EECS 16B Designing Information Devices and Systems II
Spring 2019 A. Sahai, J. Roychowdhury, K. Pister Discussion 1B
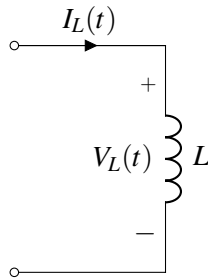
1 Inductors

Figure 1: Example Inductor Circuit

At the most fundamental level, an inductor is simply a coil of wire. The voltage across the inductor is related to its current as follows:

$$V_L(t) = L\frac{dI_L(t)}{dt},$$

where L is the *inductance* of the inductor. The SI unit of inductance is Henry (H).

At steady state, when the current flowing through an inductor is constant, there is no voltage drop across the inductor. Similarly, if the current through the inductor is changing, there will be a voltage drop across the inductor. Note that the *current* through an inductor cannot change instantly.

The energy stored in an inductor is given by $E = \frac{1}{2}LI^2$.

2 Log-Log Plots: a Power Law Power Tool

A log-log plot is a special kind of plot, which is very useful for understanding power laws. We will be encountering many power laws when we cover frequency-domain analysis in this course. The log-log plot will become an invaluable tool, as well as the basis for *Bode plots*, which are used to determine how an electrical circuit will respond to sinusoidal inputs.

As the name suggests, a log-log plot is just a special type of plot, which uses a different kind of *scale* to the kinds of plots you're used to seeing. Here is an example of the ordinary kind of plot:
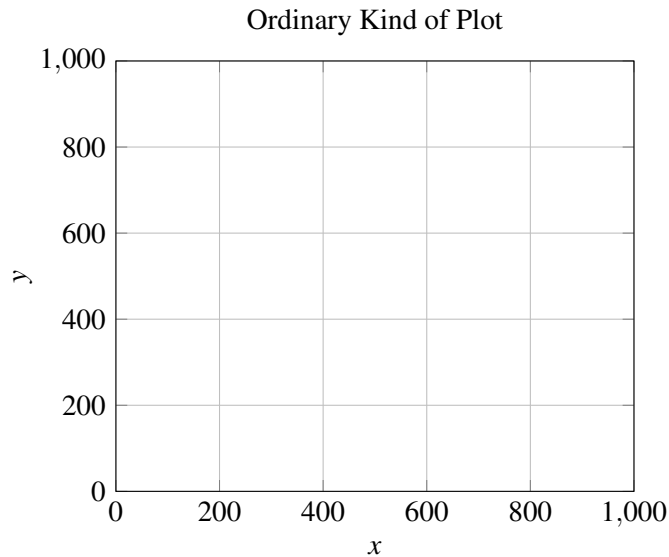
Ordinary Kind of Plot

Figure 2: An ordinary plot, with a *linear scale*.

If you look at the axes of this plot, you'll see that the magnitude of x and y increase *linearly* with the distance along the axis: we say that this plot uses a *linear scale* along the x and y axes.

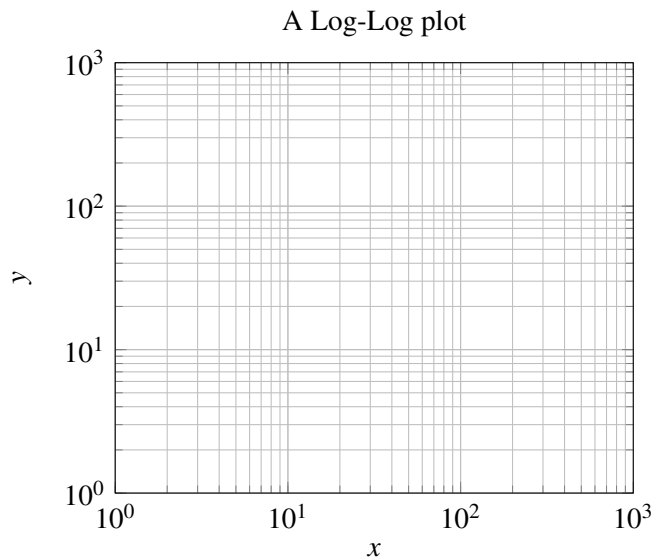Now, here is a log-log plot:

A Log-Log plot

Figure 3: A log-log plot, which uses a *logarithmic scale* on each axis.

The magnitude of x and y do not scale linearly with distance on this graph. Instead, the magnitude of $\log(x)$ and $\log(y)$ scale linearly with distance: we say that the plot uses a *logarithmic scale* (or *log scale* for short) on the x and y axes. Note that the grid lines still mark a linear sequence of values– that's why they look like they're all bunched in certain areas.

