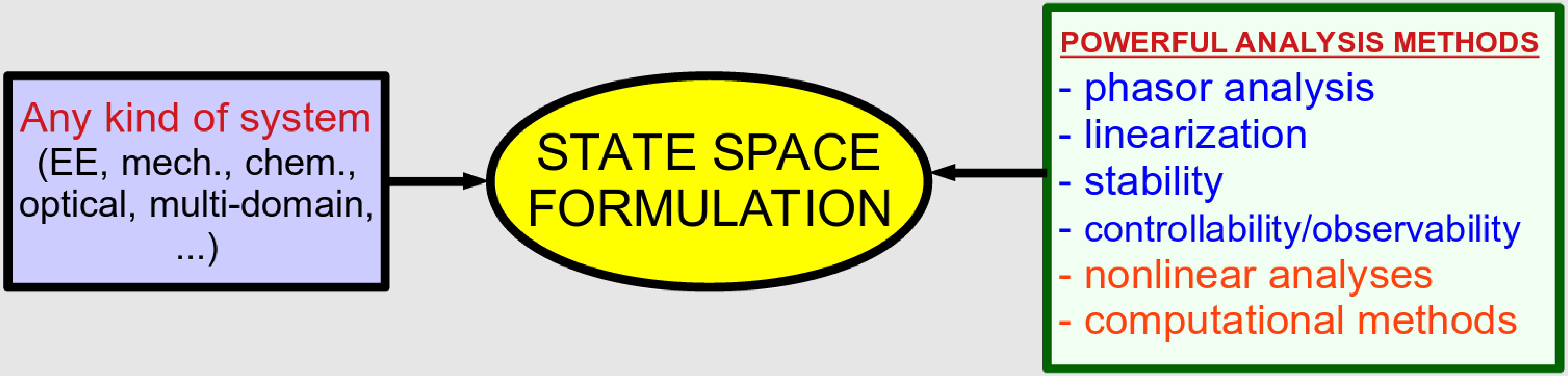

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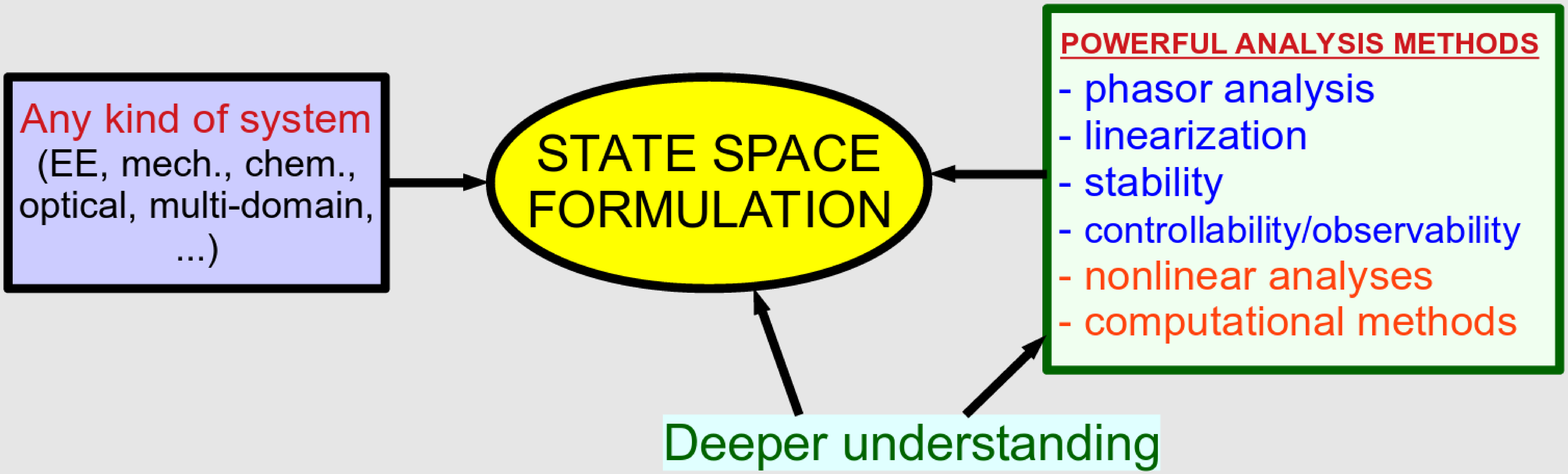

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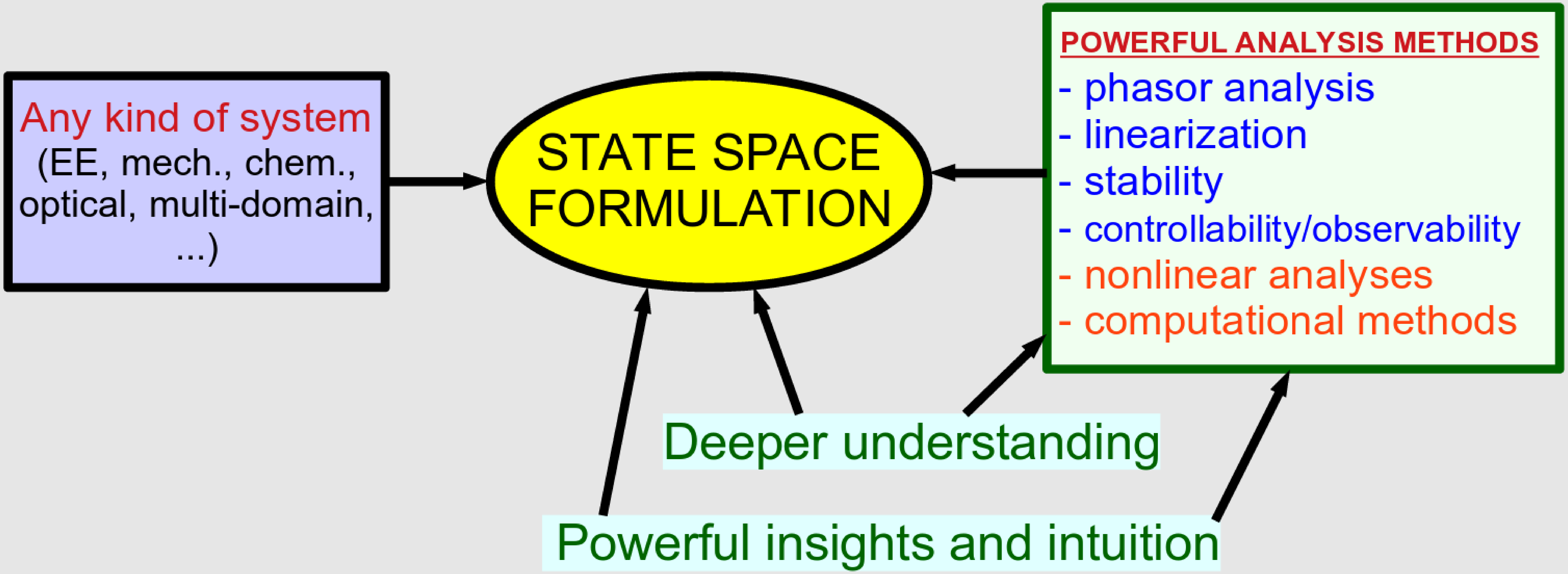

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Previous Lecture

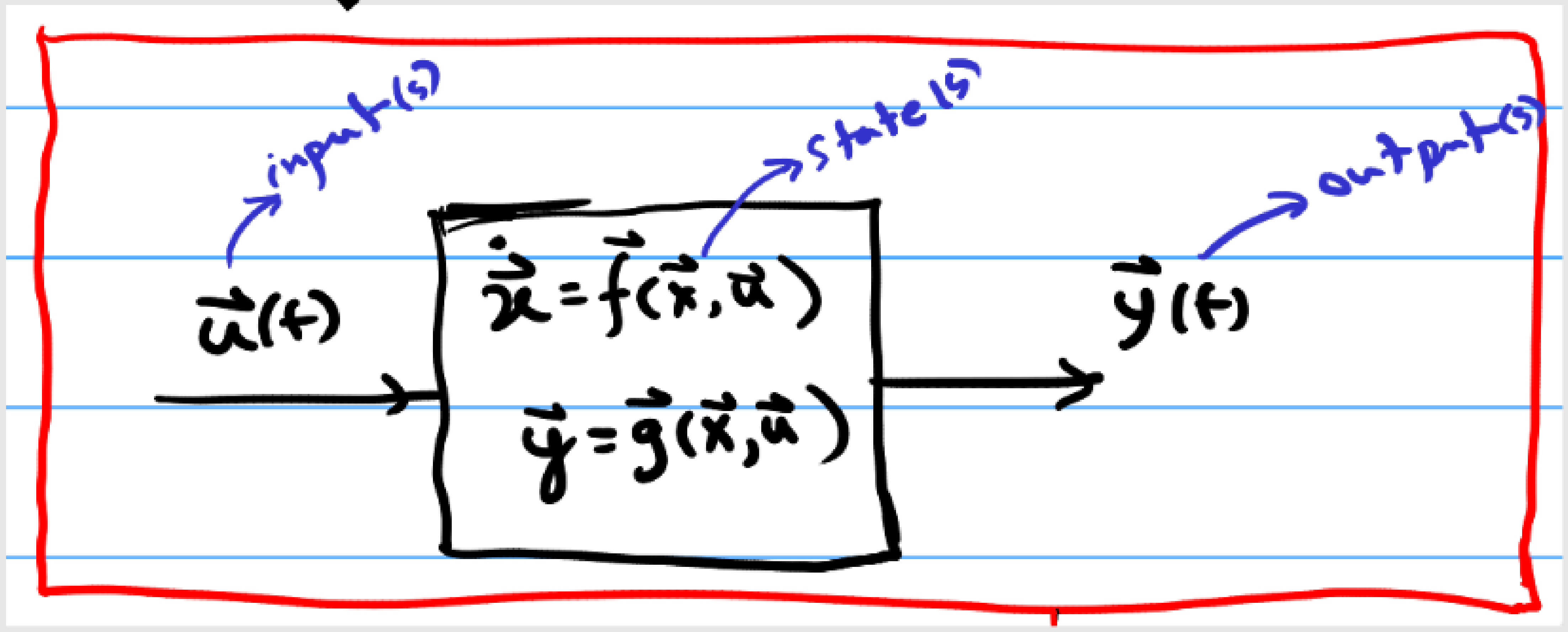
$$\frac{d}{dt}\vec{x}(t) = \vec{f}(\vec{x}(t), \vec{u}(t))$$

$$\vec{y}(t) = \vec{g}(\vec{x}(t), \vec{u}(t))$$

+ initial condition (IC)

STATE SPACE FORMULATION

Any kind of system (EE, mech., chem., optical, multi-domain, ...)



