INTERPOLATION

1. MOTIVATION: AUDIO TRANSMISSION EXAMPLE : SAMPLING AND INTERPOLATION
2. PWL AND ZOH INTERPOLATION
3. INTERPOLATION USING BASIS FUNCTIONS
4. SINC INTERPOLATION
5. INTERPOLATION USING (GLOBAL) POLYNOMIALS

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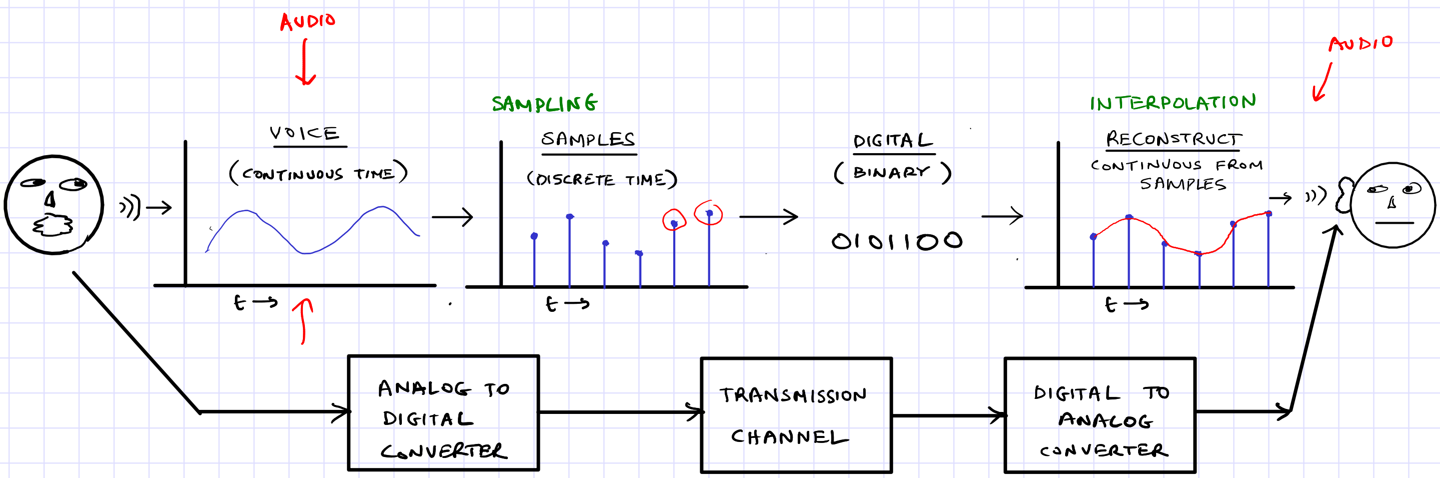
1. MOTIVATION



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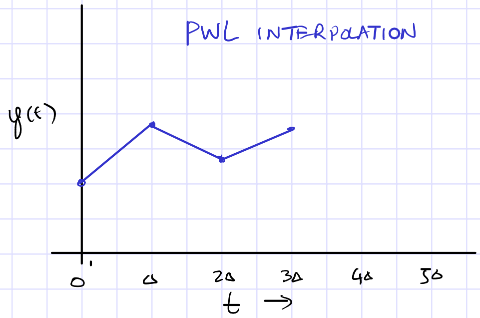
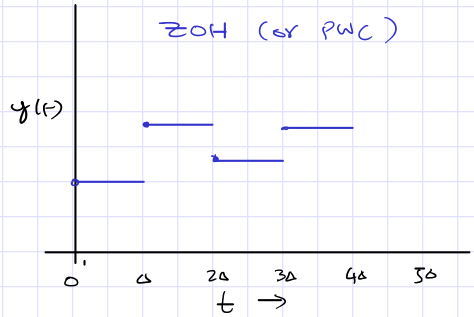
2. PIECEWISE LINEAR (PWL) and ZEROTH-ORDER HOLD (ZOH) INTERPOLATION



