LINEAR TIME-INVARIANT (LTI) SYSTEMS (CONTD.)

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→ **EXAMPLE:** $x[t+1] = a\,x[t] + b\,u[t]$, $\quad y[t] = c\,x[t] + d\,u[t]$

→ I.C. $\quad x[0] = 0$

→ IMPULSE INPUT $\delta[t] = \begin{cases} 1, & t=0 \\ 0, & \text{otherwise} \end{cases}$

$$y[0] = c\,x[0] + d\,u[0] = d\,u[0] = d\,\delta[0] = d$$

$$x[1] = a\,x[0] + b\,u[0] = b \quad ; \quad y[1] = cb + d\,u[1] = cb$$

$$x[2] = a\,x[1] + b\,u[1] = ab \quad ; \quad y[2] = cab$$

$$x[3] = a\,x[2] + b\,u[2] = a^2 b \quad ; \quad y[3] = ca^2 b$$

$$\vdots$$

$$x[t] = a^{t-1} b \quad ; \quad y[t] = c a^{t-1} b$$

→ $y[t] = \sum_{i=0}^{t} u[i]\,h[t-i]$ (assuming 1) causal & 2) $u[t]=0$ for $t<0$)

→ if $u[i] = \delta[i]$

→ $y[t] = \underset{1}{u[0]}\,h[t] = h[t]$ ✓

→ FOR THIS SYSTEM:

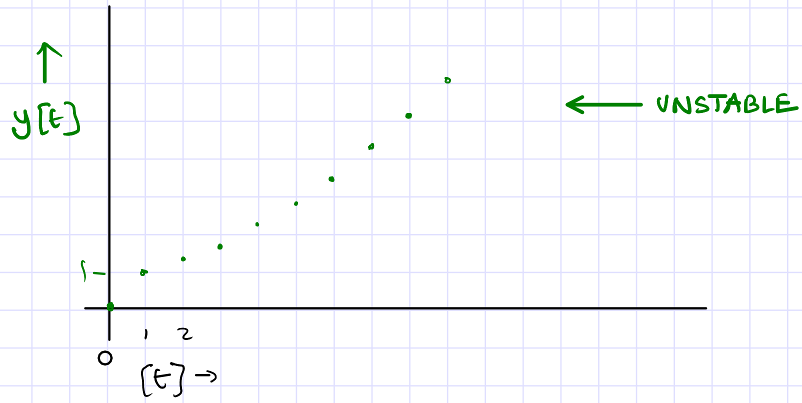
$$h[t] = \begin{cases} d, & \text{if } t=0 \\ c a^{t-1} b, & \text{for } t>0 \end{cases}$$

→ **COMPOUND INTEREST EXAMPLE**

→ $s[t+1] = (1 + r/_{12})\, s[t] + u[t]$, $y[t] = s[t]$

→ $a = (1 + r/_{12})$, $b = 1$, $c = 1$, $d = 0$

→ $h[t] = \begin{cases} 0, & t = 0 \\ (1 + r/_{12})^{t-1}, & t > 0 \end{cases}$



→ GIVEN **ONLY** h[t] for some LTI system

→ CAN YOU DETERMINE IF THE SYSTEM IS BIBO STABLE?

↳ FIGURE OUT FOR YOURSELF

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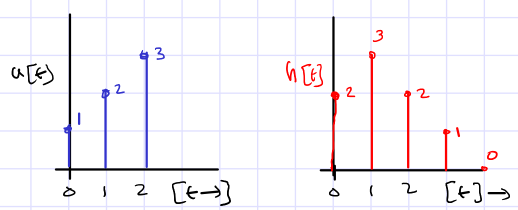
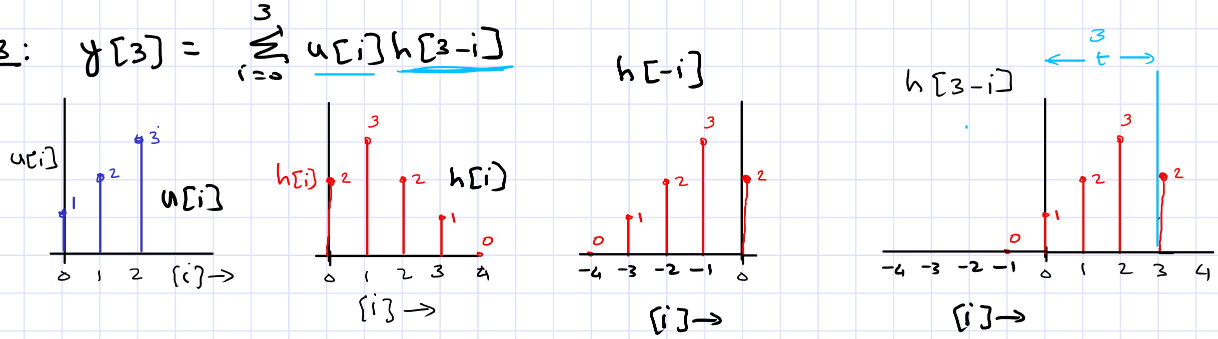
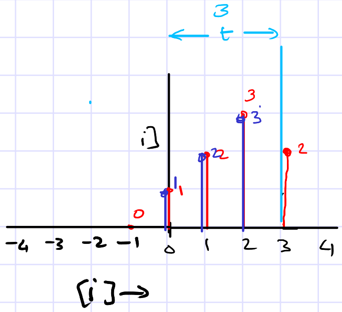
6. CONVOLUTION, ILLUSTRATED GRAPHICALLY