Previous Lecture

- examples
  - RLC circuit
  - pendulum

$$\frac{d}{dt}\vec{x}(t) = \vec{f}(\vec{x}(t), \vec{u}(t)) \qquad \vec{y}(t) = \vec{g}(\vec{x}(t), \vec{u}(t))$$

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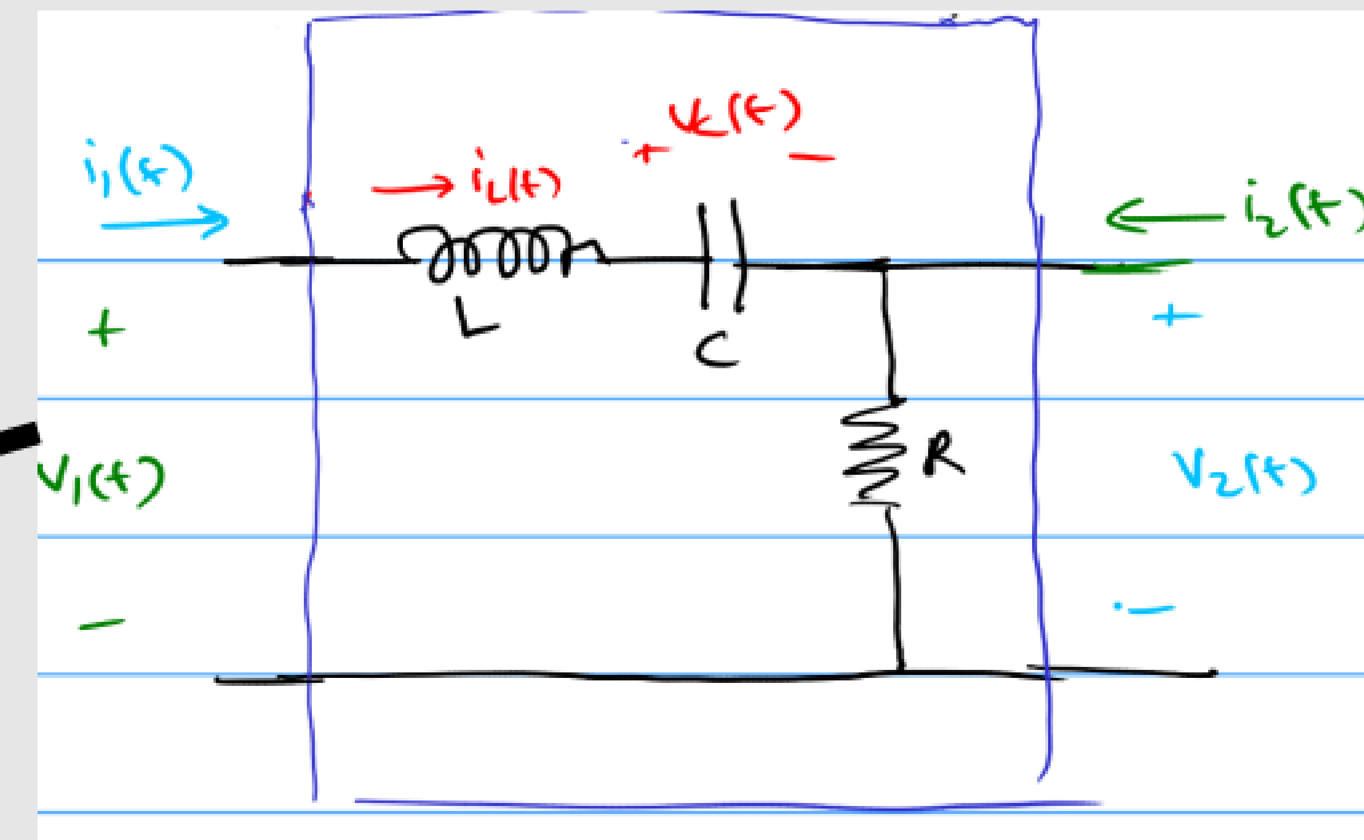
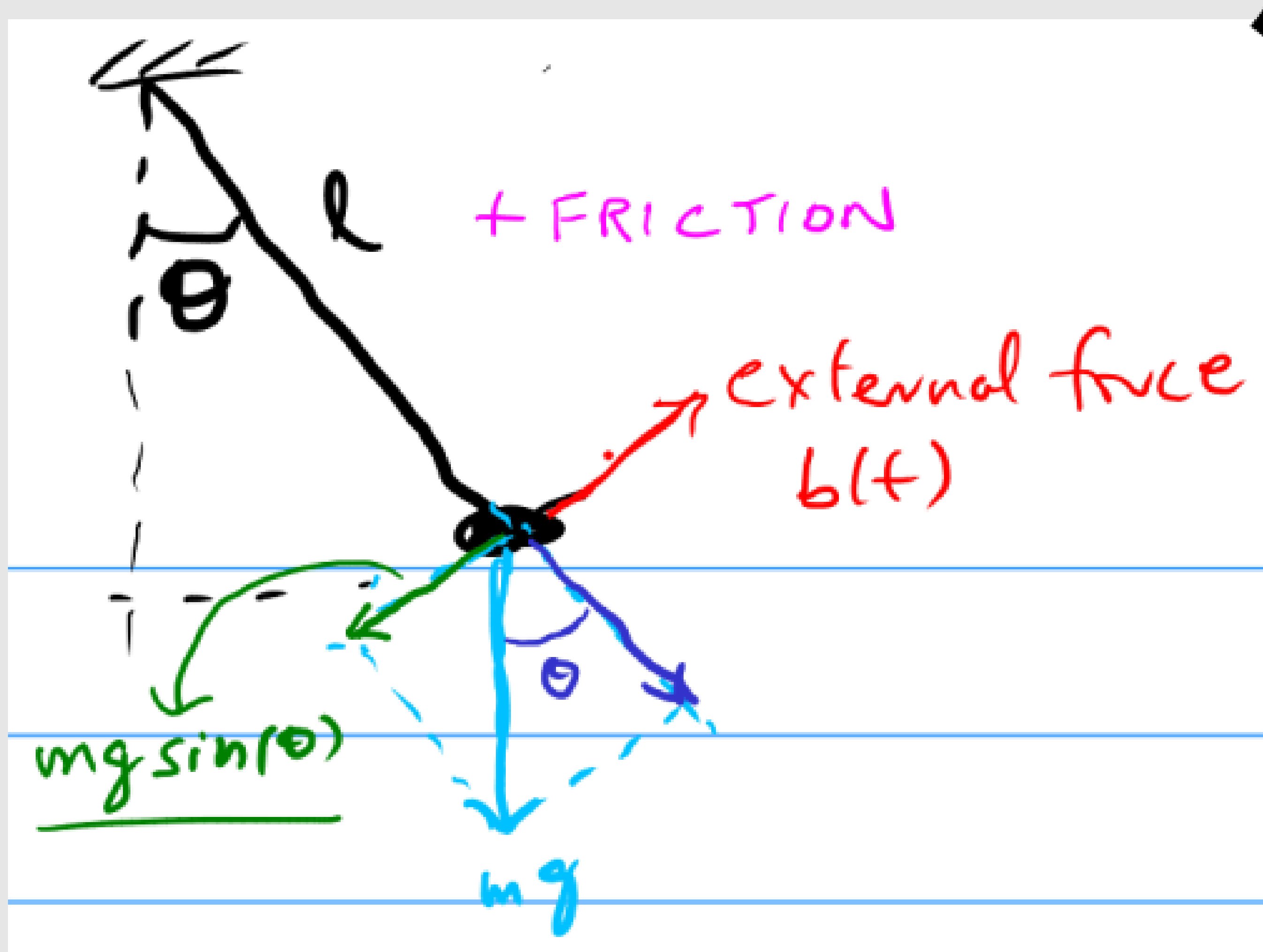
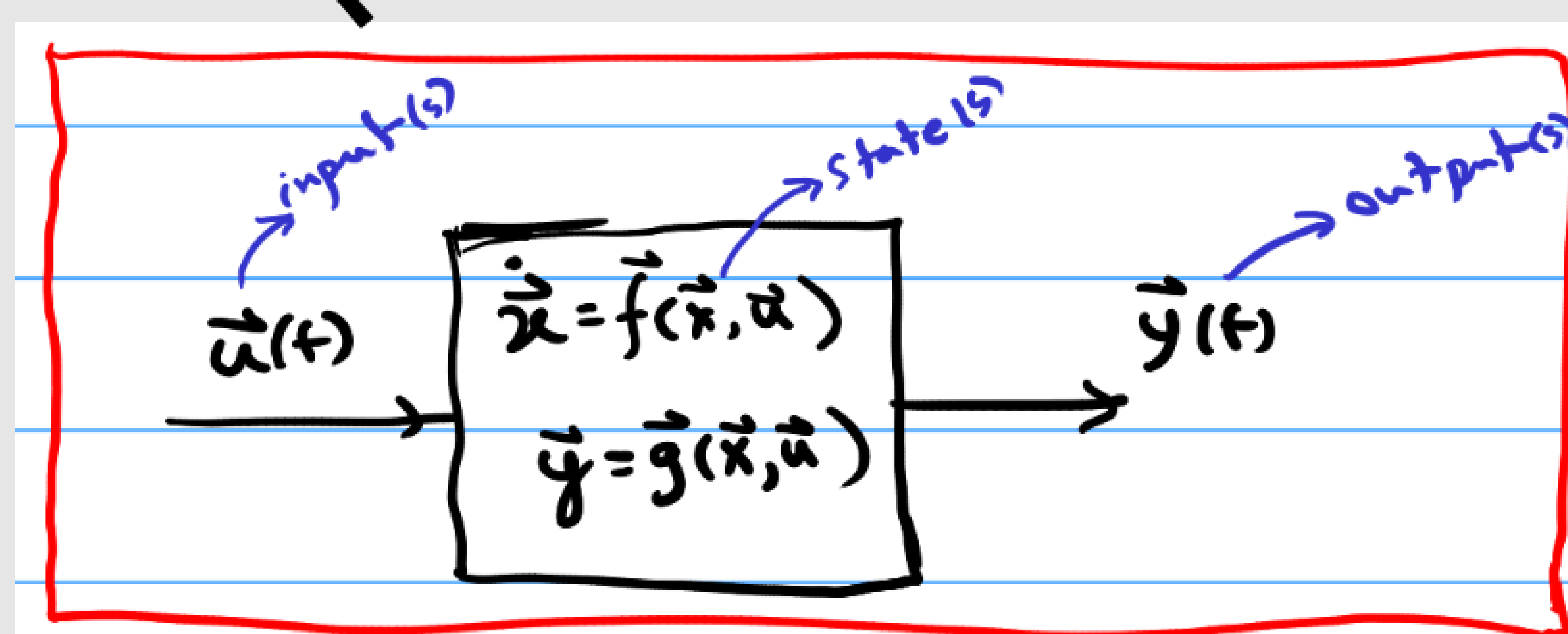






Mechanical Example: Pendulum

- (move to xournal)

