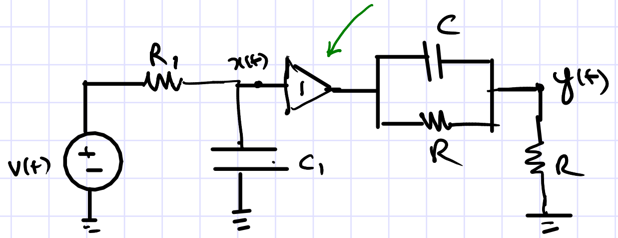
BODE PLOTS FOR SYSTEMS WITH MORE THAN 1 POLE/ZERO

→ EXAMPLE CIRCUIT WITH 2 POLES AND 1 ZERO (ALL REAL)



$$\tilde{X} = \frac{\tilde{V}}{1 + j\omega R_1 C_1} \quad ; \quad \tilde{Y} = \frac{\tilde{X}\, R}{R + (R \| Z_C)} = \frac{\tilde{X} R}{R + \frac{R Z_C}{R + Z_C}} = \frac{\tilde{X}}{1 + \frac{Z_C}{R + Z_C}} = \frac{\tilde{X}(R + Z_C)}{R + 2Z_C}$$

$$= \frac{\tilde{X}\left(R + \frac{1}{j\omega C}\right)}{R + \frac{2}{j\omega C}} = \frac{\tilde{X}(1 + j\omega RC)}{2 + j\omega RC}$$

$$\Longrightarrow \quad \tilde{Y} = \tilde{V}\, \frac{1}{2}\, \frac{(1 + j\omega RC)}{(1 + j\omega R_1 C_1)\left(1 + j\omega \frac{RC}{2}\right)}$$

$$= \tilde{V}\, \frac{RC\Big(j\omega + \overbrace{\tfrac{1}{RC}}^{\text{call this } z_1}\Big)}{R_1C_1\Big(j\omega + \underbrace{\tfrac{1}{R_1C_1}}_{p_1}\Big)\, \frac{RC}{2}\Big(j\omega + \underbrace{\tfrac{1}{(RC/2)}}_{p_2}\Big)}$$

$$= \tilde{V}\, \overbrace{\frac{2\cancel{RC}}{(R_1C_1)(\cancel{RC})}}^{\text{call this } K}\, \frac{(j\omega + z_1)}{(j\omega + p_1)(j\omega + p_2)}$$

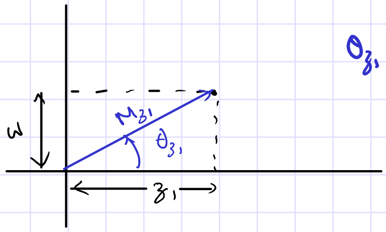
→ TRANSFER FN. $H(\omega) = \dfrac{\tilde{Y}}{\tilde{V}}$ HAS THE FORM:

$$\boxed{\frac{K\,(j\omega + \overbrace{z_1}^{\text{zero \#1}})}{(j\omega + \underbrace{p_1}_{\text{pole \#1}})\,(j\omega + \underbrace{p_2}_{\text{pole \#2}})}}$$

→ BODE PLOT OF $H(\omega) \triangleq \dfrac{K\,(j\omega + z_1)}{(j\omega + p_1)(j\omega + p_2)}$

→ First, express each pole/zero term in polar co-ordinates

→ $j\omega + z_1$:



$\theta_{z_1} = \tan^{-1}\left(\frac{\omega}{z_1}\right)$; $M_{z_1} = \sqrt{\omega^2 + z_1^2}$

$\Rightarrow j\omega_1 + z_1 = M_{z_1}\, e^{j\theta_{z_1}}$

→ Similarly: $j\omega + p_1 = M_{p_1} e^{j\theta_{p_1}}$, $M_{p_1} = \sqrt{\omega^2 + p_1^2}$, $\theta_{p_1} = \tan^{-1}\left(\frac{\omega}{p_1}\right)$

$j\omega + p_2 = M_{p_2} e^{j\theta_{p_2}}$, $M_{p_2} = \sqrt{\omega^2 + p_2^2}$, $\theta_{p_2} = \tan^{-1}\left(\frac{\omega}{p_2}\right)$

$$\Rightarrow H(\omega) = \frac{K\; M_{z_1}(\omega)\; e^{j\theta_{z_1}(\omega)}}{M_{p_1}(\omega)\, e^{j\theta_{p_1}(\omega)}\quad M_{p_2}(\omega)\, e^{j\theta_{p_2}(\omega)}}$$

$$\Rightarrow \boxed{H(\omega) = \frac{K\, M_{z_1}(\omega)}{M_{p_1}(\omega)\; M_{p_2}(\omega)}\; e^{j\left[\theta_{z_1}(\omega) - \theta_{p_1}(\omega) - \theta_{p_2}(\omega)\right]}}$$

$$\Rightarrow |H(\omega)| = \frac{K\, M_{z_1}(\omega)}{M_{p_1}(\omega)\; M_{p_2}(\omega)} \qquad \angle H(\omega) = \theta_{z_1}(\omega) - \theta_{p_1}(\omega) - \theta_{p_2}(\omega)$$

$$\log_{10}(|H(\omega)|) = \log_{10}(K) + \log_{10}(M_{z_1}(\omega)) - \log_{10}(M_{p_1}(\omega)) - \log_{10}(M_{p_2}(\omega))$$