When is it useful to use logarithmic scales, rather than linear scales, on a plot? Well, one good time is when you want to visualize many orders of magnitude on a single plot. In class you learned about the

wide range of frequencies that electrical engineers find useful: from millihertz shifts of the earth's crust to gigahertz radar signals, and far beyond into the terahertz frequencies of visible light. Trends between these frequencies are hard to see on a linear scale plot, but are easier to visualize on a log scale plot.

Another instance in which log scales are useful is when you are dealing with quantities that follow a *power law*. A power law is a relationship between two variables (say x and y) of the form

$$y = ax^p$$

where a, the coefficient, and p, the power, are some constants. You've seen some power laws already. In EE16A, for example, you saw the relationship between the energy stored by a capacitor and the voltage across it, $U = \frac{1}{2}CV^2$, which is a power law where $p = 2$. When plotted on a linear scale, power laws can take on all sorts of shapes. For example, if $p = 2$, then the power law will look like a parabola; if $p = -1$, then it will look like an inverse. If p is some non-integer value like $p = -0.75$, there is no easy way to plot y on a linear scale without using a computer to help us plot, perhaps by using a package like `matplotlib` or `pgfplots`.

The story is very different with log scales. How would a power law look when plotted using a log scale? To find out, let's take the logarithm of both sides of the power law:

$$\log(y) = \log(ax^p) = \log(a) + p\log(x)$$

When x and y have a power law relationship, it means that $\log(x)$ and $\log(y)$ have a linear relationship. This means that, on a log-log plot, all power laws look like straight lines, no matter what p is!

Let's take a minute to do Exercise 1 (Log-Log Plot) before continuing.

3   Impedance magnitudes of capacitors and inductors

Normally, the relationship between voltage and current for inductors and capacitors is rather complicated: this is the reason that transient analysis can be an involved process. However, when the voltages or currents involved are *sinusoids* - that is, some combination of sines and cosines - the situation becomes a bit simpler. Take the capacitor, for example. Suppose that some voltage $V_C(t) = V_0 \sin(2\pi f t)$ is being applied across a capacitor. Then the current will be

$$I_C(t) = C\frac{d}{dt}V_C(t) = V_0 2\pi C f \cos(2\pi f t). \tag{1}$$

In other words, when the voltage across a capacitor is a sinusoid, the current through it will also be a sinusoid, but the *scaling* of the sinusoid will have changed. For example, the voltage sinusoid here is scaled by V_0, while the current sinusoid is scaled by $V_0 2\pi f C$. The ratio of the two constants is related to the concept of an *impedance*, which is a generalized notion of resistance that we can apply to capacitors and inductors as well as resistors.

This makes sense because usually when take the ratio of V and I, we find $\frac{V}{I} = R$. Now we are extending this concept to capacitors and inductors. We can think of the constants that scale the current and voltage sinusoids as the magnitudes of those sinusoids. So, when we take the magnitude of $\frac{V_C(t)}{I_C(t)}$, we will get the magnitude of the impedance, which we'll call $|Z|$. Specifically, in this case this ratio is the *magnitude* of the impedance of the capacitor. We write this as:

$$|Z_C| = \frac{|V_C(t)|}{|I_C(t)|} = \frac{V_0}{V_0 2\pi C f} = \frac{1}{2\pi C f}$$

In an entirely analogous way, we can write the magnitude of the impedance of an inductor as

$$|Z_L| = 2\pi L f.$$

As a quick exercise, look at Exercise 2 on this worksheet and use the same steps we used to find $|Z_C|$ to derive $|Z_L|$.

Since the impedance is a single number representing the current-voltage ratio of capacitors and inductors, you might think of $|Z_C|$ and $|Z_L|$ as the *resistance* of capacitors and inductors. However, unlike simple resistors, we can see that they have *frequency-dependent resistance*. Furthermore, the relationships of $|Z_C|$ to f, and $|Z_L|$ to f, are both power laws (our power exponent on f in these cases are simply $p = -1$ and $p = 1$), so it's convenient to draw them on a log-log plot.

As an exercise, you'll be drawing $|Z_C|$ and $|Z_L|$ for some different capacitors and inductors in the blank log-log plot provided in Exercise 3 of this worksheet.

1. **Power Laws on a Log-Log Plot**

   Take a look at figure below. On the left are three power laws plotted on a linear scale, and on the right is a blank log-log plot.