→ Newton's eqn. of motion: $F = ma$ or $a = \frac{F}{m}$

→ total tangential force =

force due to gravity : $-mg\sin(\theta)$

\+ force " " friction : $-k \cdot$ velocity

\+ externally applied force : $b(t)$

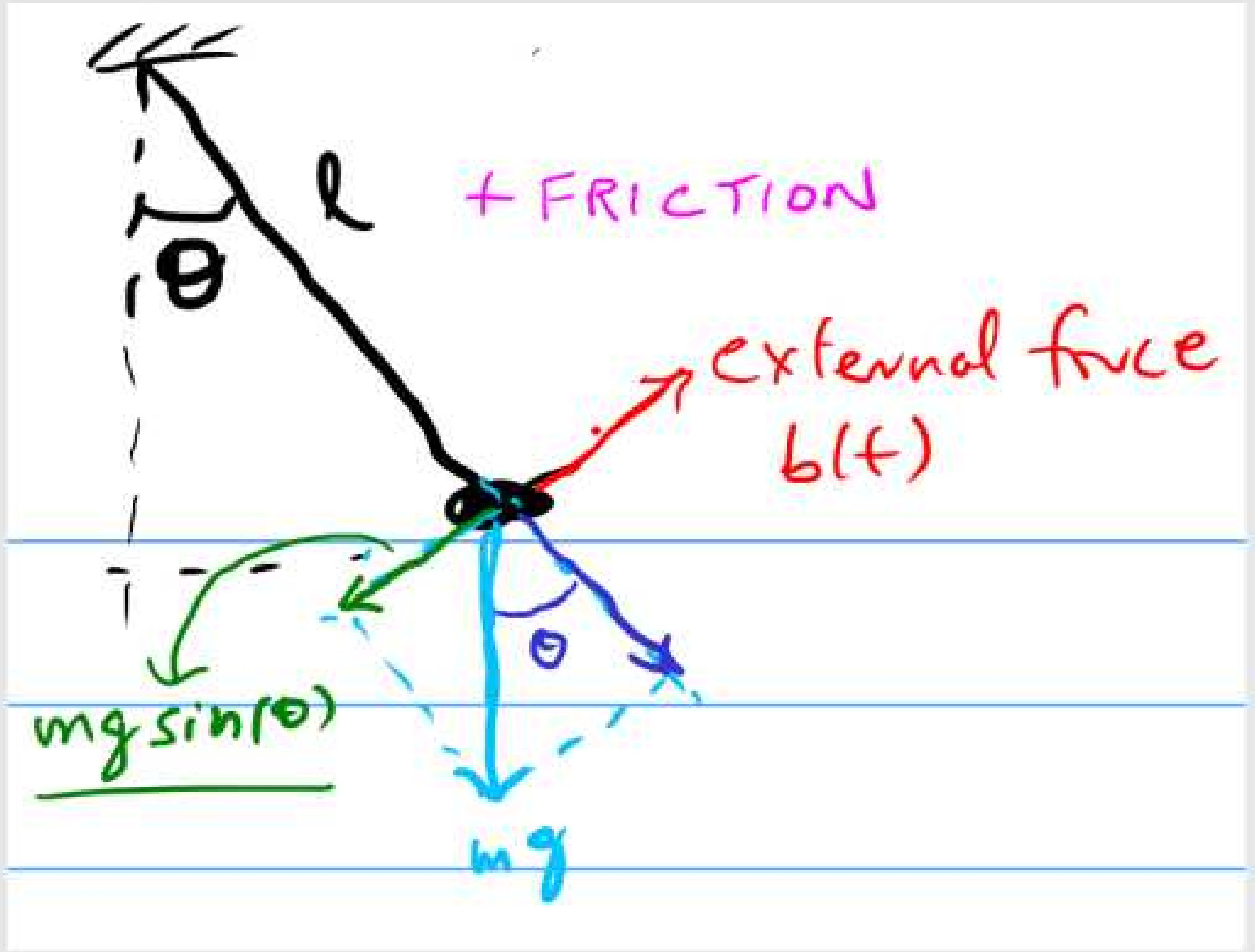
→ arc-length (from bottom) $= y = l\theta$ ← θ in radians

→ velocity $= \frac{dy}{dt} = l\,\underbrace{d\theta/dt}_{v_\theta}$

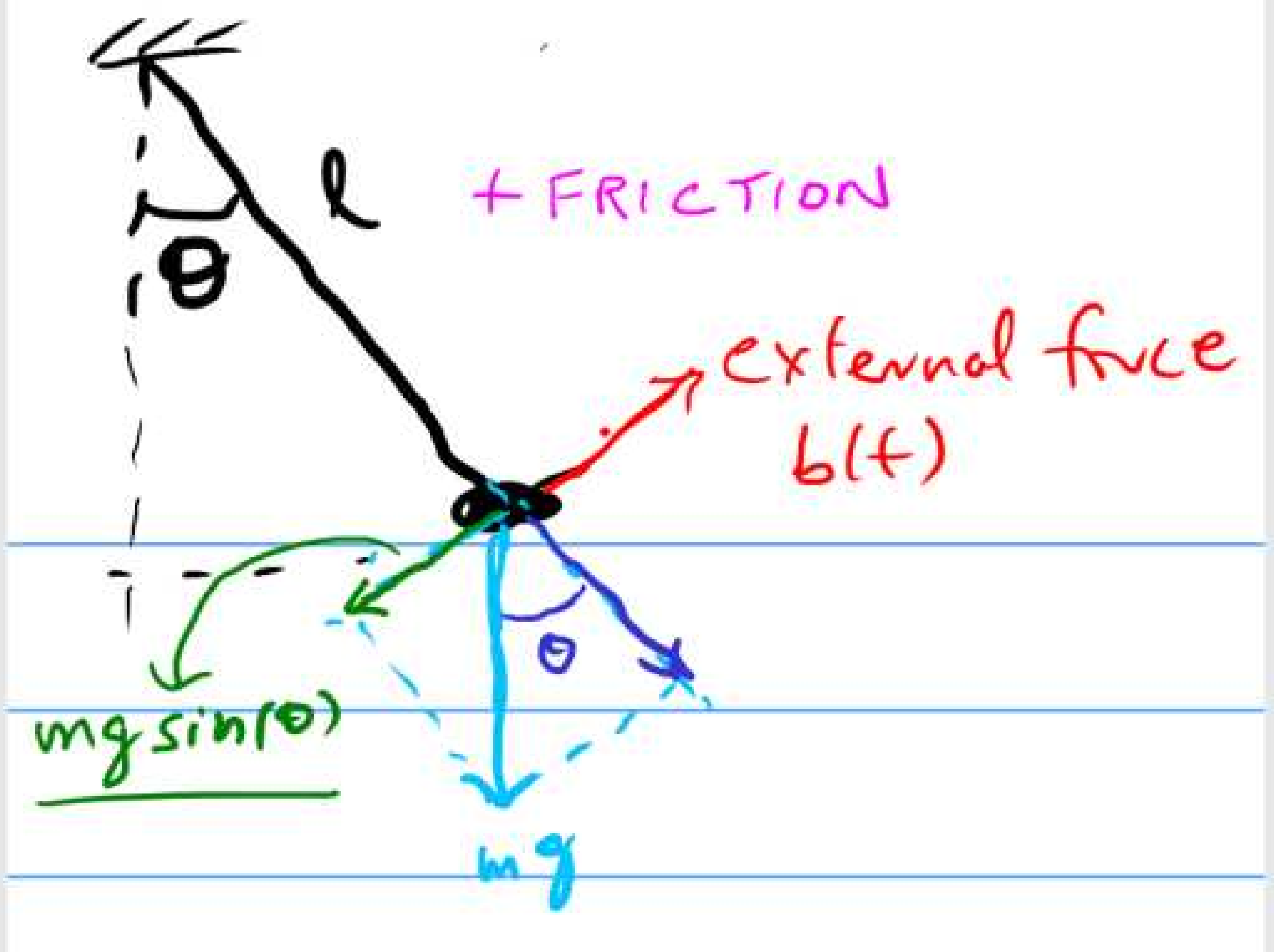
→ acceleration $= \frac{d^2y}{dt^2} = l\frac{d^2\theta}{dt^2} = l\frac{dv_\theta}{dt}$

→ total force : $-mg\sin(\theta) - kl\frac{d\theta}{dt} + b(t)$

→ $a = F/m \implies l\frac{d^2\theta}{dt^2} = -g\sin(\theta) - \frac{kl}{m}\frac{d\theta}{dt} + \frac{b(t)}{m}$



Mechanical Example: Pendulum



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$$\boxed{\frac{d^2\theta}{dt^2} = -\frac{g}{l}\sin(\theta) - \frac{k}{m}\frac{d\theta}{dt} + \frac{b(t)}{ml}}$$

$$\vec{x} = \begin{bmatrix}\theta \\ v_\theta\end{bmatrix}, \quad \vec{u} = [b(t)]$$

$$\frac{d\vec{x}}{dt} = \begin{bmatrix} v_\theta \\ -\frac{g}{l}\sin(\theta) - \frac{k}{m}v_\theta + \frac{b(t)}{ml}\end{bmatrix}$$

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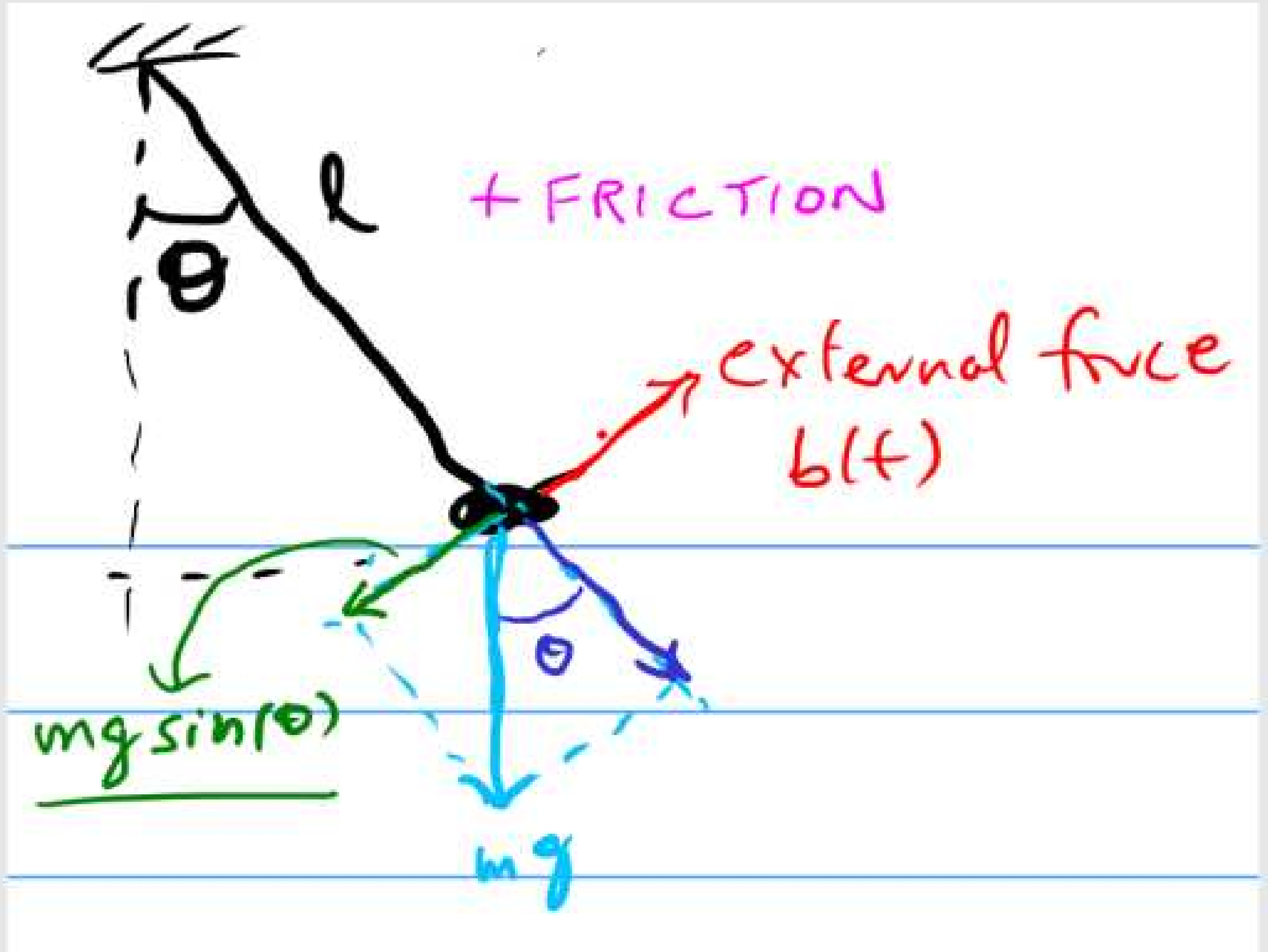

