$$\frac{d\vec{x}(t)}{dt} = A\vec{x}(t) + \vec{b}\,u(t) \qquad \vec{z} = T\vec{x} \iff \underline{\vec{x} = T^{-1}\vec{z}}$$

$$\frac{d}{dt}\left[T^{-1}\vec{z}(t)\right] = AT^{-1}\vec{z} + \vec{b}\,u(t)$$

$$T^{-1}\frac{d}{dt}\vec{z}(t) = AT^{-1}\vec{z} + \vec{b}\,u(t)$$

$$\frac{d}{dt}\vec{z}(t) = \underbrace{TAT^{-1}}_{\hat{A}}\vec{z} + \underbrace{T\vec{b}}_{\hat{\vec{b}}}\,u(t)$$

orig. sys.: $\rightarrow$ $\vec{x}[t+1] = A\vec{x}[t] + \vec{b}\,u[t]$ $\rightarrow$ (1) ; $y[t] = \vec{c}^{T}\vec{x}[t] + d\,u[t]$

observer: $\hat{x}[t+1] = A\hat{x}[t] + \vec{b}\,u[t] + \vec{\ell}\left(\vec{c}^{T}\hat{x}[t] - y[t]\right)$

$+d\,u[t]$

$$\hat{x}[t+1] = A\hat{x}[t] + \vec{b}\,u[t] + \vec{\ell}\left(\vec{c}^{T}\hat{x}[t] - \vec{c}^{T}\vec{x}[t] - d\,u[t]\right)$$

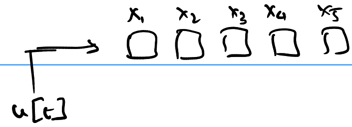
$+d\,u[t]$

$\rightarrow$ $\hat{x}[t+1] = A\hat{x}[t] + \vec{b}\,u[t] + \vec{\ell}\,\vec{c}^{T}\underbrace{\left(\hat{x}[t] - \vec{x}[t]\right)}_{\vec{\epsilon}[t]}$ (2)

$$(2)-(1)\quad \underbrace{\hat{x}[t+1] - \vec{x}[t+1]}_{\vec{\epsilon}[t+1]} = A\underbrace{\left(\hat{x}[t] - \vec{x}[t]\right)}_{\vec{\epsilon}[t]} + \vec{\ell}\,\vec{c}^{T}\vec{\epsilon}[t]$$

$$\Rightarrow \quad \vec{\epsilon}[t+1] = A\,\underline{\vec{\epsilon}[t]} + \vec{\ell}\,\vec{c}^{T}\,\underline{\vec{\epsilon}[t]}$$

$$\Rightarrow \quad \boxed{\vec{\epsilon}[t+1] = \left(A + \ell\,\vec{c}^{T}\right)\vec{\epsilon}[t]}$$



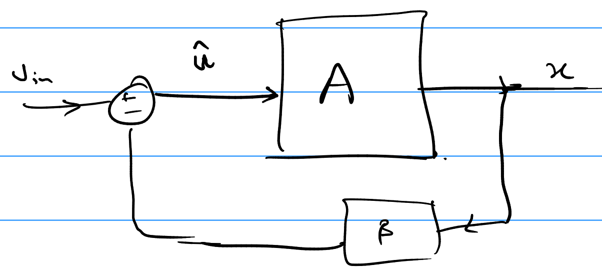
$0 - 255$

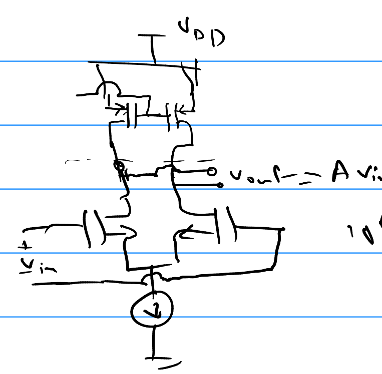
$x_i \in [0, \dots, 255]$

$$\vec{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_5 \end{bmatrix}$$

$$\begin{bmatrix} x_1[t+1] \\ x_2[t+1] \\ x_3[t+1] \\ x_4[t+1] \\ x_5[t+1] \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1[t] \\ x_2[t] \\ x_3[t] \\ x_4[t] \\ x_5[t] \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} u[t]$$

$A \sim$





10^6

$\hat{u} = v_{in} - \beta x$

$x = A\hat{u}$

$x = A(v_{in} - \beta x) \implies x(1 + \beta A) = A v_{in}$

$$x = \frac{A}{1 + \beta A} v_{in} \simeq \frac{A}{\beta A} v_{in} = \frac{v_{in}}{\beta}$$

10^{-3}

$\beta A \gg 1$

$A > 10^4$