   Plot the following three power law equations on the right-hand side, log-log plot below:

$$y = x \tag{2}$$

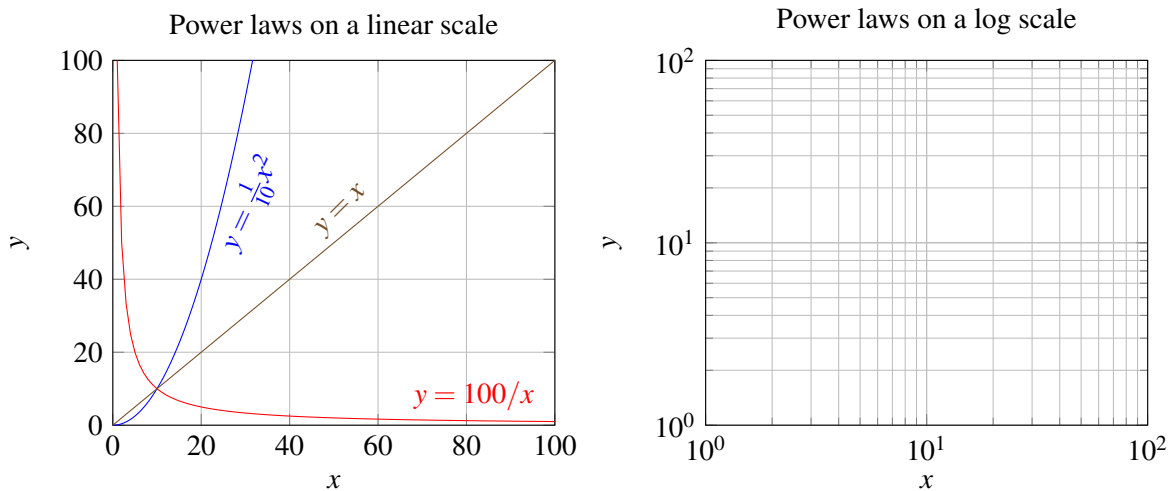$$y = 100x^{-1} \tag{3}$$

$$y = \frac{1}{10}x^2 \tag{4}$$

Figure 4: (left) Three power laws plotted on a linear scale. (right) The same three power laws, which **you** have plotted on a log scale. Go you!

2. **Impedance of an Inductor**

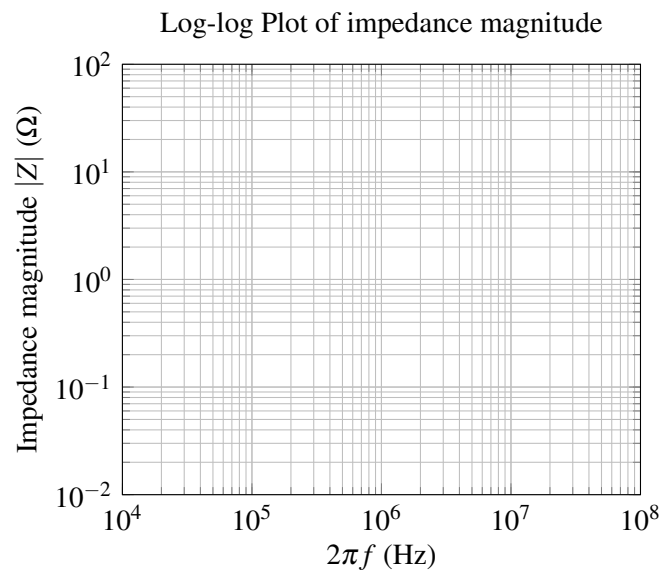   Suppose that some current $I_L(t) = I_0 \sin(2\pi f t)$ is being applied across an inductor.

   (a) What is the voltage across the inductor, $V_L(t)$?

   (b) What are the magnitudes of the current and voltage sinusoids, $|I_L(t)|$ and $|V_L(t)|$?

   (c) What is the impedance of the inductor, $|Z_L|$?

3. **Impedances on a log-log plot**

   Earlier, we found that the magnitude of the impedances for inductors and capacitors obey power laws with respect to frequency. In this exercise, you'll be drawing some of these on a log-log plot.

   (a) Come up with an equation that describes the relationship between $\log_{10}(|Z_C|)$ and $\log_{10}(2\pi f)$, and another that describes the relationship between $\log_{10}(|Z_L|)$ and $\log_{10}(2\pi f)$.

   (b) Using the empty log-log plot provided below, plot the impedance magnitude of the following four components:

      i. a 10 μF capacitor;

ii. a 100 nF capacitor;

iii. a 10 μH inductor;

iv. a 100 nH inductor.

Log-log Plot of impedance magnitude

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