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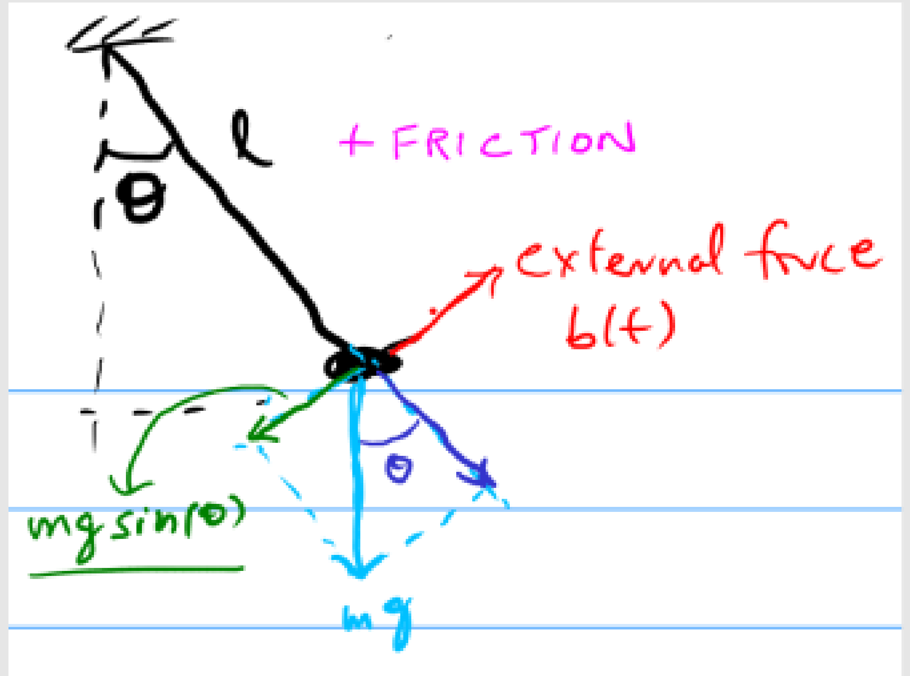
- The system is nonlinear (because of sin(...))

Pendulum: simplification for small θ

- $$\frac{d\vec{x}}{dt} = \begin{bmatrix} v_\theta \\ -g/l \sin(\theta) - k/m \, v_\theta + \frac{b(t)}{ml} \end{bmatrix}$$



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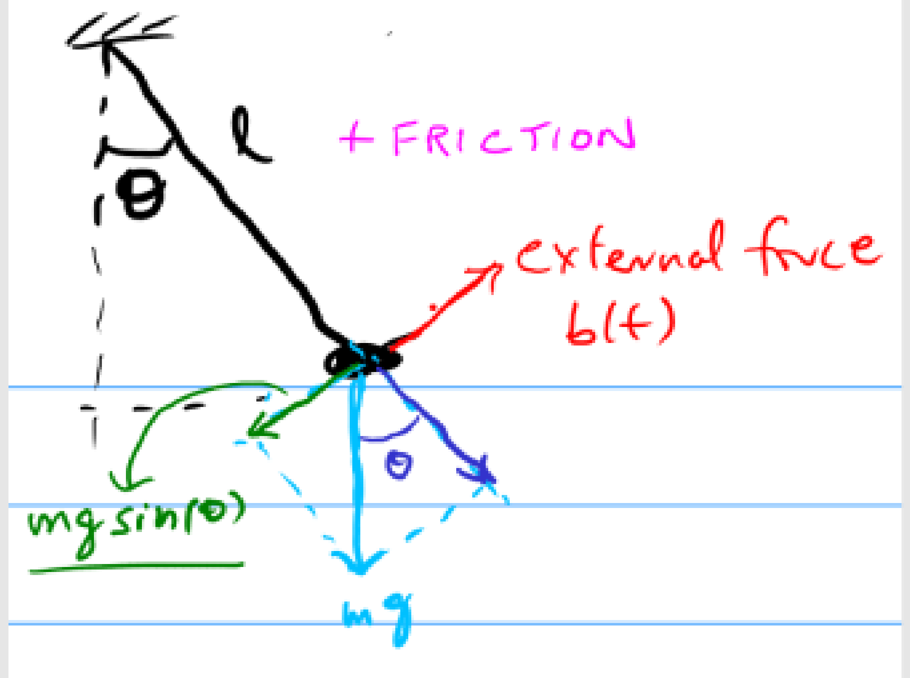

– if θ is small (check in MATLAB)

→ $\sin(\theta) \simeq \theta$ (in radians)

→ example of linearization

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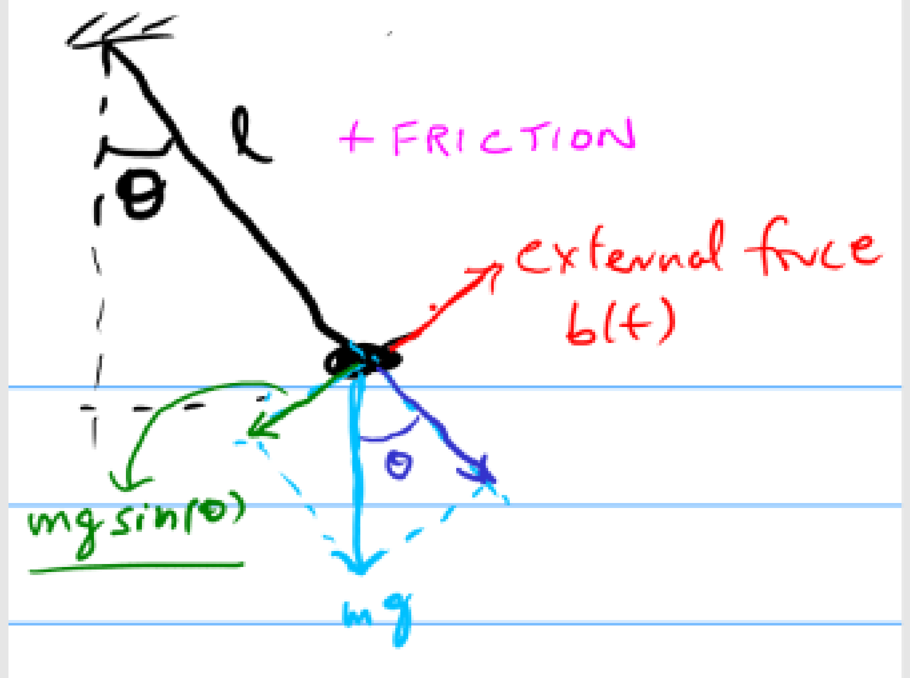

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$$\frac{d\vec{x}}{dt} = \begin{bmatrix} 0 & 1 \\ -g/l & -\frac{k}{m} \end{bmatrix} \vec{x}(t) + \begin{bmatrix} 0 \\ \frac{1}{ml} \end{bmatrix} \vec{u}(t)$$

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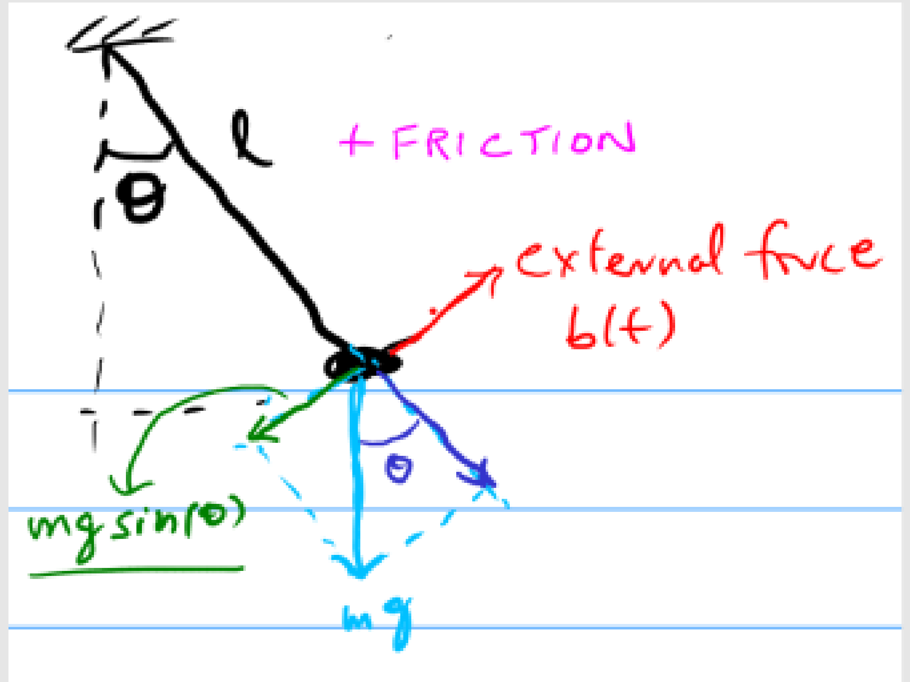

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- Does this look familiar?