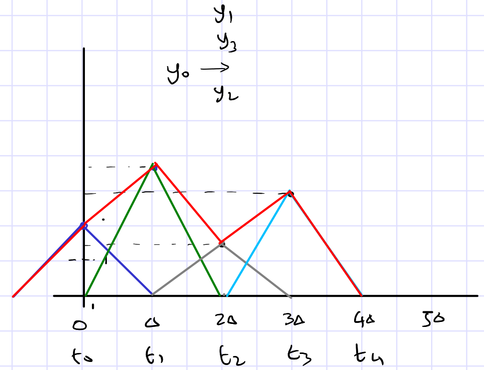


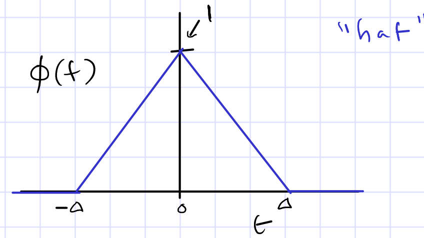
→ CONTINUOUS (NO "JUMPS")

→ DERIVATIVE DISCONTINUOS
  → HAS "KINKS"

→ "JUMPS" (DISCONTINUITIES)

3. INTERPOLATION USING BASIS FUNCTIONS





$$\phi(t) = \begin{cases} 0, & t < -\Delta \\ 1 + \frac{t}{\Delta}, & -\Delta \le t \le 0 \\ 1 - \frac{t}{\Delta}, & 0 \le t \le \Delta \\ 0, & t > \Delta \end{cases}$$

$$y(t) = y_0\, \phi(t - \underset{\downarrow}{\overset{0}{t_0}}) + y_1\, \underbrace{\phi(t - \overset{1\cdot\Delta}{t_1})}$$

$$+\; y_2\, \phi(t - \overset{2\Delta}{t_2}) \;+\; y_3\, \phi(t - \underset{3\Delta}{t_3})$$

$$y(t) = \sum_{i=0}^{3} y_i\, \phi(t - \underset{i\Delta}{t_i}) \quad ;$$

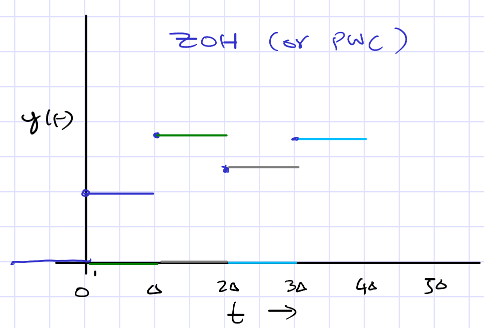
→ Given N samples: $(t_0, y_0) \;\cdots\; (t_{N-1}, y_{N-1})$

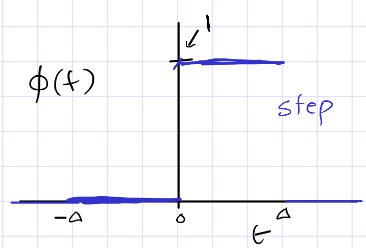
$$\underline{y_{PWL}(t) = \sum_{i=0}^{N-1} y_i\, \phi(t - t_i)}$$

$t_0, \ldots, t_{N-1}$ are equally spaced at Δ

→ THE POWER OF THE BASIS EXPANSION?