$$y[t] = u[t] \circledast h[t] = \sum_{i=0}^{t} u[i]\, h[t-i]$$ (assuming 1) causal, 2) $u[t] = 0 \;\forall t < 0$)



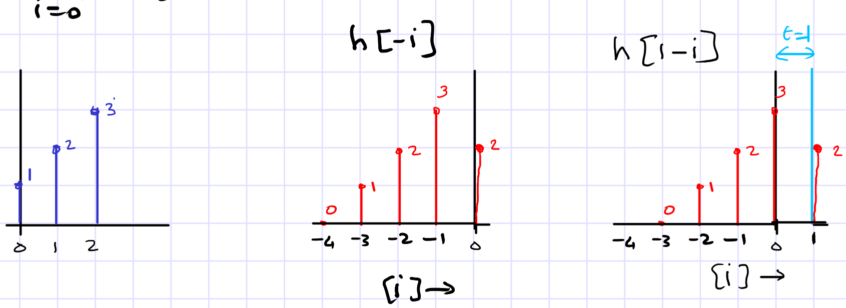
t=3: $y[3] = \sum_{i=0}^{3} u[i]\, h[3-i]$

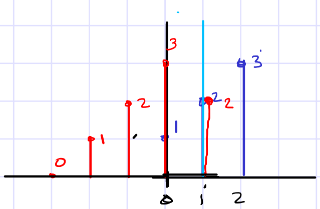




$y[3] = 1 + 4 + 9 = 14$

$$y[1] = \sum_{i=0}^{1} u[i]\, h[1-i]$$

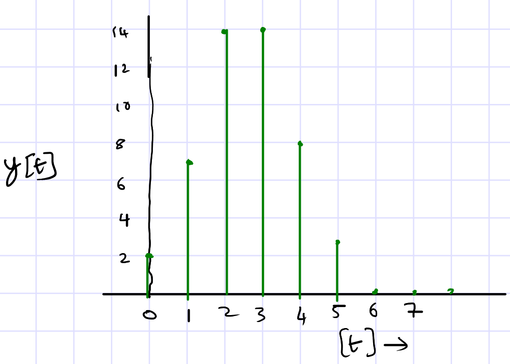


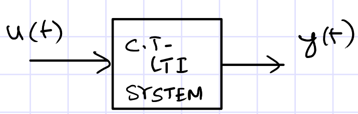


$y[1] = 3 + 4 = 7$

$y[0] = 2$; $y[1] = 7$; $y[2] = 14$; $y[3] = 14$; $y[4] = 8$; $y[5] = 3$

$y[6]$ and above $= 0$





→ OUTLINE OF FLOW FOR C.T. SYSTEMS

→ DIRAC δ function : $\delta(t) = \begin{cases} \infty, & t = 0 \\ 0, & \text{everywhere else} \end{cases}$

$$\int_{-\epsilon}^{\epsilon} \delta(\tau)\, d\tau = 1 \quad \text{for any } \epsilon > 0$$

→ Apply $\delta(t)$ as input to the system

→ Record $y(t)$ ← this is the impulse response $h(t)$

→ Given any $u(t)$, if $u(t) \mapsto y(t)$, then:

→ $$y(t) = \int_{-\infty}^{\infty} u(\tau)\, h(t-\tau)\, d\tau = u(t) \circledast h(t)$$

↑ CONTINUOUS TIME CONVOLUTION

→ <u>Causality for C.T. systems</u>:

→ LTI. C.T. system is causal iff $h(t) = 0 \;\; \forall t < 0$

→ $$y(t) = \int_{-\infty}^{t} u(\tau)\, h(t-\tau)\, d\tau$$