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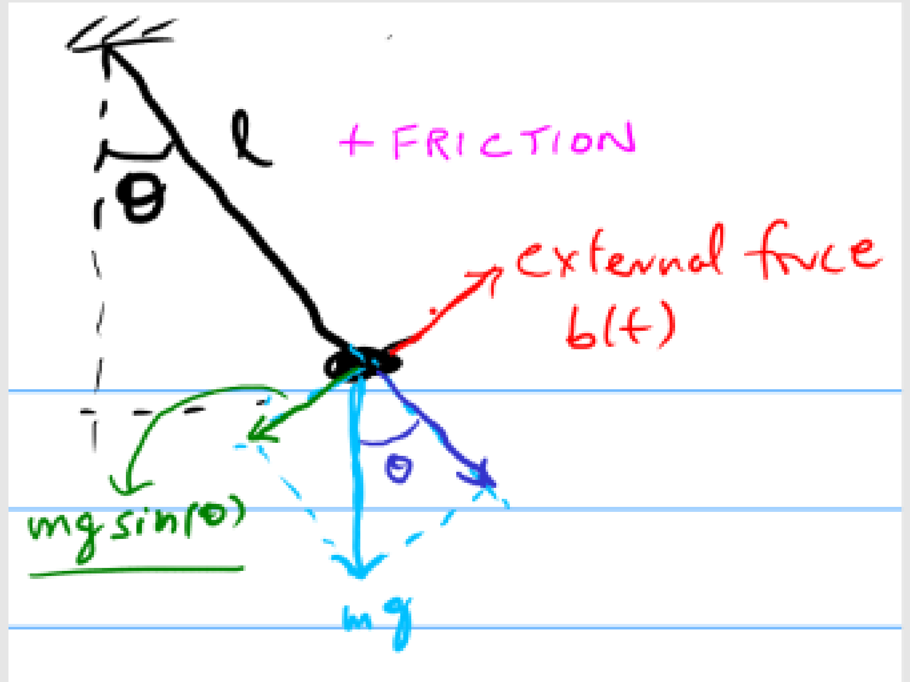

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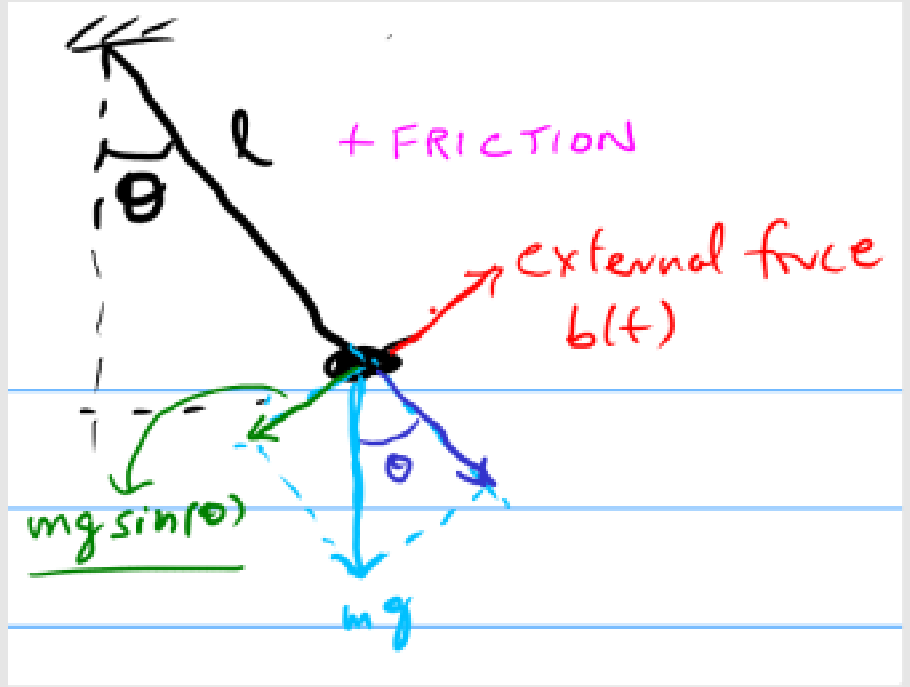

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  - **INSIGHT**: RLC ckt and damped pendulum are “the same”!

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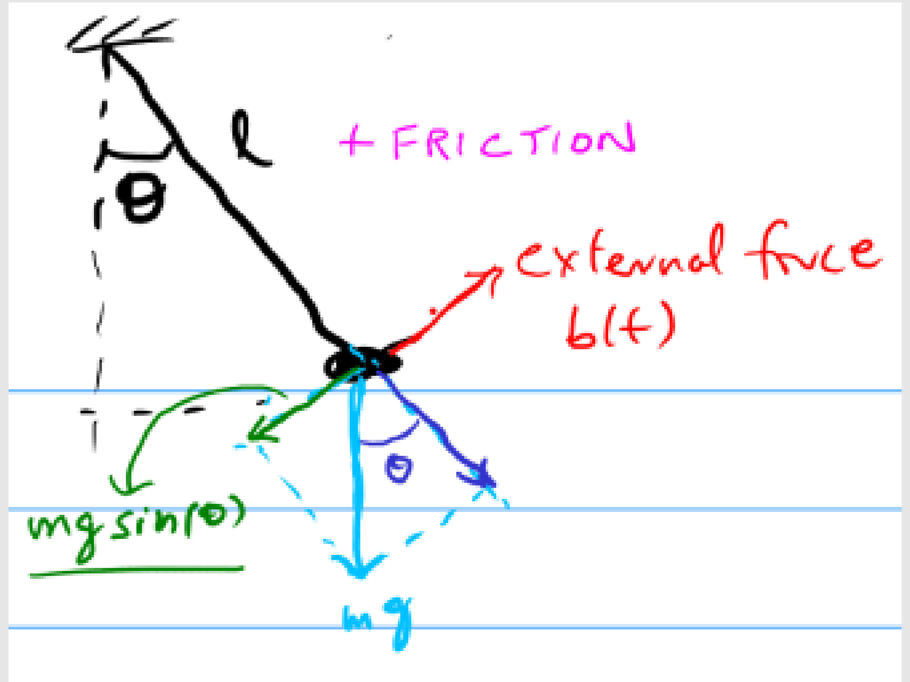

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Filters

Pendulum: simplification for small θ

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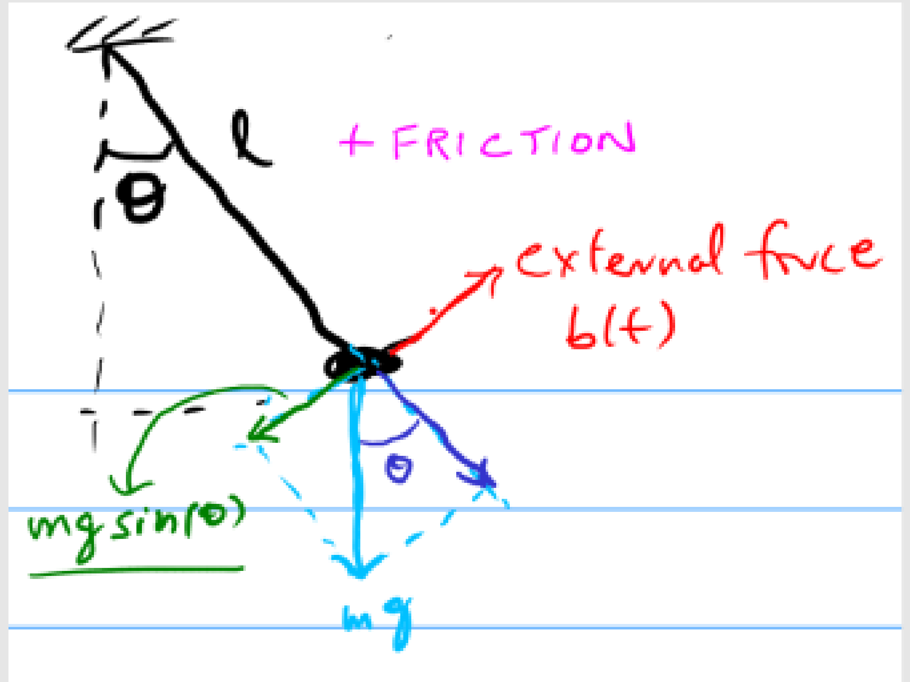

– if θ is small (check in MATLAB)

$\rightarrow$ $\sin(\theta) \simeq \theta$ (in radians)

$\rightarrow$ example of linearization

$$\frac{d\vec{x}}{dt} = \begin{bmatrix} 0 & 1 \\ -g/l & -\frac{k}{m} \end{bmatrix} \vec{x}(t) + \begin{bmatrix} 0 \\ \frac{1}{ml} \end{bmatrix} \vec{u}(t)$$

- Does this look familiar?

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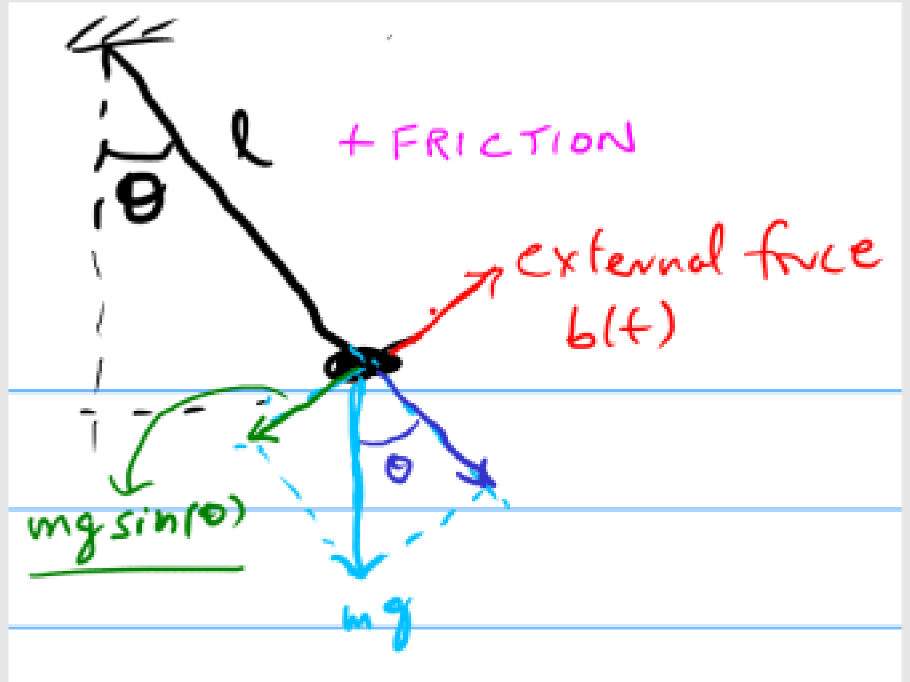