$$\longrightarrow \quad y_{PWL}(t) = \sum_{i=0}^{N-1} y_i\, \phi(t - t_i)$$





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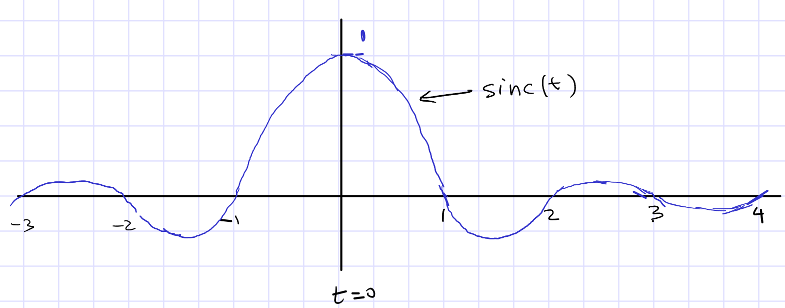
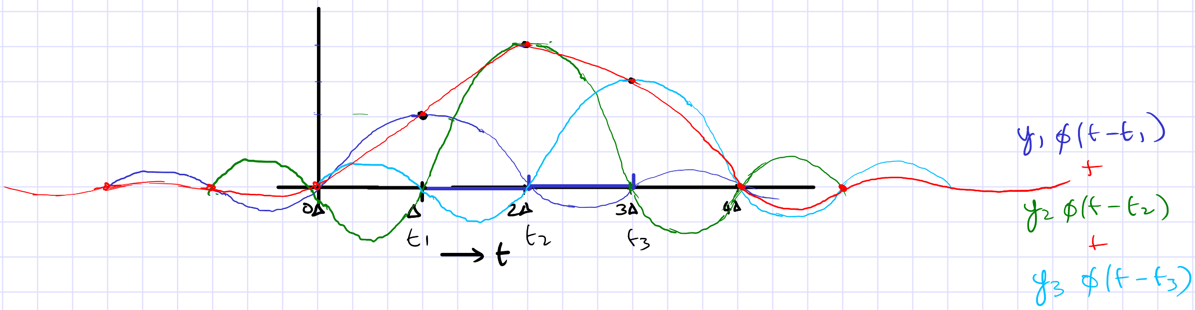
4. SINC INTERPOLATION

$$\rightarrow \quad \text{sinc}(t) \triangleq \begin{cases} 1, & \text{if } t = 0 \\ \frac{\sin(\pi t)}{\pi t}, & t \neq 0 \end{cases}$$



$$\phi(t) = \mathrm{sinc}(t/\Delta)$$

$$y(t) = \sum_{i=0}^{N-1} y_i \, \phi(t - t_i)$$

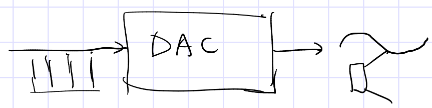


→ <u>ADVANTAGES</u> (OVER PWL & PWC)?

→ "SMOOTH" (NO JUMPS OR KINKS) → <u>ALL</u> DERIVATIVES CONTINUOUS

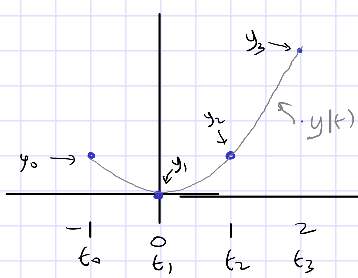
→ BAND-LIMITED : IMPORTANT

↳ WILL TALK ABOUT IT LATER (WHEN WE DO DFTS)



5. <u>INTERPOLATION USING GLOBAL POLYNOMIALS</u>

DATA: (−1, 1), (0, 0), (1, 1), (2, 4) : 4 points = N



$$\longrightarrow y(t) = \underline{a_0} + \underline{a_1} t + \underline{a_2} t^2 + \underline{a_3} t^3$$

$$\underbrace{\begin{aligned}
y_0 &= y(t_0) = \underline{a_0} + \underline{a_1} t_0 + \underline{a_2} t_0^2 + \underline{a_3} t_0^3 \\
y_1 &= y(t_1) = \underline{a_0} + \underline{a_1} t_1 + \underline{a_2} t_1^2 + \underline{a_3} t_1^3 \\
y_2 &= y(t_2) = \underline{a_0} + \underline{a_1} t_2 + \underline{a_2} t_2^2 + \underline{a_3} t_2^3 \\
y_3 &= y(t_3) = \underline{a_0} + \underline{a_1} t_3 + \underline{a_2} t_3^2 + \underline{a_3} t_3^3
\end{aligned}}$$

$$\underbrace{\begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix}}_{\vec{y}} = \underbrace{\begin{bmatrix} 1 & t_0 & t_0^2 & t_0^3 \\ 1 & t_1 & t_1^2 & t_1^3 \\ 1 & t_2 & t_2^2 & t_2^3 \\ 1 & t_3 & t_3^2 & t_3^3 \end{bmatrix}}_{\text{VAN DER MONDE MATRIX } V} \underbrace{\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix}}_{\vec{a}}$$

IF V is invertible

$$\vec{y} = V \vec{a}$$

$$\Longrightarrow \vec{a} = V^{-1} \vec{y}$$