+ for zero — for pole — for pole

→ <u>LOOK AT THE TERMS</u> :

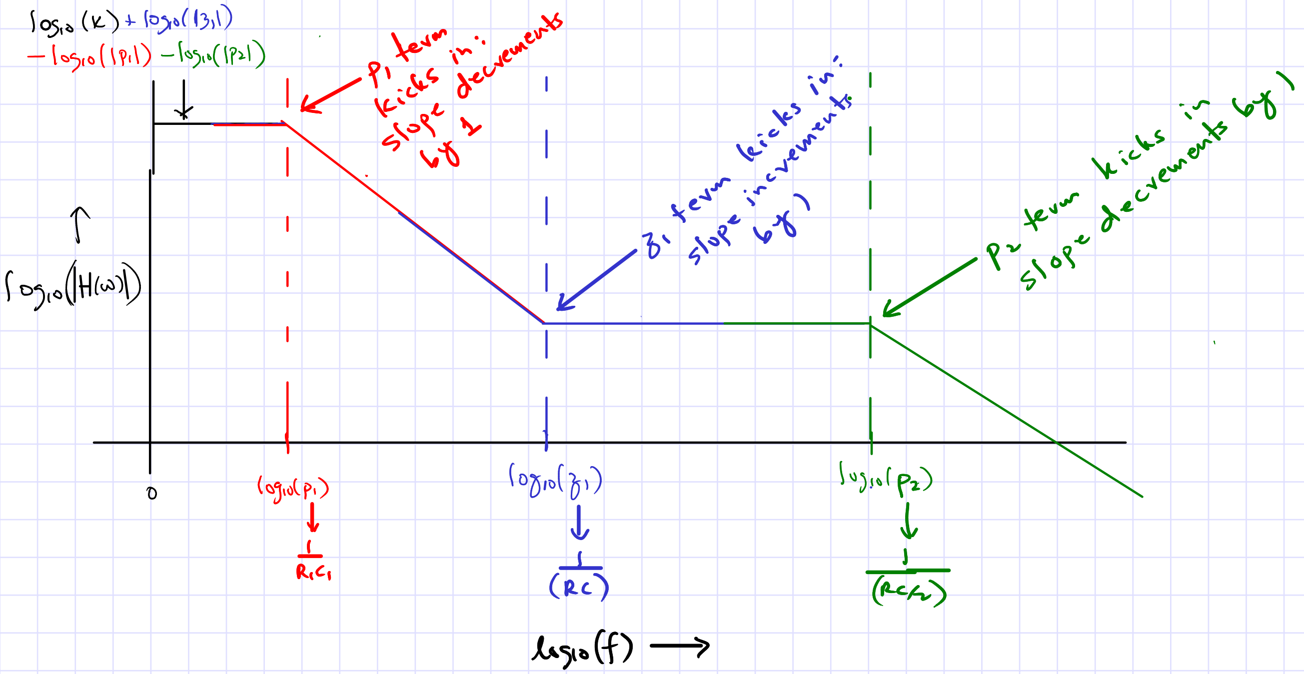
→ $\log_{10}(K)$: constant for all ω

→ $+\log_{10}(M_{z_1}(\omega))$: $+\log_{10}(|z_1|)$ for $\omega \ll z_1$, $+\log_{10}(\omega)$ for $\omega \gg z_1$

→ $-\log_{10}(M_{p_1}(\omega))$: $-\log_{10}(|p_1|)$ for $\omega \ll p_1$, $-\log_{10}(\omega)$ for $\omega \gg p_1$

→ $-\log_{10}(M_{p_2}(\omega))$: $-\log_{10}(|p_2|)$ for $\omega \ll p_2$, $-\log_{10}(\omega)$ for $\omega \gg p_2$

→ AND THE TERMS ADD, MAKING PLOTTING EASY



⟶ **PLOTTING THE ANGLE** $\angle H(\omega) = \Theta_{z_1}(\omega) - \Theta_{p_1}(\omega) - \Theta_{p_2}(\omega)$

⟶ LOOK AT EACH TERM:

⟶ $\Theta_{z_1}(\omega)$: 0° for $\omega \ll z_1$, $+90^\circ$ for $\omega \gg z_1$, $+45^\circ$ at $\omega = z_1$

⟶ $-\Theta_{p_1}(\omega)$: 0° for $\omega \ll p_1$, -90° for $\omega \gg p_1$, -45°

⟶ $-\Theta_{p_2}(\omega)$: 0° for $\omega \ll p_2$, -90° for $\omega \gg p_2$, -45°