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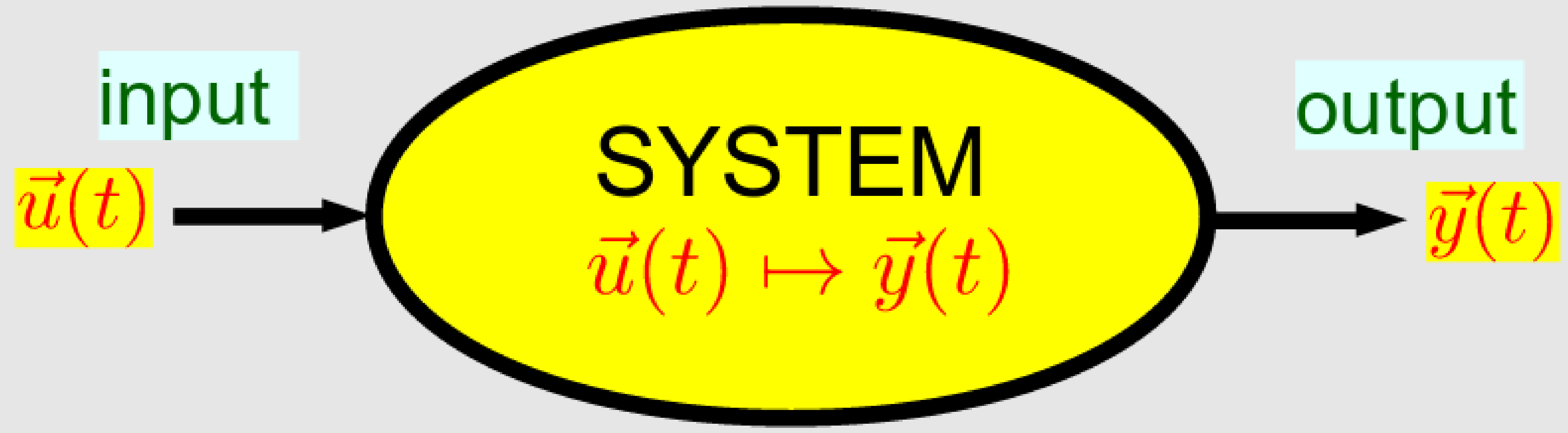
Filters

Filters?? → MEMS filters! (smaller, better, cheaper)

Recap of Linearity

- The concept of LINEARITY is extremely important
  - **it is fundamentally a systems concept**



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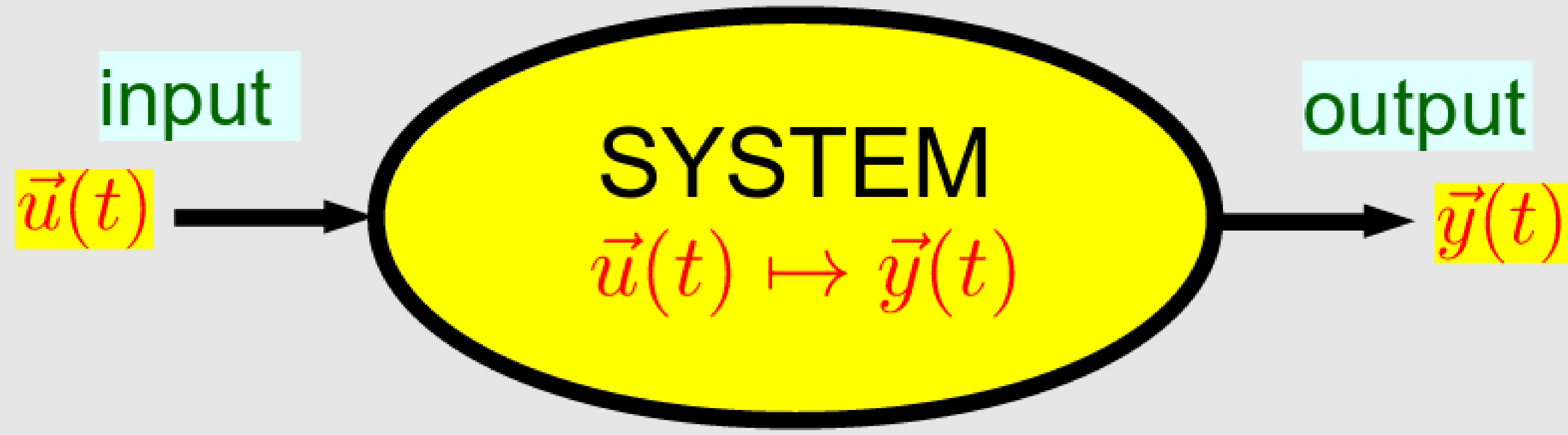

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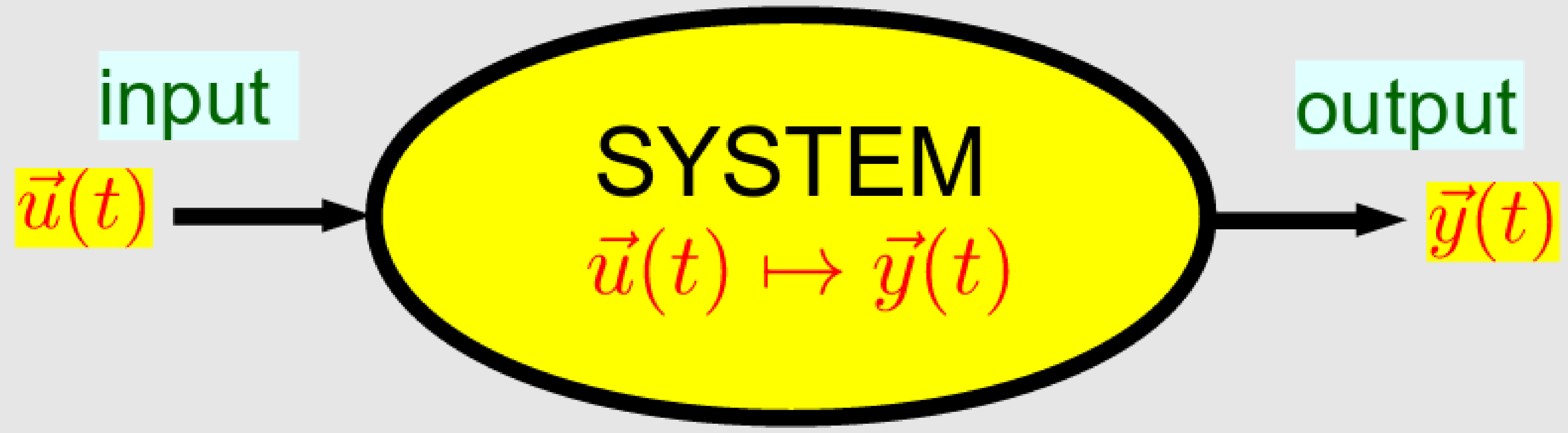

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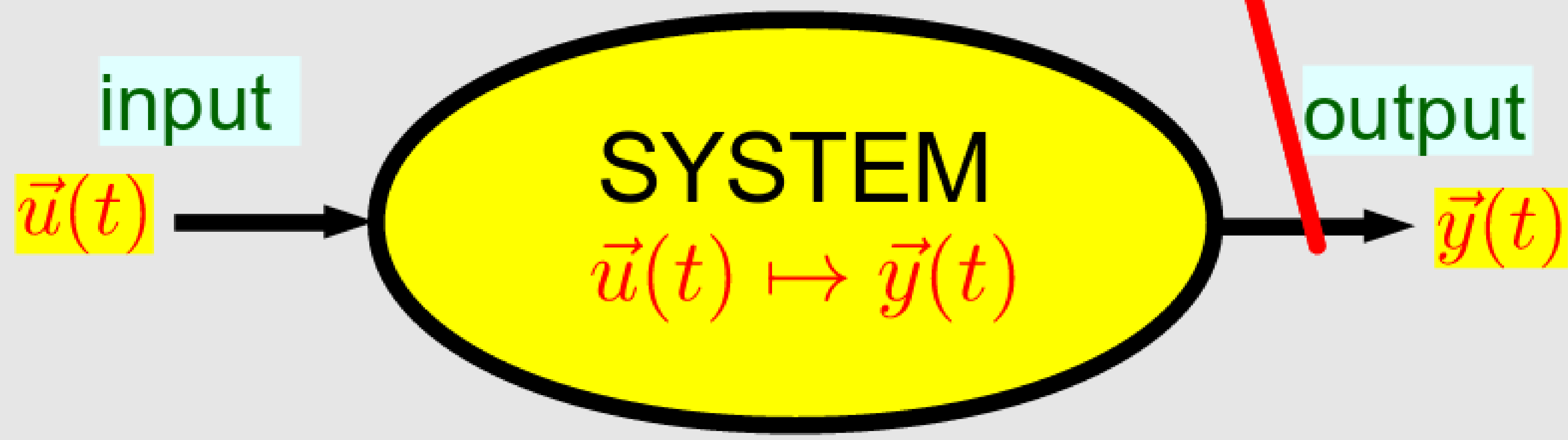

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  - **check scaling**
    - if $\vec{u}(t) \mapsto \vec{y}(t)$, then $(\alpha\vec{u}(t)) \mapsto (\alpha\vec{y}(t))$, $\forall \vec{u}(t)$ and $\forall \alpha \in \Re$

Linearity of State Space Formulations

- general S.S.F: $\frac{d}{dt}\vec{x}(t) = \vec{f}(\vec{x}(t), \vec{u}(t))$, $\vec{y}(t) = \vec{g}(\vec{x}(t), \vec{u}(t))$

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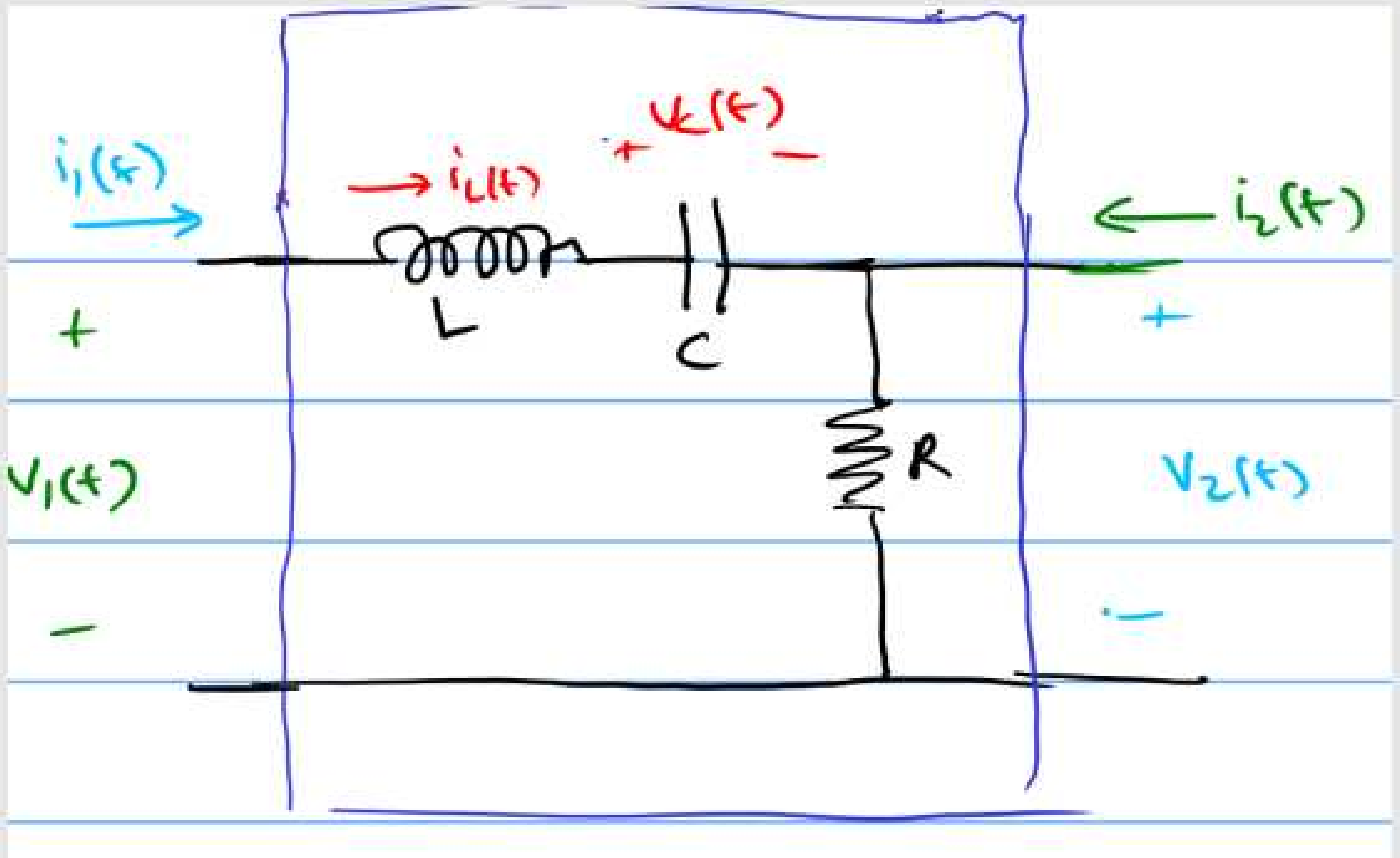
- Proof?

