Discussion 6A

1. Changing behavior through feedback

In this question, we discuss how feedback control can be used to change the effective behavior of a system.

(a) Consider the scalar system:

$$x(i+1) = 0.9x(i) + u(i) + w(i) \tag{1}$$

where $u(i)$ is the control input we get to apply based on the current state and $w(i)$ is the external disturbance.

Is the system stable? If $|w(i)| \leq \varepsilon$, what can you say about $|x(i)|$ at all time if you further assume that $u(i) = 0$ and the initial condition $x(0) = 0$? How big can $|x(i)|$ get?

(b) Suppose that we decide to choose a control law $u(i) = kx(i)$ to apply in feedback. For what values of λ can you get the system to behave like:

$$x(i+1) = \lambda x(i) + w(i) \tag{2}$$

vis-a-vis the disturbance $w(i)$? How would you pick k?

(c) For the previous part, which k would you choose to minimize how big $|x(i)|$ can get?

(d) What if instead of a 0.9, we had a 3 in the original (1). What, if anything, would change?

(e) Now suppose that we have a vector-valued system with a vector-valued control:

$$\vec{x}(i+1) = A\vec{x}(i) + B\vec{u}(i) + \vec{w}(i) \tag{3}$$

where we further assume that B is an invertible square matrix.

Suppose we decide to apply linear feedback control using a square matrix K so we choose $\vec{u}(i) = K\vec{x}(i)$. For what values of matrix G can you get the system to behave like:

$$\vec{x}(i+1) = G\vec{x}(i) + \vec{w}(i) \tag{4}$$

vis-a-vis the disturbance $\vec{w}(i)$? How would you pick K given knowledge of A, B and the desired goal dynamics G?

2. Eigenvalues Placement in Discrete Time

Consider the following linear discrete time system

$$\vec{x}(t+1) = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix} \vec{x}(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t) + \vec{w}(t) \tag{5}$$

(a) Is this system controllable from $u(t)$?

(b) Is the linear discrete time system stable?

(c) Derive a state space representation of the resulting closed loop system using state feedback of the form $u(t) = \begin{bmatrix} k_1 & k_2 \end{bmatrix} \vec{x}(t)$

(d) Find the appropriate state feedback constants, k_1, k_2 in order the state space representation of the resulting closed loop system to place the eigenvalues at $\lambda_1 = -\frac{1}{2}, \lambda_2 = \frac{1}{2}$

(e) Is the system now stable?

(f) Suppose that instead of $\begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t)$ in (5), we had $\begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t)$ as the way that the discrete-time control acted on the system. Is this system controllable from $u(t)$?

(g) For the part above, suppose we used $[k_1, k_2]$ to try and control the system. What would the eigenvalues be? Can you move all the eigenvalues to where you want? Give an intuitive explanation of what is going on.

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