Are these Systems Linear?



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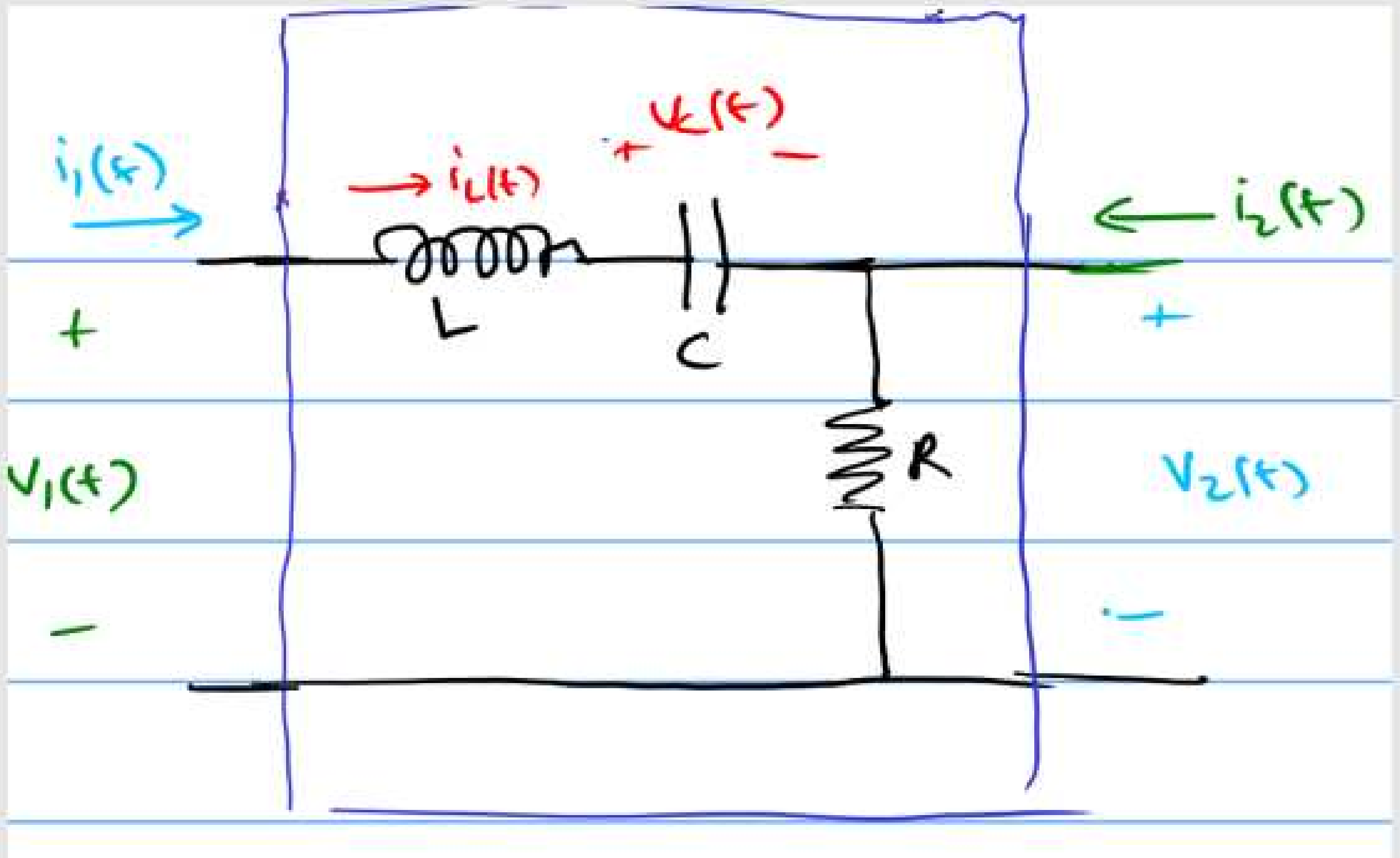

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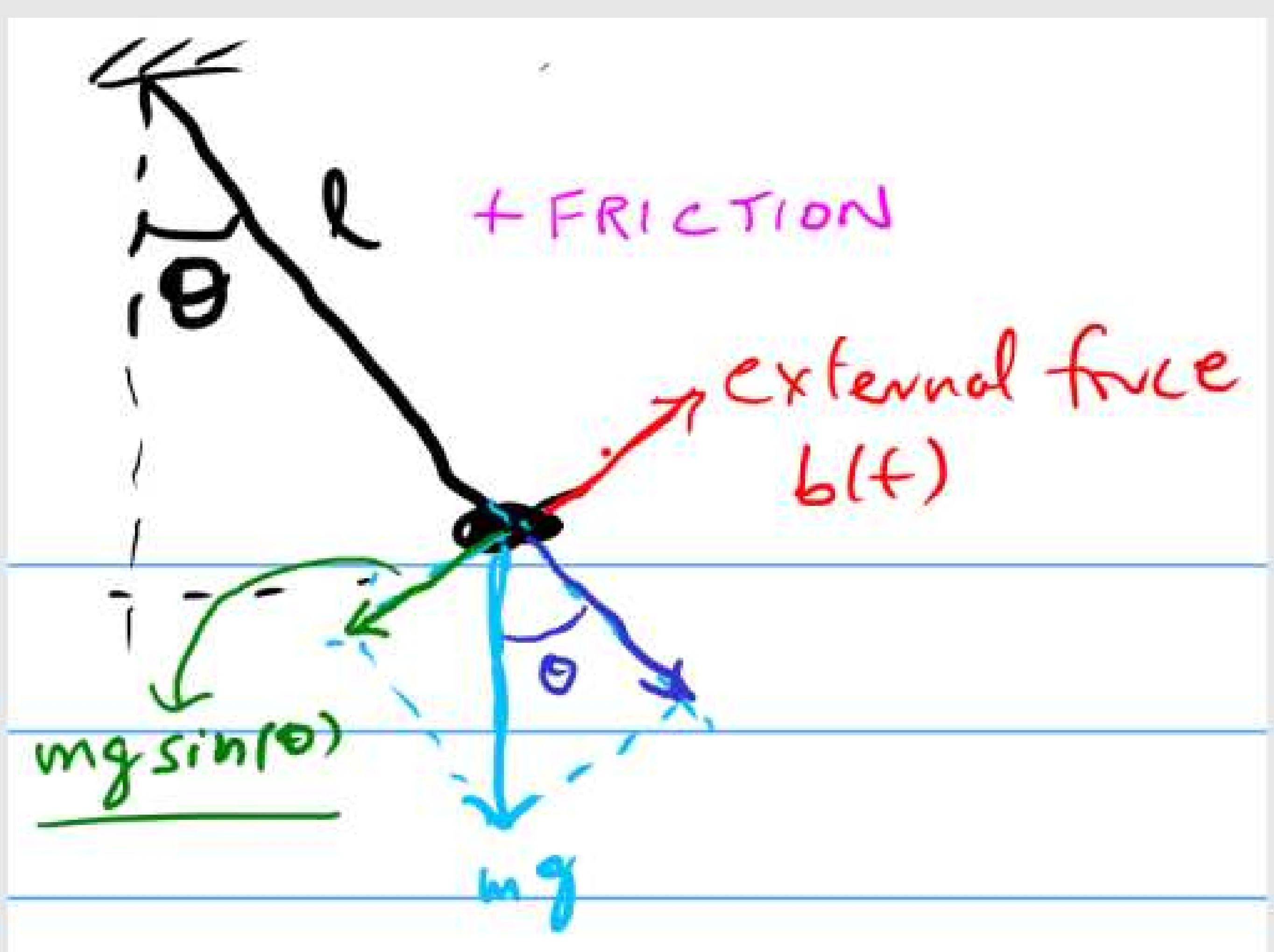

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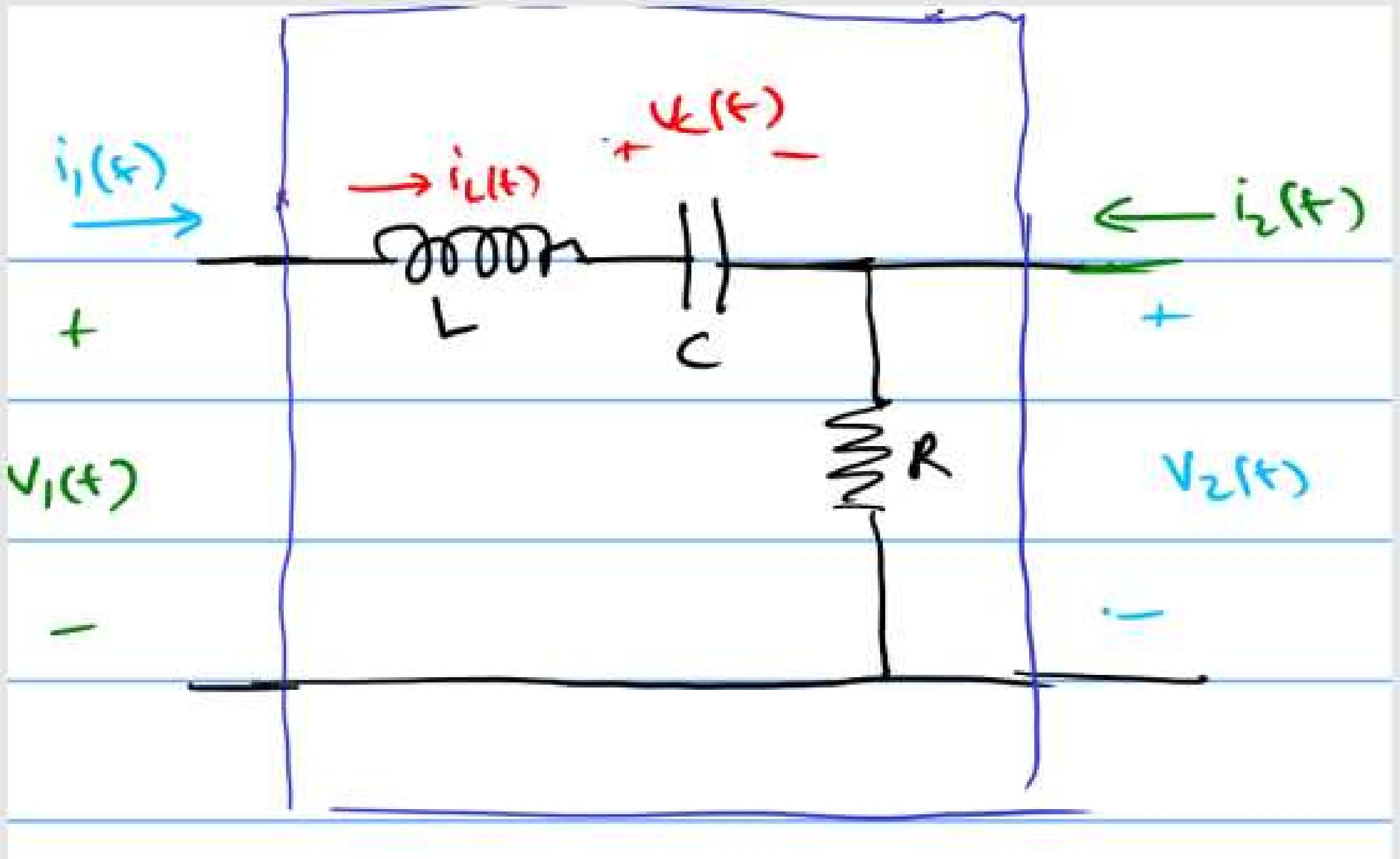

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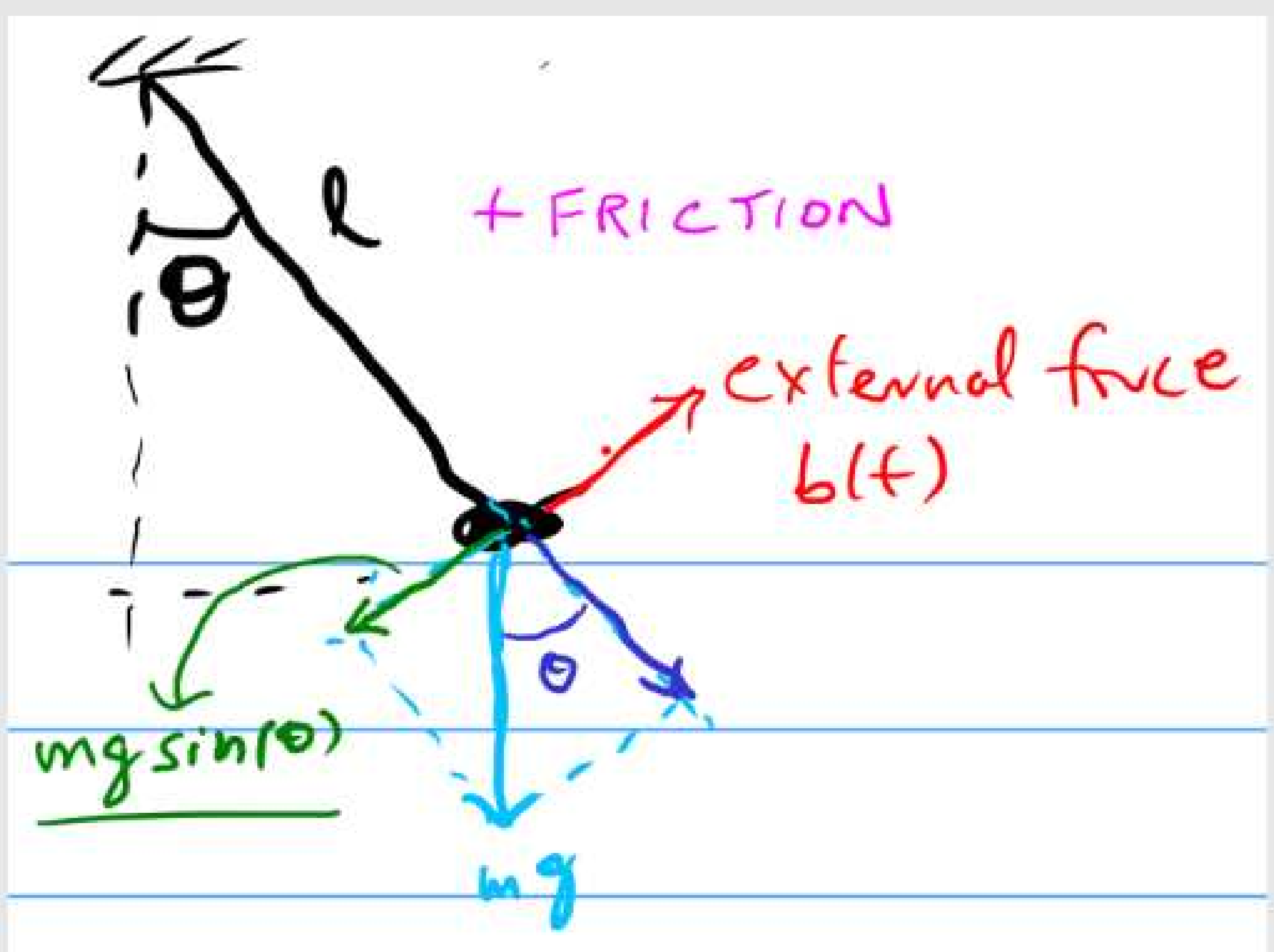

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State Space F. for Discrete Time Systems

- What is a discrete-time system?
  - example: compound interest
    - ➔ (move to xournal)

— Example: compound interest

— Principal P

— Annual rate of interest r, compounded monthly. ← DISCRETE (integer, not real number)

— Savings $S[t] = S[t-1] + \frac{r}{12} S[t-1]$, $t \to 1, 2, 3, \ldots$

with $S[0] = P$ ← INITIAL CONDITION

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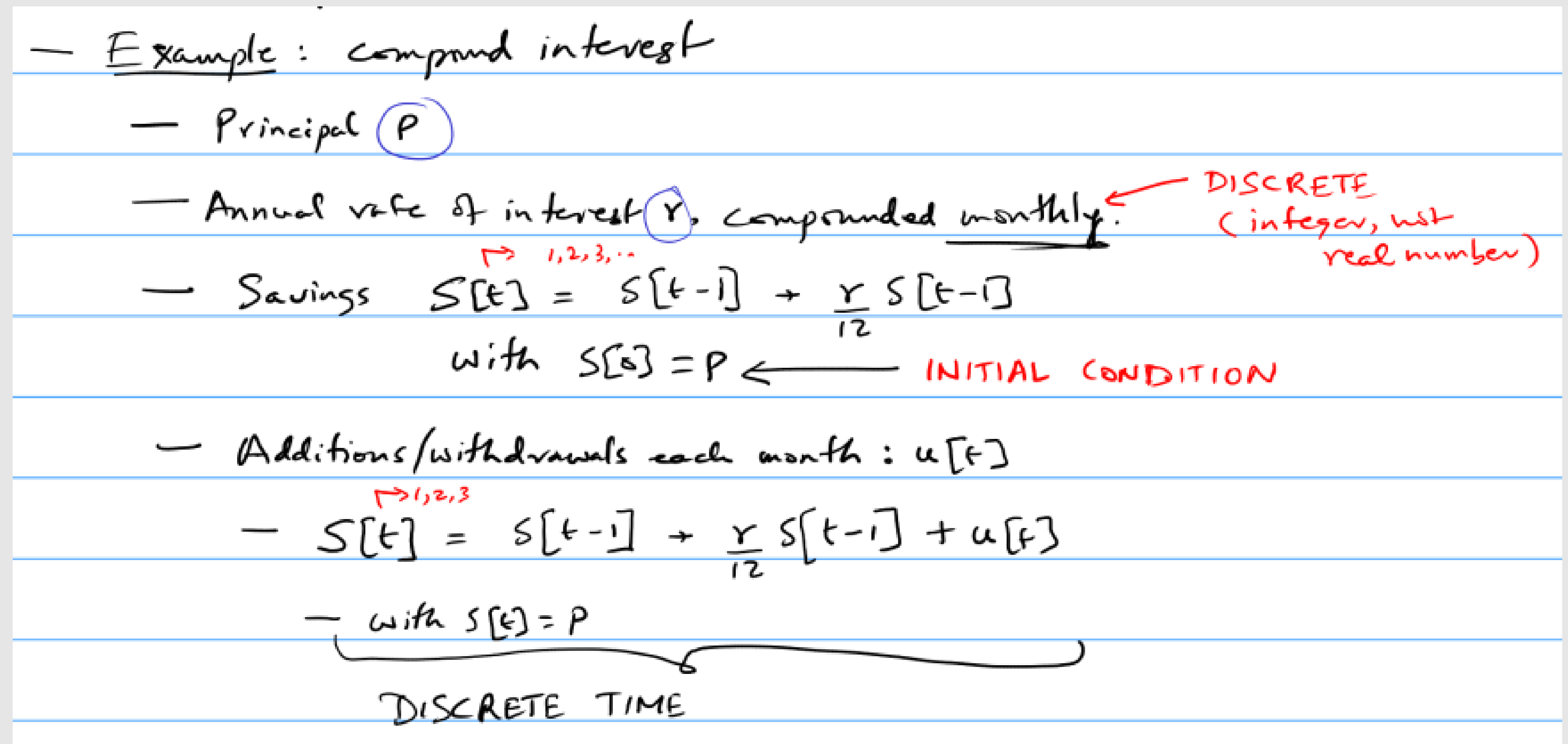

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STATE SPACE FORMULATION

- Discrete time sometimes more natural (eg, for finance, social dynamics, ...)

Another D.T. Example: Profs. and PhDs.

- (move to xournal)

- $p[t]$: no. of profs. in the US, year t $\quad (t = 1, 2, 3, \ldots)$
- $r[t]$: no. of PhDs in year t
- γ: fraction of PhDs who become professors
- δ: fraction in each profession retiring
- $u[t]$: average number of PhD students graduated per prof. per year

  can be manipulated by the professor (controlled by, eg, funding)

- Q: how do $p(t)$ and $r(t)$ evolve with time?

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$$\vec{x}(t) \triangleq \begin{bmatrix} p[t] \\ r[t] \end{bmatrix}; \quad \vec{f}(\vec{x}) = \begin{bmatrix} p(1-\delta) + \gamma r \\ r(1-\delta-\gamma) + pu \end{bmatrix}$$

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VECTOR D.T. STATE-SPACE FORMULATION

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VECTOR D.T. STATE-SPACE FORMULATION

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